Chapter 1

SIMILARITY

LONG QUESTIONS

Q. 1

Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

SOLUTION:

Let the base, height and area of the first triangle be b_1, h_1, and A_1 respectively.

Let the base, height and area of the second triangle be b_2, h_2 and A_2 respectively.

$$\frac{A_1}{A_2} = \frac{(b_1 \times h_1)}{(b_2 \times h_2)}$$

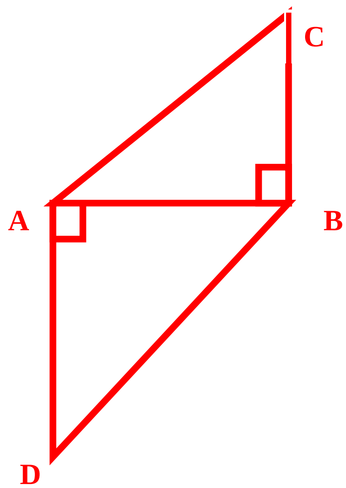
$$\frac{A_1}{A_2} = \frac{9 \times 5}{10 \times 6} = \frac{45}{60}$$

$$\frac{A_1}{A_2} = \frac{3}{4}$$

Ans.: The ratio of areas of triangles is 3:4

Q. 2

In the adjoining figure, BC ⊥ AB, AD ⊥ AB, BC = 4, AD = 8, then find $\frac{A\,(\Delta\,ABC)}{A\,(\Delta\,ADB)}$



SOLUTION:

ΔABC and ΔADB have same base AB.

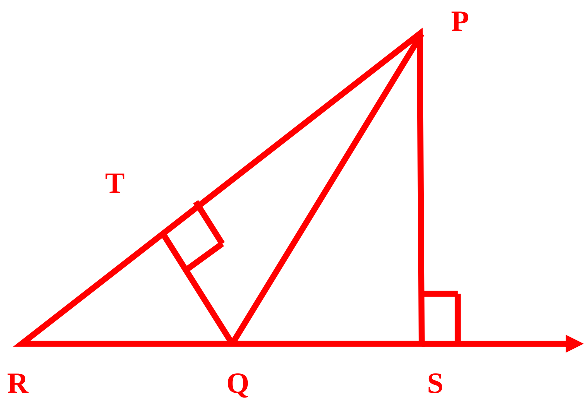
$$\frac{A\,(\Delta\,ABC)}{A\,(\Delta\,ADB)} = \frac{BC}{AD} = \frac{4}{8}$$

$$\frac{A\,(\Delta\,ABC)}{A\,(\Delta\,ADB)} = \frac{1}{2}$$

Ans.: Ratio of area of ΔABC to ΔADB is 1:2

Q. 3

In the figure, seg PS ⊥ seg RQ, seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12, then find QT.



SOLUTION:

In ΔPQR, PR is the base and QT is the corresponding height.

Also, RQ is the base and PS is the corresponding height.

$$\frac{A(\Delta PQR)}{A(\Delta PQR)} = \frac{PR \times QT}{RQ \times PS}$$

[Ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights]

$\therefore \frac{1}{1} = \frac{PR \times QT}{RQ \times PS}$

$\therefore PR \times QT = RQ \times PS$

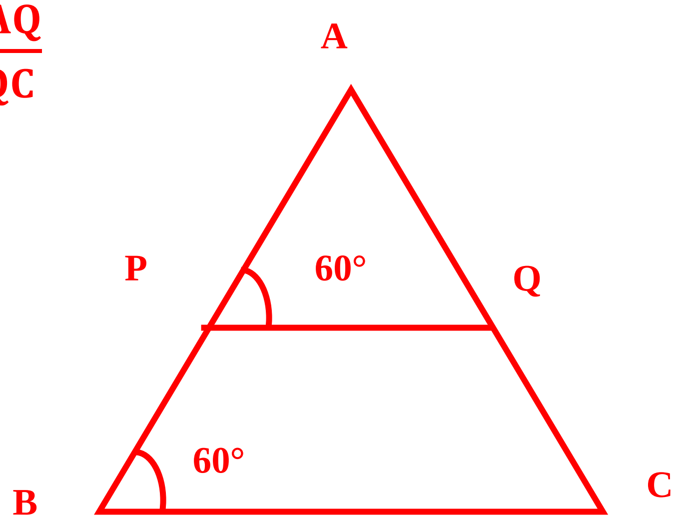
$\therefore 12 \times QT = 6 \times 6$

$\therefore QT = \frac{36}{12} = 3$

Ans.: QT = 3 units

Q. 4

Measures of some angles in the figure are given. Prove that $\frac{AP}{PB} = \frac{AQ}{QC}$



SOLUTION:

Proof

∠APQ = ∠ABC = 60° [Given]

∴ ∠APQ ≅ ∠ABC

∴ side PQ || side BC (i) [Corresponding angles test]

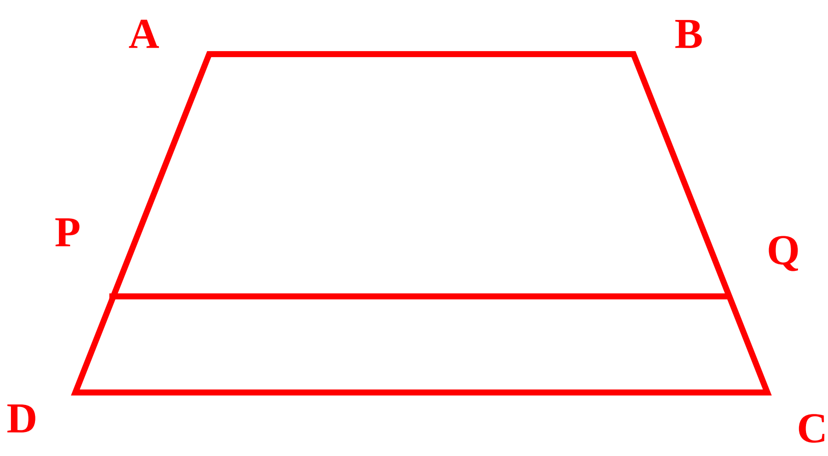
In △ABC,

side PQ || side BC [From (i)]

∴ $\frac{AP}{PB} = \frac{AQ}{QC}$ **[Basic proportionality theorem]**

Q. 5

In trapezium ABCD, side AB || side PQ || side DC, AP = 15, PD = 12, QC = 14, find BQ.



SOLUTION:

side AB || side PQ || side DC [Given]

∴ AP/PD = BQ/QC [Property of three parallel lines and their transversals]

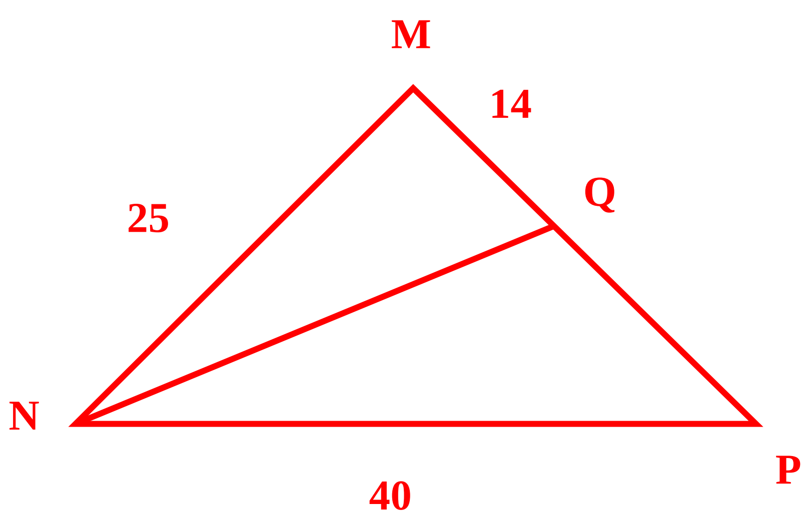
∴15/12 = BQ/14

∴ BQ = 15 × 14 / 12

∴ BQ = 17.5 units

Q. 6

Find QP using given information in the figure.



SOLUTION:

In ΔMNP, seg NQ bisects ∠N [Given]

$\therefore \frac{PN}{MN} = \frac{QP}{MQ}$

[Property of angle bisector of a triangle]

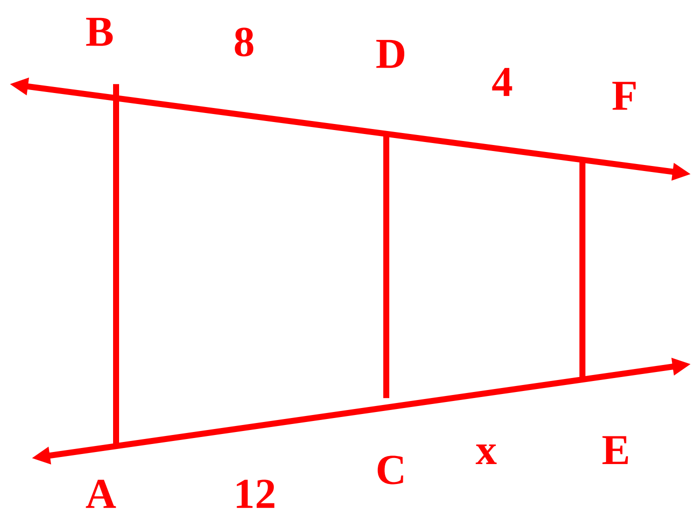
$\therefore \frac{40}{25} = \frac{QP}{14}$

$\therefore QP = \frac{(40 \times 14)}{25} = 22.4$

$\therefore$ **QP = 22.4 units**

Q. 7

In the adjoining figure, if AB || CD || FE, then find x and AE.



SOLUTION:

line AB || line CD || line FE [Given]

$\therefore \frac{BD}{DF} = \frac{AC}{CE}$

[Property of three parallel lines and their transversals]

∴ 84 = 12x

∴ x = 12 × 48

∴ x = 6 units

Now, AE AC + CE [A – C – E]

= 12 + x

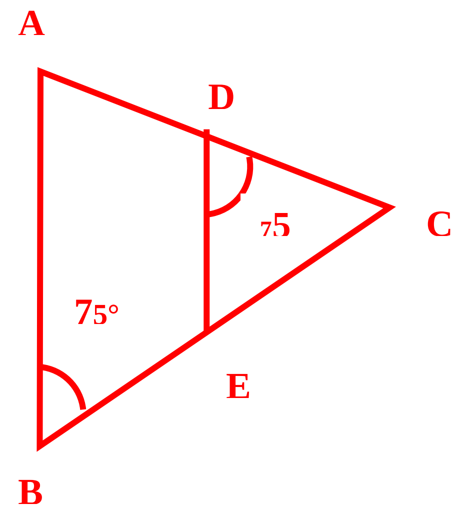
= 12 + 6

= 18 units

∴ x = 6 units and AE = 18 units

Q. 8

In the adjoining figure, ∠ABC = 75°, ∠EDC = 75°. State which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.



SOLUTION:

In ΔABC and ΔEDC,

∠ABC ≅ ∠EDC [Each angle is of measure 75°]

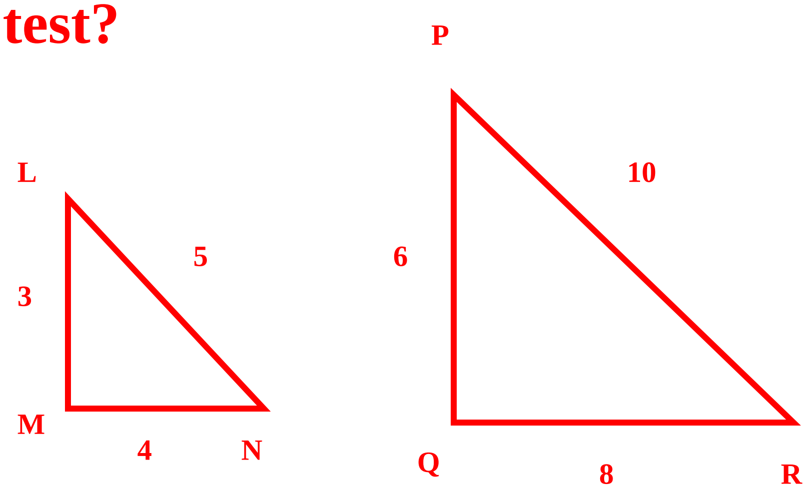
∠ACB ≅ ∠ECD [Common angle]

∴ ΔABC ~ ΔEDC [AA test of similarity]

One to one correspondence is ABC ↔ EDC

Q. 9

Are the triangles in the adjoining figure similar? If yes, by which test?



SOLUTION:

In ΔPQR and ΔLMN,

$$\frac{PQ}{LM} = \frac{6}{3} = \frac{2}{1} \qquad \text{(i)}$$

$$\frac{QR}{MN} = \frac{8}{4} = \frac{2}{1} \qquad \text{(ii)}$$

$$\frac{PR}{LN} = \frac{10}{5} = \frac{2}{1} \qquad \text{(iii)}$$

$$\therefore \frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LN} \qquad \text{[From (i), (ii) and (iii)]}$$

∴ ΔPQR ~ ΔLMN [SSS test of similarity]

Q. 10

As shown in the adjoining figure, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m, then how long will be the shadow of the bigger pole at the same time?

SOLUTION:

Here, AC and PR represents the bigger and smaller poles, and BC and QR represents their shadows respectively.

Now, $\Delta ACB \sim \Delta PRQ$ [$\because$ Vertical poles and their shadows form similar figures]

$\therefore \frac{CB}{RQ} = \frac{AC}{PR}$ **[Corresponding sides of similar triangles]**

$\therefore \frac{x}{6} = \frac{8}{4}$

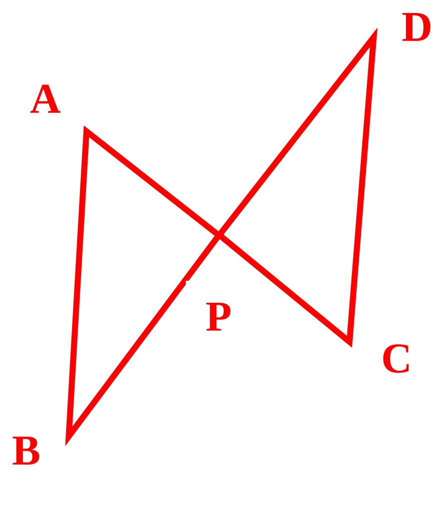
$\therefore x = \frac{8 \times 6}{4}$

$\therefore x = 12$ **m**

∴ The shadow of the bigger pole will be 12 meters long at that time.

Q. 11

In the adjoining figure, seg AC and seg BD intersect each other in point P and APCP = BPDP Prove that, ΔABP ~ ΔCDP



SOLUTION:

Proof:

In ΔABP and ΔCDP,

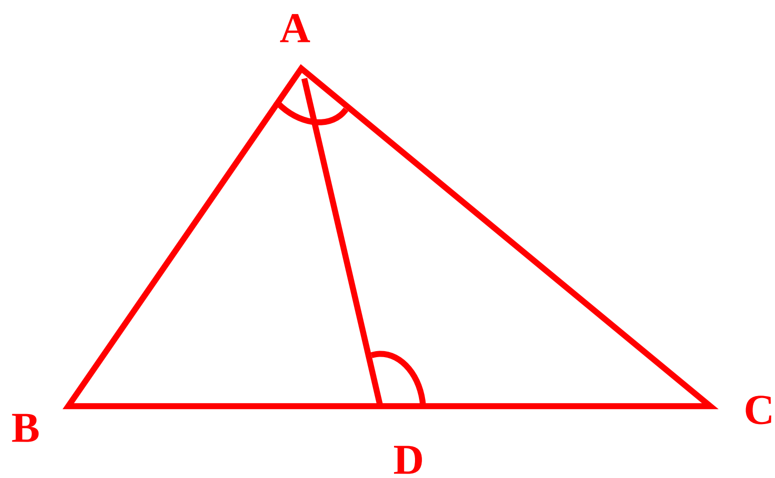
APCP = BPDP [Given]

∠APB ≅ ∠CPD [Vertically opposite angles]

∴ ΔABP ~ ΔCDP [SAS test of similarity]

Q. 12

In the adjoining figure, in ΔABC, point D is on side BC such that, ∠BAC = ∠ADC. Prove that, CA^2 = CB × CD,



SOLUTION:

Proof:

In ΔBAC and ΔADC,

∠BAC ≅ ∠ADC [Given]

∠BCA ≅ ∠ACD [Common angle]

∴ ΔBAC ~ ΔADC [AA test of similarity]

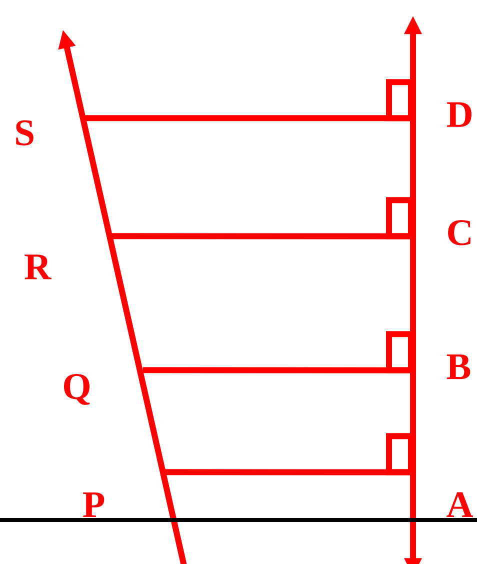
∴ $\frac{CA}{CD} = \frac{CB}{CA}$ [Corresponding sides of similar triangles]

∴ CA × CA = CB × CD

∴ CA^2 = CB × CD

Q. 13

In the figure, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD. AB = 60, BC = 70, CD = 80, PS = 280, then find PQ, QR and RS.



B

A

SOLUTION:

seg PA, seg QB, seg RC and seg SD are perpendicular to line AD. [Given]

∴ seg PA || seg QB || seg RC || seg SD (i) [Lines perpendicular to the same line are parallel to each other]

Let the value of PQ be x and that of QR be y. PS = PQ + QS [P – Q – S]

∴ 280 – x + QS

∴ QS = 280 – x (ii)

Now, seg PA || seg QB || seg SD [From (i)]

$\therefore \frac{AB}{BD} = \frac{PQ}{QS}$ **[Property of three parallel lines and their transversals]**

$\therefore \frac{AB}{BC+CD} = \frac{PQ}{QS}$ **[B – C – D]**

$\therefore \frac{60}{70+80} = \frac{x}{280-x}$

$\therefore \frac{60}{150} = \frac{x}{280-x}$

$\therefore \frac{2}{5} = \frac{x}{280-x}$

$\therefore 5x = 2(280 - x)$

$\therefore 5x = 560 - 2x$

$\therefore 7x = 560$

$\therefore x = \frac{560}{7} = 80$

∴ PQ = 80 units

QS = 280 – x [From (ii)]

= 280 – 80

= 200 units

But, QS = QR + RS [Q – R – S]

∴ 200 = y + RS

∴ RS = 200 – y (ii)

Now, seg QB || seg RC || seg SD [From (i)]

∴ $\frac{BC}{CD} = \frac{QR}{RS}$ **[Property of three parallel lines and their transversals]**

∴ $\frac{70}{80} = \frac{y}{200-y}$

∴ $\frac{7}{8} = \frac{y}{200-y}$

∴ $8y = 7(200 - y)$

∴ $8y = 1400 - 7y$

∴ $15y = 1400$

∴ $y = \frac{1400}{15} = \frac{280}{3}$

∴ $QR = \frac{280}{3}$ **units**

RS = 200 – 7 [From (iii)]

$$= 200 - \frac{280}{3}$$

$$= \frac{(200 \times 3 - 280)}{3}$$

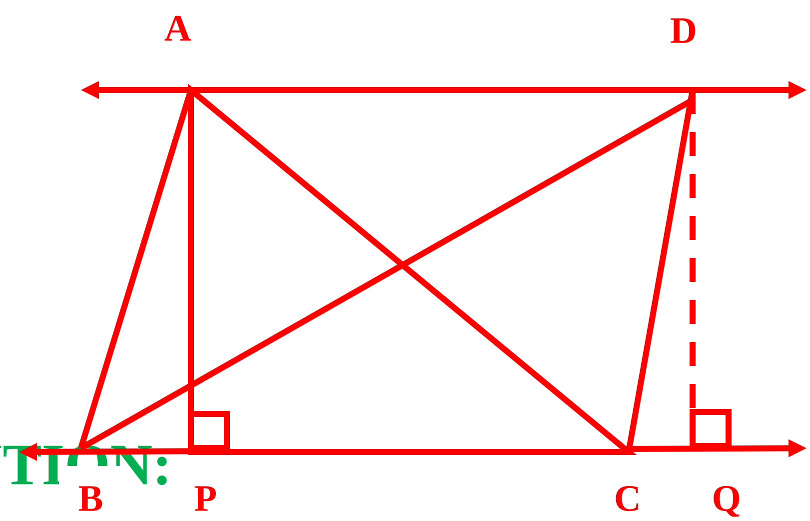
$$= \frac{(600 - 280)}{3} = \frac{320}{3}$$

∴ RS = 320/3 units

Ans.: PQ = 80 units, QR = 280/3 units, RS = 320/3 units

Q. 14

In the adjoining figure, AP ⊥ BC, AD || BC, then find A (△ABC) : A (△BCD).



SOLUTION:

Draw DQ ⊥ BC, B – C – Q

AD || BC [Given]

∴ AP = DQ [Perpendicular distance between two parallel lines is the same]

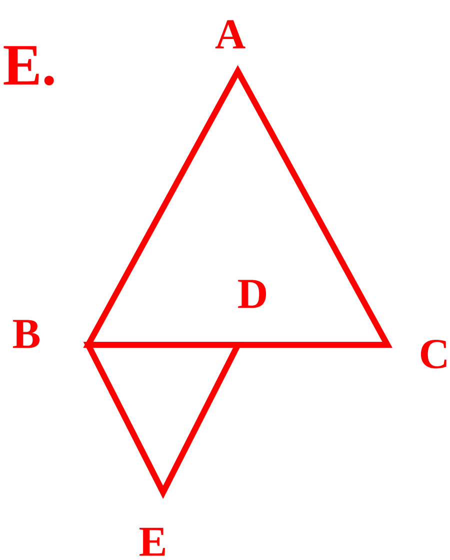
ΔABC and ΔBCD have same base BC.

$$\frac{A(\Delta ABC)}{A(\Delta BCD)} = \frac{AP}{DQ} = \frac{AP}{AP} = 1$$

Ans.: A (ΔABC): A (ΔBCD) = 1:1

Q. 15

ABC and BDE are two equilateral triangles such that D is mid-point of BC. Find the ratio of the areas of triangles ABC and BDE.



SOLUTION:

$$\frac{A\ (\Delta ABC)}{A\ (\Delta BDE)} = \frac{BC^2}{BD^2}$$

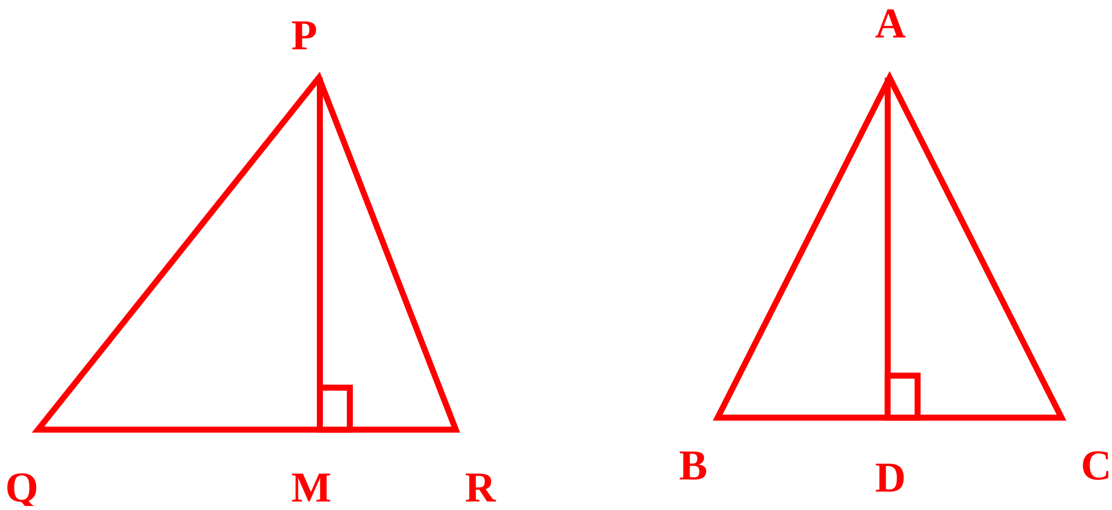
$$= \frac{2(BD)^2}{BD^2} \text{ [Since BC = 2BD]}$$

$$\frac{A\ (\Delta ABC)}{A\ (\Delta BDE)} = 4:1$$

Ans.: $\frac{A\ (\Delta ABC)}{A\ (\Delta BDE)} = 4:1$

Q. 16

ΔABC ~ ΔPQR. Area of ΔABC = 64 cm² and area of ΔPQR = 144 cm². Find the altitude PM, if AD = 8 cm.



SOLUTION:

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AD^2}{PM^2}$$

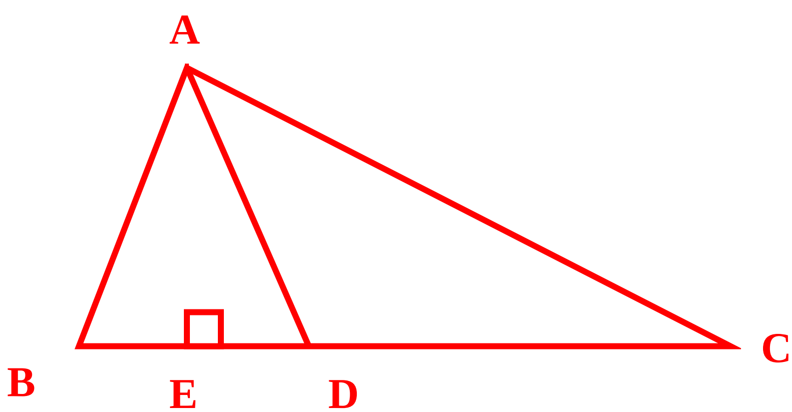
Ratio of areas of two triangles is equal to the ratio of the squares of corresponding heights.

$$\frac{81}{121} = \frac{9^2}{PM^2}$$

Ans.: PM = 11

Q. 17

In ΔABC, B-D-C and BD = 7, BC = 20, then find following ratio: $\frac{A\,(\Delta\, ABD)}{A\,(\Delta\, ABC)}, \frac{A\,(\Delta\, ABD)}{A\,(\Delta\, ADC)}, \frac{A\,(\Delta\, ADC)}{A\,(\Delta\, ABC)}$



SOLUTION:

Draw AE ⊥ BC, B – E – C

BC = BD + DC [B – D – C]

∴ 20 = 7 + DC

∴ DC = 20 – 7 = 13

i. Δ ABD and Δ ADC have same height AE.

$\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{DC}$ **[Triangles having equal height]**

$\therefore \frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{7}{13}$

ii. Δ ABD and Δ ABC have same height AE.

$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC}$ **[Triangles having equal height]**

$\therefore \frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{7}{20}$

iii. Δ ADC and Δ ABC have same height AE.

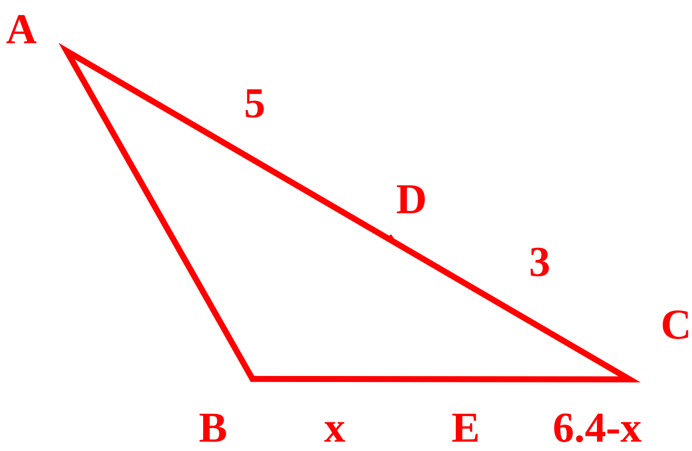
$\frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{DC}{BC}$ **[Triangles having equal height]**

$\therefore \frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{13}{20}$

Ans.: $\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{7}{13}, \frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{7}{20}, \frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{13}{20}$

Q. 18

In the adjoining figure, A – D – C and B – E – C. seg DE || side AB. If AD = 5, DC = 3, BC = 6.4, then find BE.



SOLUTION:

In ΔABC, seg DE || side AB [Given]

$\therefore \frac{DC}{AD} = \frac{EC}{BE}$ **[Basic proportionality theorem]**

$\therefore \frac{3}{4} = \frac{(6.4 - x)}{x}$

∴ 3x = 5 (6.4 – x)

∴ 3x = 32 – 5x

∴ 8x = 32

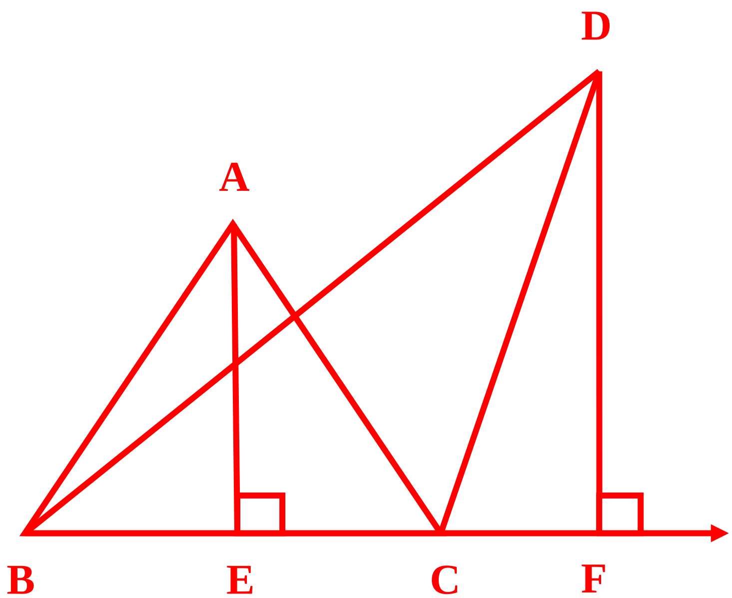
∴ x = 32/8

∴ x = 4

∴ BE = 4 units

Q. 19

In the given figure, AE ⊥ seg BC, seg DF ⊥ line BC, AE = 4, DF = 6, then find $\frac{A(\Delta ABC)}{A(\Delta DBC)}$



SOLUTION:

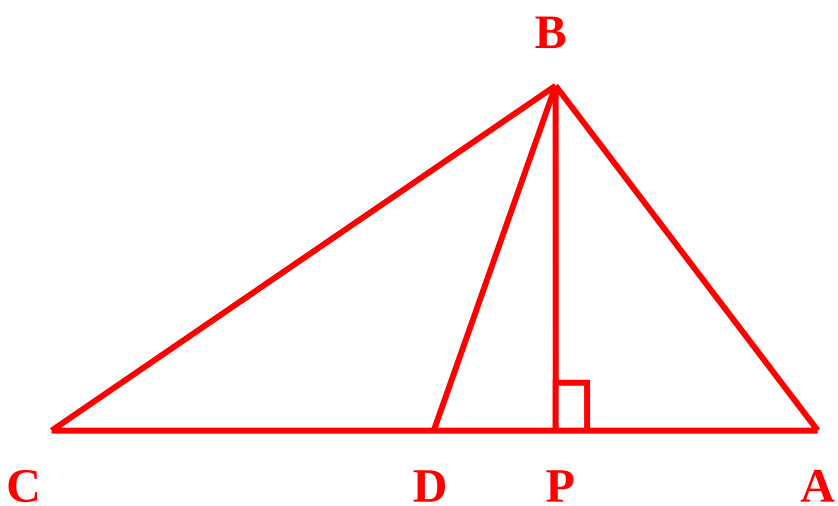
$\frac{A(\Delta ABC)}{A(\Delta DBC)} = \frac{AE}{DF}$ **……... (bases are equal; hence areas are proportional to heights)**

$$\frac{A\ (\Delta\ ABC)}{A\ (\Delta\ DBC)} = \frac{4}{6} = \frac{2}{3}$$

Q. 20

In the following figure in Δ ABC, point D is on side AC. If AC = 16, DC = 9 and BP ⊥ AC, then find the following ratios:

$$\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ABC)}, \frac{A\ (\Delta\ BDC)}{A\ (\Delta\ ABC)}, \frac{A\ (\Delta\ ABD)}{A\ (\Delta\ BDC)}$$



SOLUTION:

In Δ ABC, point P and D are on side AC, hence B is the common vertex of Δ ABD, Δ BDC, Δ ABC and Δ APB and their sides AD, DC, AC, and AP are

collinear. Heights of all the triangles are equal. Hence, areas of these triangles are proportional to their bases. AC = 16, DC = 9

∴ AD = 16 – 9 = 7

$$\therefore \frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{AD}{AC} = \frac{7}{16}$$

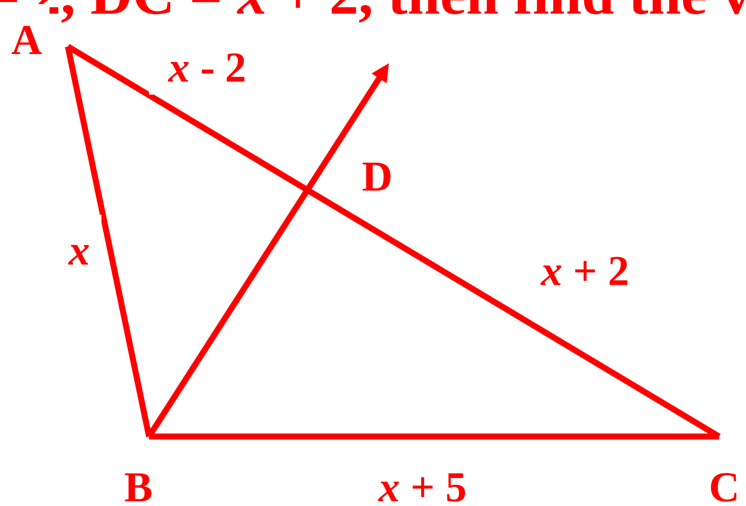
$$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{DC}{AC} = \frac{9}{16}$$

$$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{AD}{DC} = \frac{7}{9}$$

Triangles having Equal heights

Q. 21

In Δ ABC, seg BD bisects ∠ ABC. If AB = *x*, BC = *x* + 5, AD = *x* – 2, DC = *x* + 2, then find the value of *x*.



SOLUTION:

In Δ ABC, ray BD bisects $\angle$ ABC

$\therefore$ by property of angle bisector of triangle,

$$\frac{AB}{BC} = \frac{AD}{DC}$$

$$\therefore \frac{x}{x+5} = \frac{x-2}{x+2}$$

$$\therefore x\,(x+2) = (x-2)(x+5)$$

$$\therefore x^2 + 2x = x\,(x+5) - 2\,(x+5)$$

$$\therefore \cancel{x^2} + 2x = \cancel{x^2} + 5x - 2x - 10$$

$$\therefore 2x = 3x - 10$$

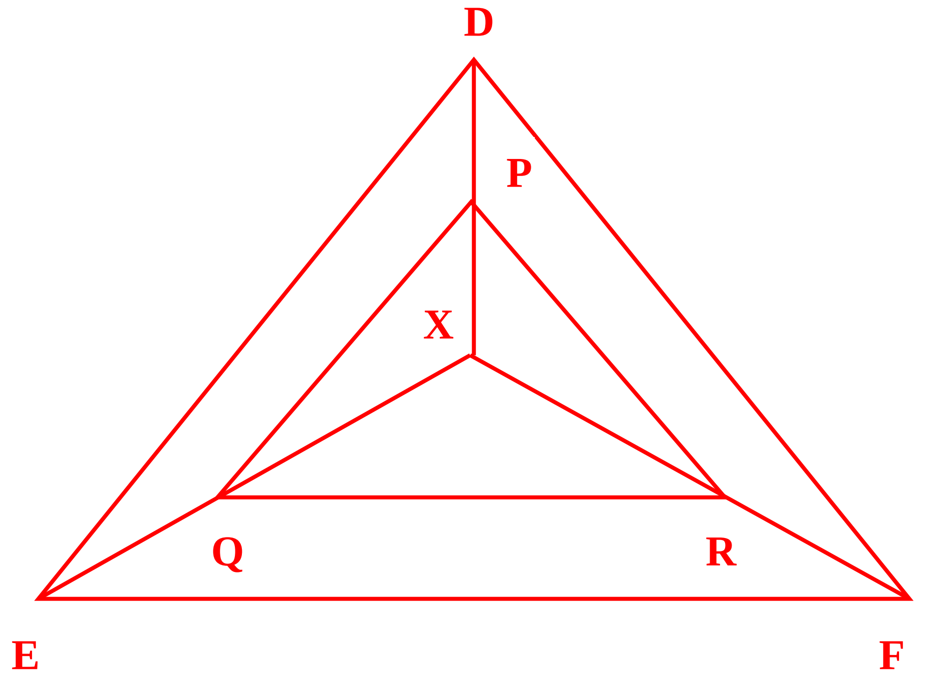
$$\therefore 2x - 3x = -10$$

$$\therefore -x = -10$$

$$\therefore x = 10$$

Q. 22

In the figure, X is any point in the interior of triangle. seg PQ || seg DE, seg QR || seg EF. Prove that seg PR || seg DF.



SOLUTION:

In Δ XDE, PQ || DE ... Given

$\therefore \frac{XP}{PD} = \frac{XQ}{QE}$ **... (I) (Basic proportionality theorem)**

In Δ XEF, QR || EF ... Given

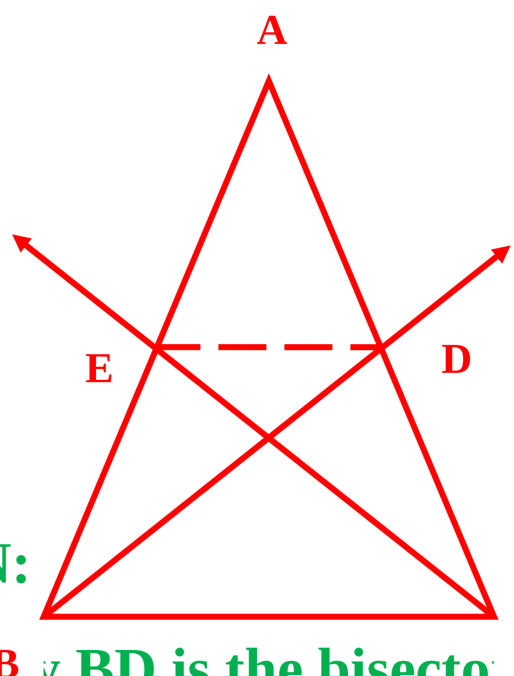
$\therefore \frac{XQ}{QE} = \frac{XR}{RF}$ **... (II) (Basic proportionality theorem)**

$\therefore \frac{XP}{PD} = \frac{XR}{RF}$ **… From (I) & (II)**

∴ seg PR ∥ seg DF … (Converse of basic proportionality theorem)

Q. 23

In Δ ABC, ray BD bisects ∠ ABC and ray CE bisects ∠ ACB. If seg AB ≅ seg AC; then prove that ED ∥ BC.



SOLUTION:

In Δ ABC, ray BD is the bisector of ∠ ABC

∴ by property of angle bisector of a triangle,

$\frac{AB}{BC} = \frac{AD}{DC}$ **… (1)**

In Δ ABC, ray CE is the bisector of ∠ ACB

∴ by property of angle bisector of a triangle,

$\frac{AC}{BC} = \frac{AE}{EB}$ **… (2)**

seg AB ≅ seg AC (Given) … (3)

∴ $\frac{AB}{BC} = \frac{AC}{BC}$ **… From (1), (2) & (3) … (4)**

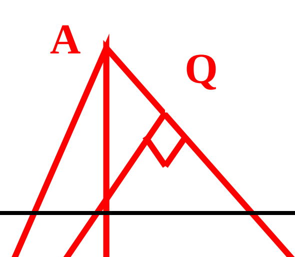
In Δ ABC, $\frac{AE}{EB} = \frac{AD}{DC}$ **… From (1), (2) & (4)**

By converse of basic proportionality theorem, seg ED || side BC

i.e. ED || BC

Q. 24

In Δ ABC, AP ⊥ BC, BQ ⊥ AC. B – P – C, A – Q – C; then prove that Δ CPA ~ Δ CQB. If AP = 7, BQ = 8, BC = 12; then find AC.



SOLUTION:

In Δ CPA and Δ CQB,

∠ CPA ≅ ∠ CQB … (Each measures 90º)

∠ ACP ≅ ∠ BCQ … (Common angle)

∴ Δ CPA ≅ Δ CQB … (AA test of similarity)

∴ $\frac{AP}{BQ} = \frac{AC}{BC}$ … (Corresponding sides of similar triangles are in proportion)

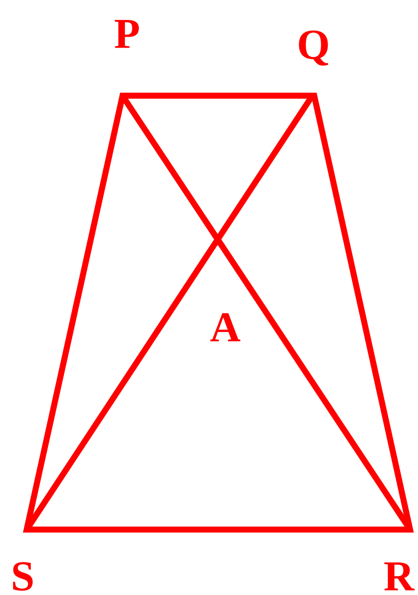
∴ $\frac{7}{8} = \frac{AC}{12}$

∴ AC x 8 = 7 x 12

∴ AC = 10.5

Q. 25

In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ; then prove that, SR = 5PQ.



SOLUTION:

Side PQ || side SR and line QS is the transversal

∴ ∠ PQS ≅ ∠ RSQ … (Alternate angles theorem)

i.e. ∠ PQA ≅ ∠ RSA … (Q – A – S) … (1)

In Δ PQA and Δ RSA,

∠ PQA ≅ ∠ RSA … from (1)

$\angle$ PAQ ≅ $\angle$ RAS … (Vertically opposite angles)

$\therefore$ Δ PQA ~ Δ RSA … (AA test of similarity)

$\therefore \frac{PQ}{RS} = \frac{AQ}{AS} = \frac{AP}{AR}$ **… (Corresponding sides of similar triangles are in proportion) … (2)**

AR = 5AP … (Given) … (3)

Substituting (3) in (2) we get,

$\frac{PQ}{SR} = \frac{AQ}{AS} = \frac{\cancel{AP}}{5\cancel{AP}}$

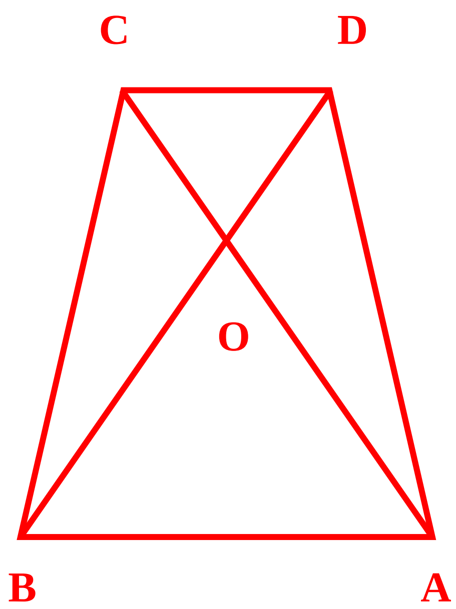
$\therefore \frac{PQ}{SR} = \frac{AQ}{AS} = \frac{1}{5}$ **… (4)**

$\therefore \frac{PQ}{SR} = \frac{1}{5}$ **… From (4)**

$\therefore$ SR = 5PQ

Q. 26

In trapezium ABCD, side AB || side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15, then find OD.



SOLUTION:

Side AB || side DC and line DB is the transversal,

∠ CDB ≅ ∠ ABD … (Alternate angles theorem)

i.e. ∠ CDO ≅ ∠ ABO … (B – O – D) … (1)

In Δ COD and Δ AOB,

∠ CDO ≅ ∠ ABO … From (1)

∠ COD ≅ ∠ AOB … (Vertically opposite angles)

∴ Δ COD ~ Δ AOB … (AA test of similarity)

∴ $\frac{OD}{OB} = \frac{DC}{AB}$ … (Corresponding sides of similar triangles are in proportion)

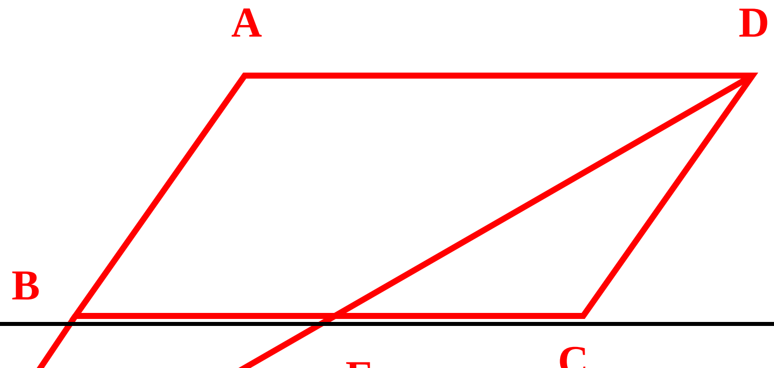
∴ $\frac{OD}{15} = \frac{6}{20}$

∴ OD x 20 = 6 x 15

∴ OD = 4.5

Q. 27

□ ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that DE x BE = CE x TE



SOLUTION:

Seg AB || seg CD … (Opposite sides of parallelogram are parallel)

i.e. seg AT || seg CD … (A – B – T)

Line TD is the transversal,

∴ ∠ ATD ≅ ∠ CDT … (Alternate angles theorem)

i.e. ∠ BTE ≅ ∠ CDE … (A – B – T, T – E – D) … (1)

In Δ BET & Δ CED,

∠ BTE ≅ ∠ CDE … From (1)

∠ BET ≅ ∠ CED … (Vertically opposite angles)

∴ Δ BET ~ Δ CED … (AA test of similarity)

$\therefore \frac{BE}{CE} = \frac{TE}{DE}$ **… (Corresponding sides of similar triangles are in proportion)**

$\therefore$ **BE x DE = CE x TE**

Q. 28

The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas.

SOLUTION:

Let the corresponding sides of two similar triangles be s_1 and s_2 and their respective areas be A_1 and A_2

$s_1 : s_2 = 3 : 5$ **… (Given)**

$\therefore \frac{s_1}{s_2} = \frac{3}{5}$ **… (1)**

The two triangles are similar

$\therefore \frac{A_1}{A_2} = \frac{{s_1}^2}{{s_2}^2}$ **… (Theorem of areas of similar triangles)**

$\therefore \frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$

$$\therefore \frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2$$ **... (From 1)**

$$\therefore \frac{A_1}{A_2} = \frac{9}{25}$$

∴ $A_1 : A_2 = 9 : 25$

Ans.: The ratio of the areas of the two similar triangles is 9 : 25

Q. 29

Areas of two similar triangles are 225 sq.cm. and 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of bigger triangle.

SOLUTION:

Let the areas of two similar triangles be A1 and A2 and their respective sides be s1 and s2.

A1 = 225 sq.cm., A2 = 81 sq.cm. and s2 = 12 cm

The two triangles are similar

∴ by theorem of areas of similar triangles,

$$\frac{A_1}{A_2} = \frac{s_1{}^2}{s_2{}^2}$$

$$\therefore \frac{225}{81} = \frac{s_1{}^2}{12^2}$$

$$\therefore s_1{}^2 = \frac{225 \times 12^2}{81}$$

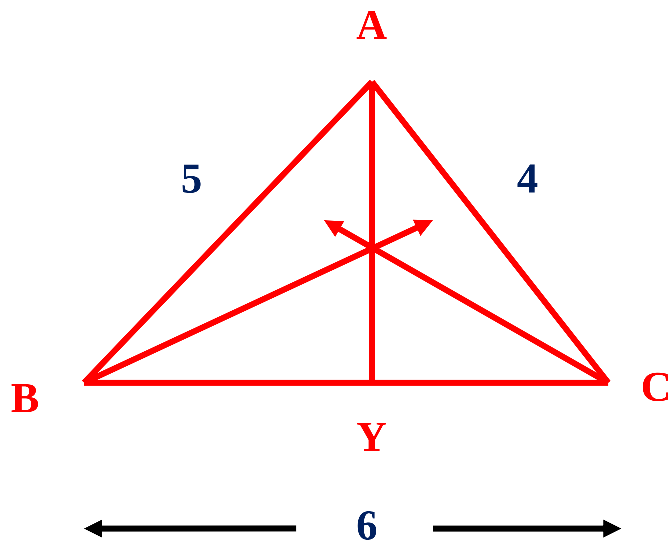
$\therefore s_1 = \frac{15 \times 12}{9}$ **… (Taking square roots on both the sides)**

$$\therefore s_1 = 20 \text{ cm}$$

Ans.: The corresponding side of the bigger triangle is 20 cm.

Q. 30

In the figure, bisectors of ∠ B and ∠ C of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$



SOLUTON:

In Δ ABY, ray BX bisects ∠ ABY

∴ by property of angle bisector of triangle,

$$\frac{AB}{BY} = \frac{AX}{XY} \qquad \ldots (1)$$

In Δ ACY, ray CX bisects ∠ ACY

∴ by property of angle bisector of triangle,

$$\frac{AC}{CY} = \frac{AX}{XY} \qquad \ldots (2)$$

$$\frac{AB}{BY} = \frac{AC}{CY} = \frac{AX}{XY} \qquad \ldots \text{From (1) and (2)}$$

$$\therefore \frac{5}{BY} = \frac{4}{CY} = \frac{AX}{XY}$$

By theorem on equal ratios,

$$\frac{5+4}{BY+CY} = \frac{AX}{XY}$$

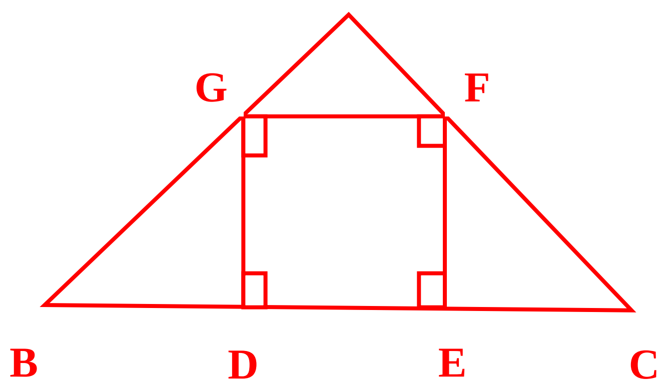
$\therefore \frac{9}{BC} = \frac{AX}{XY}$ **… (B – Y – C)**

$\therefore \frac{9}{6} = \frac{AX}{XY}$

$\therefore \frac{AX}{XY} = \frac{3}{2}$

Q. 31

In figure, the vertices of square DEFG are on the sides of Δ ABC. ∠ A A 90°. Then prove that DE^2 = BD x EC.



SOLUTION:

□ DEFG is a square

∴ DE = EF = GF = GD … (Sides of square) … (1)

∠ GDE = ∠ DEF = 90° … (Angles of a square)

$\therefore$ seg GD $\perp$ side BC and seg EF $\perp$ side BC

In Δ BAC and Δ BDG,

$\angle$ BAC $\cong$ $\angle$ BDG … (Each measures 90º)

$\angle$ ABC $\cong$ $\angle$ DBG … (Common angle)

$\therefore$ Δ BAC ~ Δ BDG … (AA test of similarity) … (2)

Similarly Δ BAC ~ Δ FEC … (3)

$\therefore$ Δ BDG ~ Δ FEC … From (2) and (3)

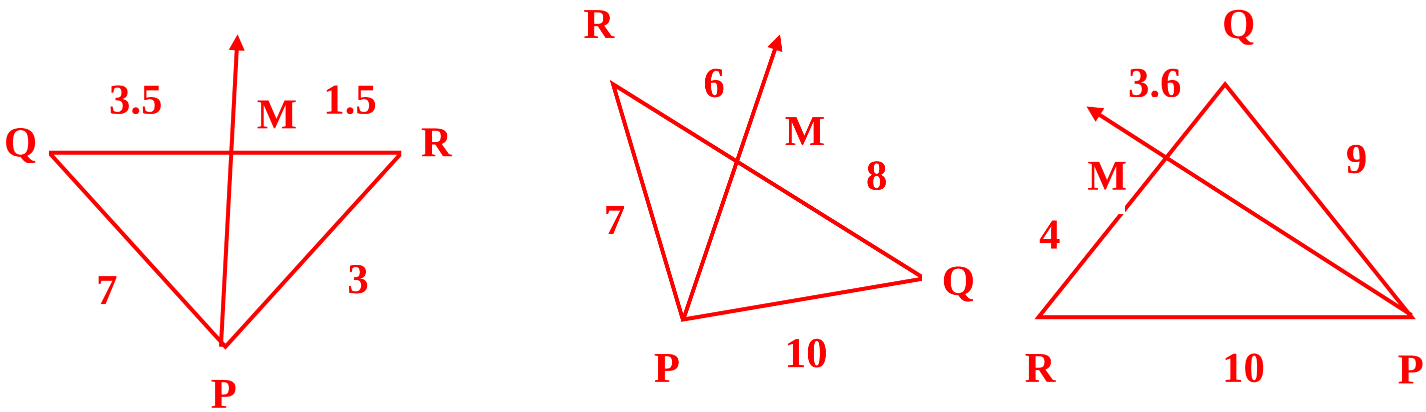
$\therefore$ $\frac{BD}{EF} = \frac{GD}{EC}$ … (Corresponding sides of similar triangles are in proportion)

$\therefore$ $\frac{BD}{DE} = \frac{DE}{EC}$ … From (1)

$\therefore$ $DE^2 = BD \times EC$

Q. 32

Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of ∠ QPR.



SOLUTION:

(i) In Δ PQR,

$$\frac{PQ}{PR} = \frac{7}{3} \qquad \ldots \text{(i)}$$

$$\frac{QM}{RM} = \frac{3.5}{1.5} = \frac{35}{15} = \frac{7}{3} \qquad \ldots \text{(ii)}$$

$$\therefore \frac{PQ}{PR} = \frac{QM}{RM} \qquad \ldots \text{From (i) \& (ii)}$$

∴ **Ray RM is the bisector of ∠ QPR. (Converse of angle bisector theorem)**

(ii) In Δ PQR,

$\frac{PQ}{PR} = \frac{10}{7}$ ... (i)

$\frac{QM}{RM} = \frac{8}{6} = \frac{4}{3}$ ... (ii)

$\therefore \frac{PQ}{PR} \neq \frac{QM}{RM}$ ... From (i) & (ii)

∴ Ray RM is not the bisector of ∠ QPR.

(iii) In Δ PQR,

$\frac{PQ}{PR} = \frac{9}{10}$ ... (i)

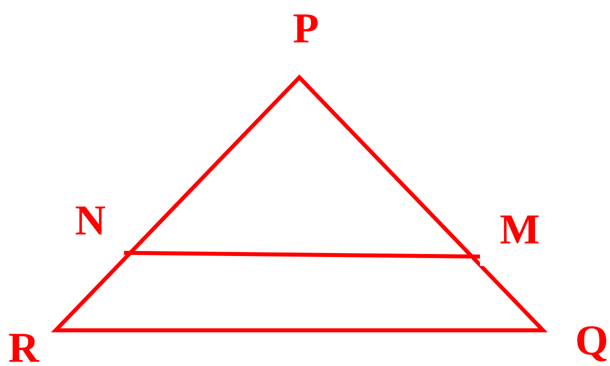
$\frac{QM}{RM} = \frac{3.6}{4} = \frac{36}{40} = \frac{9}{10}$ ... (ii)

$\therefore \frac{PQ}{PR} = \frac{QM}{RM}$ ... From (i) & (ii)

∴ Ray RM is the bisector of ∠ QPR (Converse of angle bisector theorem)

Q. 33

In Δ PQR, PM = 15, PQ = 25, NR = 8. State whether line NM is parallel to side RQ. Give reason.



SOLUTION:

PN + NR = PR [P – N – R]

∴ PN + 8 = 20

∴ PN = 12

Also, PM + MQ = PQ [P – M – Q]

∴ 15 + MQ = 25

∴ MQ = 10

$$\frac{PN}{NR} = \frac{12}{8} = \frac{3}{2} \quad \ldots (i)$$

$$\frac{PM}{MQ} = \frac{15}{10} = \frac{3}{2} \quad \ldots (ii)$$

In Δ PQR,

$\frac{PN}{NR} = \frac{PM}{MQ}$ … From (i) & (ii)

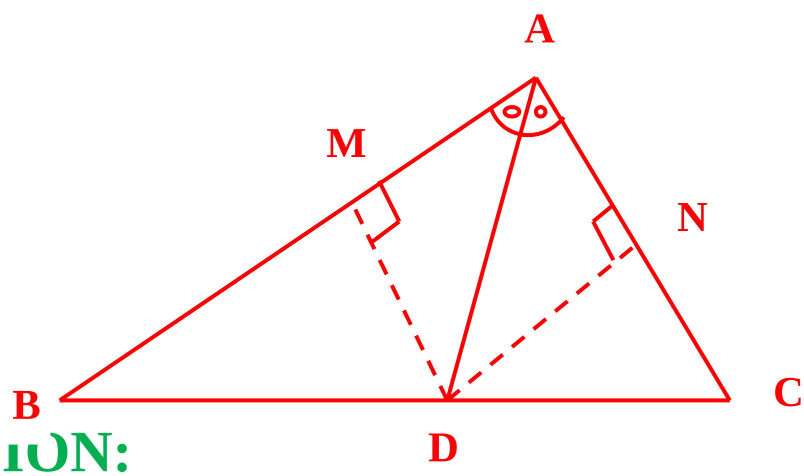
∴ Line NM || side RQ (Converse of basic proportionality theorem)

Q. 34

Use the following properties and write the proof as per the given diagram.

i. The areas of two triangles of equal height are proportional to their bases.

ii. Every point on the bisector of an angle is equidistant from the sides of the angle.



SOLUTION:

Given: In Δ CAB, ray AD bisects ∠A

To prove: $\frac{AB}{AC} = \frac{BD}{DC}$

Construction: Draw seg DM ⊥ seg AB A – M – B and seg DN ⊥ seg AC, A – N – C.

Proof:

In ΔABC, Point D is on angle bisector of ∠A. [Given]

∴ DM = DN [Every point on the bisector of an angle is equidistant from the sides of the angle]

$\frac{A\,(\Delta\ ABD)}{A\,(\Delta\ ACD)} = \frac{AB \times DM}{AC \times DN}$ **[Ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights] … (i)**

∴ $\frac{A\,(\Delta\ ABD)}{A\,(\Delta\ ACD)} = \frac{AB}{AC}$ **… (ii) [From (i)]**

Also, ΔABD and ΔACD have equal height.

∴ $\frac{A\,(\Delta\ ABD)}{A\,(\Delta\ ACD)} = \frac{BD}{CD}$ **(iii) [Triangles having equal height]**

∴ $\frac{AB}{AC} = \frac{BD}{DC}$ **[From (ii) and (iii)]**

Q. 35

i. Draw three parallel lines.

ii. Label them as l, m, n.

iii. Draw transversals t1 and t2.

iv. AB and BC are intercepts on transversal t1.

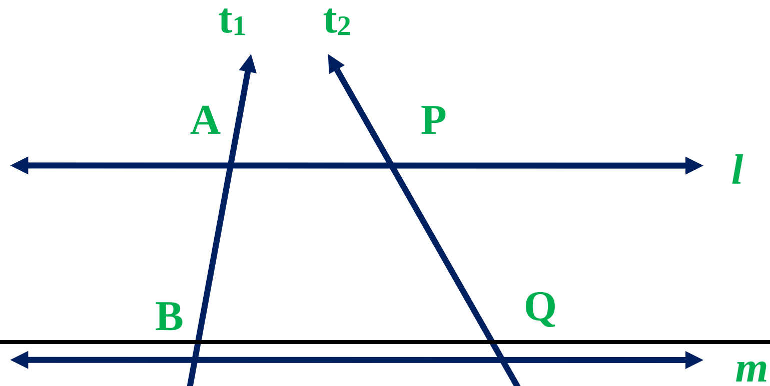
v. PQ and QR are intercepts on transversal t2.

vi. Find ratios $\frac{AB}{BC}$ and $\frac{PQ}{QR}$

AB = 1.5 cm, BC = 2.1 cm, PQ = 1.7 cm, QR = 2.3 cm

You will find that they are almost equal. Verify that they are equal.

SOLUTION:



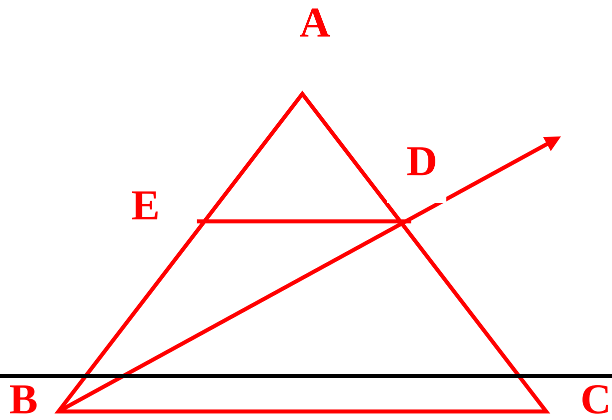
$$\frac{AB}{BC} = \frac{1.5}{2.1} = 0.714 = 0.7$$

$$\frac{PQ}{QR} = \frac{1.7}{2.3} = 0.739 = 0.7$$

$$\therefore \frac{AB}{BC} = \frac{PQ}{QR}$$

Q. 36

In ΔABC, ray BD bisects $\angle ABC$. A – D – C, side DE || side BC, A – E – B, then prove that $\frac{AB}{BC} = \frac{AE}{EB}$



SOLUTION:

In Δ ABC, ray BD bisects ∠ B … Given

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$ **… (i) Angle bisector theorem**

In Δ ABC, DE || BC … Given

$\therefore \frac{AE}{EB} = \frac{AD}{DC}$ **… (ii) Basic proportionality theorem**

$\therefore \frac{AB}{BC} = \frac{AE}{EB}$ **… From (i) & (ii)**

Q. 37

ΔLMN ~ ΔPQR, 9 × A (ΔPQR) = 16 × A (ΔLMN). If QR = 20, then find MN.

SOLUTION:

9 × A(ΔPQR) = 16 × A(ΔLMN) [Given]

$\therefore \frac{A\ (\Delta\ LMN)}{A\ (\Delta\ PQR)} = \frac{9}{16}$ **… (i)**

Now, ΔLMN ~ ΔPQR [Given]

$$\therefore \frac{A\ (\Delta\ LMN)}{A\ (\Delta\ PQR)} = \frac{MN^2}{QR^2}$$ **… (ii) [Theorem of areas of similar triangles]**

$$\therefore \frac{MN^2}{QR^2} = \frac{9}{16}$$ **… [From (i) and (ii)]**

$$\therefore \frac{MN}{QR} = \frac{3}{4}$$ **… [Taking square root of both sides]**

$$\therefore \frac{MN}{20} = \frac{3}{4}$$

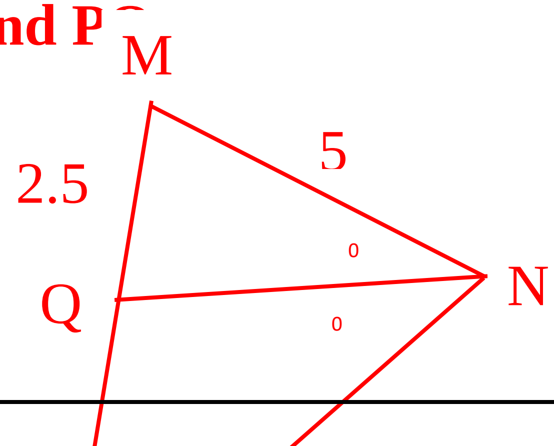
$$\therefore MN = \frac{20 \times 3}{4}$$

∴ MN = 15 units

Ans: MN = 15 units

Q. 38

In ΔMNP, NQ is a bisector of ∠ N. If MN = 5, PN = 7, MQ = 2.5, then find P



SOLUTION:

In Δ MNP, ray NQ bisects ∠ MNP

∴ By property of triangle bisector of triangle,

$$= \frac{MQ}{QP}$$

$$\therefore \frac{MN}{NP} = \frac{MQ}{QP}$$

$$\therefore \frac{5}{7} = \frac{2.5}{QP}$$

$$\therefore \frac{5}{7} = \frac{2.5}{QP}$$

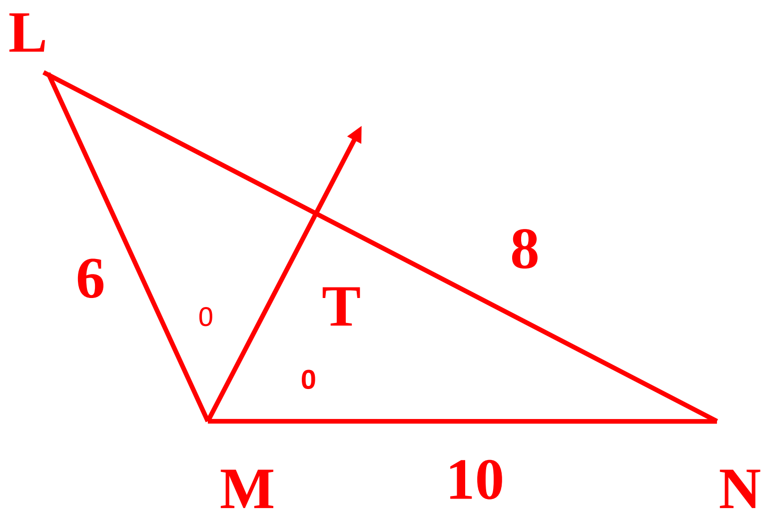
$$\therefore QP = \frac{7 \text{ X } 2.5}{5}$$

∴ QP = 3.5

Ans.: QP = 3.5

Q. 39

In Δ LMN, ray MT bisects ∠ LMN. If LM = 6, MN = 10, TN = 8 then find LT



SOLUTION:

In ΔLMN, ray MT bisects ∠ LMN

∴ By property of angle bisector of triangle,

$$\frac{LM}{MN} = \frac{LT}{TN}$$

$$\therefore \frac{6}{10} = \frac{LT}{8}$$

∴ LT x = 6 X 8

$\therefore$ LT $= \frac{6 \text{ X } 8}{108}$

Ans.: LT= 4.8

Q. 40

Δ LMN ~ Δ PQR, 9 x A (ΔPQR) = 16 x A (Δ LMN)

If QR = 20, then, find MN

SOLUTION:

Δ LMN ~ Δ PQR

9 x A (ΔPQR) = 16 x A (Δ LMN)

$\therefore \frac{A(\Delta\, LMN)}{A(\Delta PQR)} = \frac{9}{16}$

Now, $\frac{A(\Delta\, LMN)}{A(\Delta PQR)} = \frac{MN^2}{QR^{\,2}}$

{Theorem of area of similar triangles}

$\therefore \frac{9}{16} = \frac{MN^2}{QR^{\,2}}$

$\therefore \frac{MN}{QR} = \frac{3}{4}$

(Taking square roots of both sides)

$\therefore \frac{MN}{20} = \frac{3}{4}$

$\therefore MN = \frac{20 \times 3}{4}$

$\therefore$ **MN = 15**

Ans.: MN = 15

Q. 41

A (Δ ABC) and A (Δ DEF) are equilateral triangles. If A (Δ ABC) : A(Δ DEF) = 1 : 2 and AB = 4. Find DE.

SOLUTION:

A (Δ ABC) and A (Δ DEF) are equilateral triangles.

$\therefore \angle A = \angle B = \angle C = \angle D = \angle E = \angle F$

(Angles of equilateral triangles)

In Δ ABC and Δ DEF

$\angle A \cong \angle D$ … **(Each measures 60^0)**

$\angle B \cong \angle E$

$\therefore \Delta ABC \sim \Delta DEF$ … (AA test of similarity)

By theorem of areas of similar triangle

$\frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$ ……… (1)

A (Δ ABC) : A(Δ DEF) = 1 : 2 and AB = 4

(Given) …….. (2)

$\therefore \frac{1}{2} = \frac{4^2}{DE^2}$

$\therefore DE^2 = 4^2 \times 2$

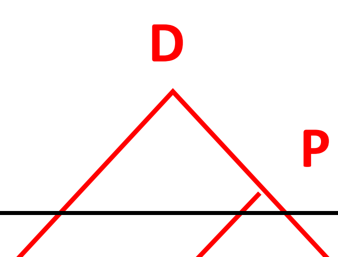
$\therefore DE = 4\sqrt{2}$ … (Taking square roots of both sides)

Ans.: $DE = 4\sqrt{2}$

Q. 42

In figure seg PQ || seg DE A (Δ PQF) = 20 units,

PF = 2 DP, then find A (□ DPQE) by completing the following activity



SOLUTION:

A (Δ PQF) = 20 units, PF = 2DP,

Let us assume DP = *x*

∴ PF = 2*x*

DF = DP + [PF] = [*x*] + [2 *x*] = 3*x*

In Δ FDE and Δ FPQ

∠ FDE = ∠ [FPQ] ....(Corresponding angles)

∠ FED = ∠ [FPQ] ....(Corresponding angles)

∴ Δ FDE ~ Δ FPQ

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{DF^3}{PF^2} = \frac{(3X)^3}{(2X)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} X \boxed{20} = \boxed{45 \text{ units}}$$

$A(\square DPQE) = A(\Delta FDE) - A(\Delta FPQ)$

$= \boxed{45} - \boxed{20}$

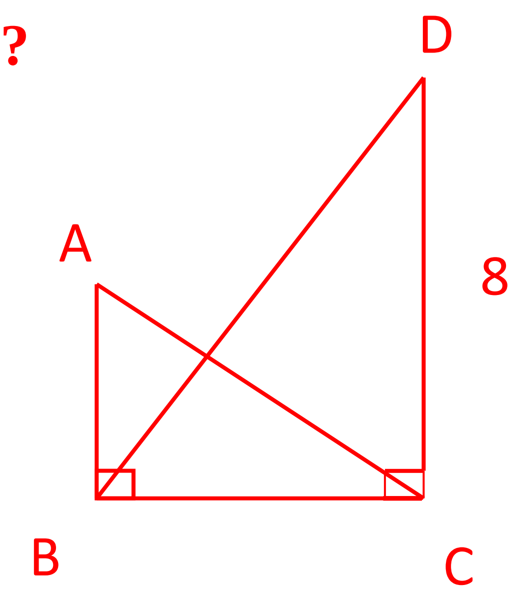
$= \boxed{25}$

(Note: Answers are in given green colour in the boxes inserted)

Q. 43

In figure $\angle ABC = \angle DCB = 90^0$ AB = 6, DC = 8;

Then $\frac{A(\Delta ABC)}{A(\Delta DCB)} = ?$



SOLUTION:

Δ ABC and Δ DCB have same base BC

Areas of triangle with equal bases are proportional to their corresponding heights

$$\therefore \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{AB}{DC}$$

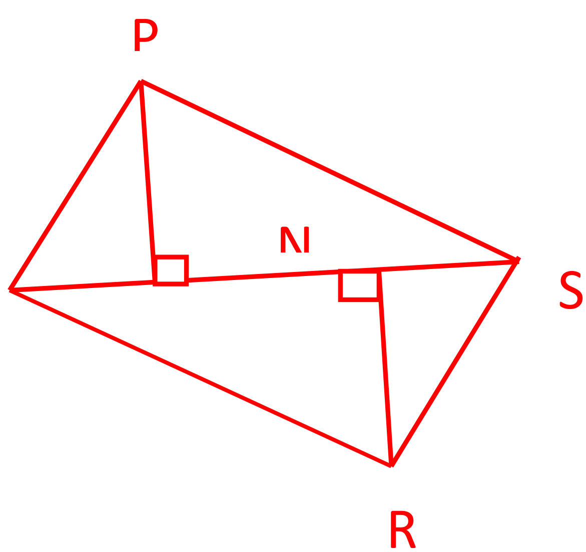
$$\therefore \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{6}{8}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{3}{4}$$

Ans. $\frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{3}{4}$

Q. 44

In figure, PM = 10 cm, A(Δ PQS) = 10 cm, A(Δ QRS) = 100 cm, then find NR



SOLUTION:

Δ PQS and Δ QRS have same base QS.

Areas of triangles with equal bases are proportional to their corresponding heights.

$$\therefore \frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{PM}{RN}$$

$$\therefore \frac{100}{110} = \frac{10}{NR}$$

$$\therefore NR = \frac{110 \times 10}{100}$$

$$\therefore NR = 11cm$$

Ans : NR = 11cm

Q. 45

Ratio of areas of two triangles with equal heights is 2:3. If base of the similar triangle is 6 cm then what is the corresponding base of the bigger triangle?

SOLUTION:

Let the areas of triangles be A_1 and A_2

Let their respective bases be b_1 and b_2

$A_1 : A_2 = 2:3$ and $b_1 = 6\ cm$ ... (Given)

The triangles are of equal height.

Area of triangle with equal heights is proportional to their corresponding bases.

$$\therefore \frac{A_1}{A_2} = \frac{b_1}{b_2}$$

$$\therefore \frac{2}{3} = \frac{6}{b_2}$$

$$\therefore b_2 = 9\ cm$$

Ans.: The corresponding base of the bigger triangle is 9 cm.

Q. 46

In ΔPQR, seg PM is a median. Angle bisectors of $\angle$PQM & $\angle$PMR intersect side PQ and side PR in

points X and Y respectively. Prove that XY || QR. Complete the proof by filling in the boxes.

SOLUTION:

In Δ PQM ray MX is bisector of ∠PMQ

$$\frac{\boxed{PM}}{\boxed{MQ}} = \frac{\boxed{PX}}{\boxed{X}}$$

$\therefore \frac{2}{3} = \frac{6}{b_2}$ **….. (I) theorem of angle bisector**

In Δ PMR ray MY is bisector of ∠PMR, then

$$\frac{\boxed{PM}}{\boxed{MQ}} = \frac{\boxed{PY}}{\boxed{YR}}$$

….(II) theorem of angle bisector

But

$\frac{MP}{MQ} = \frac{MP}{MR}$ **…. M is the mid point of QR**

Hence MQ = MR

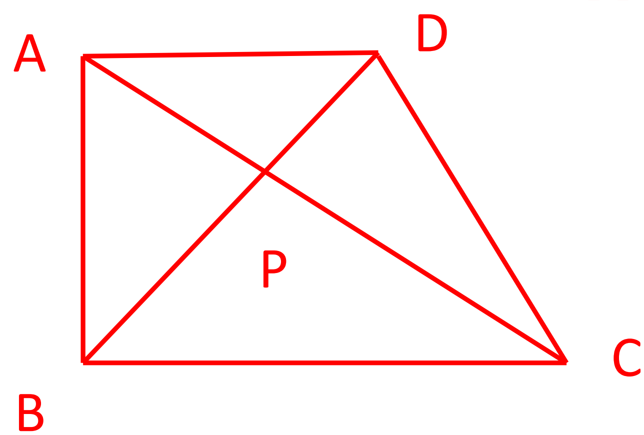
$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$

$\therefore$ XY || QR .....(Converse of basic proportionality theorem)

Ans.: Given in text boxes

Q. 47

In □ ABCD seg AD || seg BC. Diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$



SOLUTION:

seg AD || seg BC and line DB is transversal,

∠ ADP ≅ ∠ CBP (Alternate angle theorem)….. (1)

In Δ ADP and Δ CBP

∠ ADP ≅ ∠ CBP ……………………… {from (1)}

∠ APD ≅ ∠ CPB … (Vertically opposite triangles)

∴Δ ADP ~ Δ CBP …….(AA test of similarity)

∴ $\frac{AP}{CP} = \frac{PD}{BP}$ …(Corresponding sides of similar triangles in proportion)

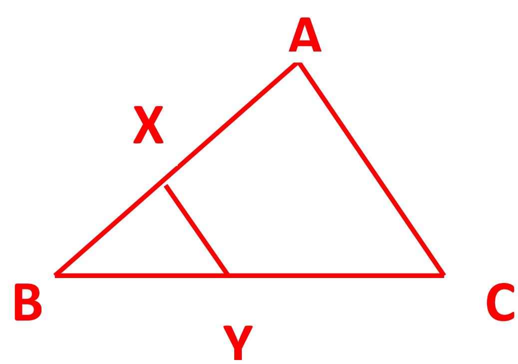
∴ $\frac{AP}{PD} = \frac{PC}{BP}$

i.e. $\frac{AP}{PD} = \frac{PC}{BP}$

Ans.: $\frac{AP}{PD} = \frac{PC}{BP}$ proved

Q. 48

In figure XY: seg AC. If 2AX = 3BX and XY = 9, complete the activity to find the value of AC



SOLUTION:

Activity

2AX = 3 BX

$\therefore \frac{AX}{BX} = \frac{3}{2}$

$\therefore \frac{AX + BX}{BX} = \frac{3 + 2}{2}$ **(by componendo)**

$\therefore \frac{AB}{BX} = \frac{5}{2}$

Δ BCA ~ Δ BYX ……..{A A test of similarity}

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$

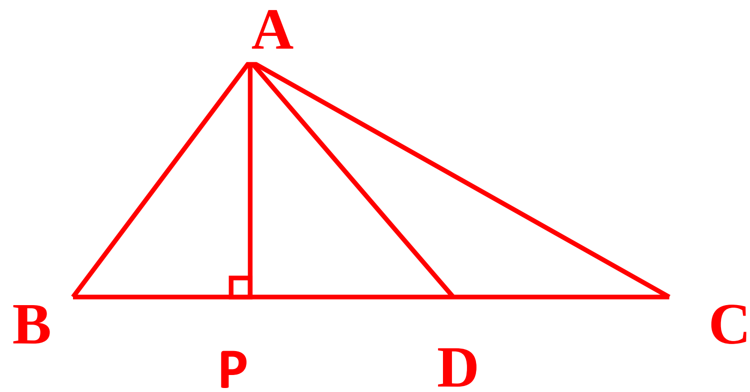
…(Corresponding sides of similar triangle)

$\therefore \frac{5}{2} = \frac{AC}{9}$

$\therefore AC = 22$ ……… (From (1)]

Q. 49

In Δ ABC point D on side BC is such that DC = 6, BC = 15. Find A (Δ ABD) : A (Δ ABC) and A (Δ ABD) : A (Δ ADC)



SOLUTION:

Point A is common vertex of Δ ABD, Δ ADC & Δ ABC their bases are collinear. Hence, heights of these triangles are equal.

BC = 15, DC = 6, ∴ BD = BC - DC = 15 - 6 = 9

$\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ABC)} = \frac{BD}{BC}$ **…heights equal hence areas proportional to bases.**

$\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ABC)} = \frac{9}{15}$

$$= \frac{3}{5}$$

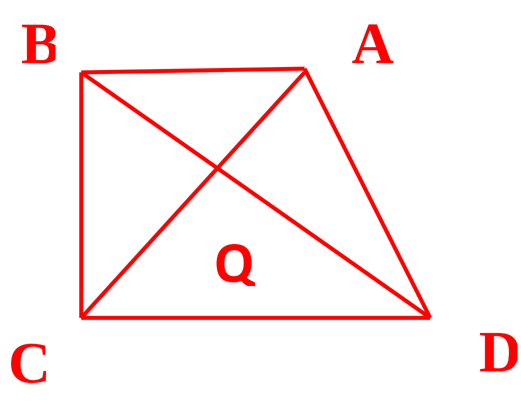
$\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ADC)} = \frac{BD}{DC}$ …… heights equal hence areas proportional to bases.

$$\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ADC)} = \frac{9}{6} = \frac{3}{2}$$

Ans: $\frac{A\ (\Delta\ ABD)}{A\ (\Delta\ ADC)} = \frac{3}{2}$

Q. 50

Diagonals of quadrilateral ABCD intersect in point Q. If 2QA = QD, then prove that DC = 2 AB. Given 2QA = QC, 2QB = QD



SOLUTION:

2QA = QC $\therefore \frac{QA}{QC} = \frac{1}{2}$ **........ (1)**

2QB = QD $\therefore \frac{QB}{QD} = \frac{1}{2}$ **........ (2)**

$\therefore \frac{QA}{QC} = \frac{QB}{QD}$ **..............from (1) and (2)**

In Δ AQB & Δ CQD

$\frac{QA}{QC} = \frac{QB}{QD}$ **...............proved**

$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3}$

∠ AQB ≅ ∠ CQD.......opposite angles

∴ Δ AQB~ Δ CQD........ (SAS test of similarity)

$\therefore \frac{AQ}{CQ} = \frac{QB}{QD} = \frac{AB}{CD}$ **...... ...corresponding side are proportional**

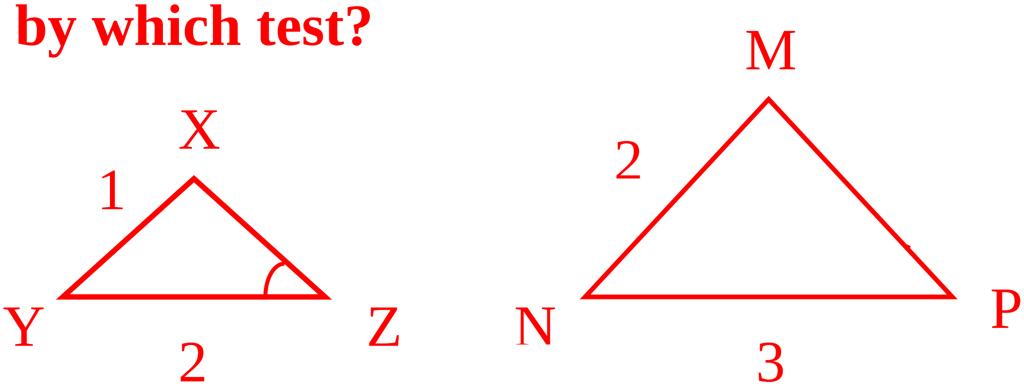
But $\frac{AQ}{CQ} = \frac{1}{2}$ $\therefore \frac{AQ}{CQ} = \frac{1}{2}$

∴ 2AB = CD

2AB = CD proved

Q. 51

Can we say that the two triangles in the figures are similar, according to the information given? If yes, by which test?



SOLUTION:

Δ XYZ & Δ MNP ,

$$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3}$$

$$\frac{YZ}{NP} = \frac{20}{30} = \frac{2}{3}$$

It is given that $\angle Z \cong \angle P$

But $\angle$ Z and $\angle$ P are not included angle by sides which are not in proportion

$\therefore \Delta$ XYZ & Δ MNP can not be said to be similar

Ans: Δ XYZ & Δ MNP can not be said to be similar