14. Algebraic Formulae-Expansion of Squares

Practice Set 50

1. Expand.

(i) $(5a + 6b)^2$

Solution :

$(5a + 6b)^2 = (5a)^2 + 2(5a) \times (6b) + (6b)^2$

$= 25a^2 + 60ab + 36b^2$

(ii) $\left(\frac{a}{2} + \frac{b}{3}\right)^2$

Solution :

$\left(\frac{a}{2} + \frac{b}{3}\right)^2 = \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right) \times \left(\frac{b}{3}\right) + \left(\frac{b}{3}\right)^2$

$= \frac{a^2}{4} + \frac{ab}{3} + \frac{b^2}{9}$

(iii) $(2p - 3q)^2$

Solution :

$(2p - 3q)^2 = (2p)^2 - 2(2p) \times (3q) + (3q)^2$

$= 4p^2 - 12pq + 9q^2$

(iv) $\left(x-\frac{2}{x}\right)^2$

Solution :

$\left(x-\frac{2}{x}\right)^2 = (x)^2 - 2(x) \times \left(\frac{2}{x}\right) + \left(\frac{2}{x}\right)^2$

$= x^2 - 4 + \frac{4}{x^2}$

(v) $(ax + by)^2$

Solution :

$(ax + by)^2 = (ax)^2 + 2\,[(ax) \times (by)] + (by)^2$

$= a^2x^2 + 2abxy + b^2y^2$

(vi) $(7m - 4)^2$

Solution :

$(7m - 4)^2 = (7m)^2 - 2\,[(7m) \times (4)] + (4)^2$

$= 49m^2 - 56m + 16$

(vii) $\left(x+\frac{1}{2}\right)^2$

Solution :

$\left(x+\frac{1}{2}\right)^2 = (x)^2 + 2\,[(x) \times \left(\frac{1}{2}\right)] + \left(\frac{1}{2}\right)^2$

$= x^2 + x + \frac{1}{4}$

(viii) $\left(a-\frac{1}{a}\right)^2$

Solution :

$$\left(a-\frac{1}{a}\right)^2 = (a)^2 - 2\left[(a)\times\left(\frac{1}{a}\right)\right] + \left(\frac{1}{a}\right)^2$$

$$= a^2 - 2 + \frac{1}{a^2}$$

2. Which of the options given below is the square of the binomial $\left(8-\frac{1}{x}\right)$.

(i) $64-\frac{1}{x^2}$ (ii) $64+\frac{1}{x^2}$

(iii) $64-\frac{16}{x}+\frac{1}{x^2}$ (iv) $64+\frac{16}{x}+\frac{1}{x^2}$

Solution:

$$\left(8-\frac{1}{x}\right)^2 = (8)^2 - 2\left[(8)\times\left(\frac{1}{x}\right)\right] + \left(\frac{1}{x}\right)^2$$

$$= 64-\frac{16}{x}+\frac{1}{x^2}$$

Ans. The correct option is (iii) $64-\frac{16}{x}+\frac{1}{x^2}$

3. Of which of the binomials given below is

$m^2n^2 + 14mnpq + 49\ p^2q^2$ the expansion?

(i) $(m + n)(p + q)$ (ii) $(mn - pq)$

(iii) $(7mn + pq)$ (iv) $(mn + 7pq)$

Solution: $m^2n^2 + 14mnpq + 49\ p^2q^2$

$= (mn)^2 + 2 \times 7 \times (mn) \times (pq) + (7pq)^2$

$= (mn)^2 + 2 \times (mn) \times (7pq) + (7pq)^2$

$= (mn + 7pq)^2$ .....$[a^2 + 2ab + b^2 = (a + b)^2]$

Ans. The correct option is (iv) $(mn + 7pq)^2$

4. Use an expansion formula to find the values.

(i) $(997)^2$

Solution:

$997 = 1000 - 3$

$\therefore\ (997)^2 = (1000 - 3)^2$

$= (1000)^2 - 2\ [(1000) \times (3)] + (3)^2$

$= 1000000 - 6000 + 9$

$= 994009$

(ii) $(102)^2$

Solution:

$102 = 100 + 2$

$\therefore (102)^2 = (100 + 2)^2$

$= (100)^2 + 2\,[(100) \times (2)] + (2)^2$

$= 10000 + 400 + 4$

$= 10404$

(iii) $(97)^2$

Solution:

$97 = 100 - 3$

$\therefore (97)^2 = (100 - 3)^2$

$= (100)^2 - 2\,[(100) \times (3)] + (3)^2$

$= 10000 - 600 + 9$

$= 9409$

(iv) $(1005)^2$

Solution:

$1005 = 1000 + 5$

$\therefore (1005)^2 = (1000 + 5)^2$

$= (1000)^2 - 2\,[(1000) \times (5)] + (5)^2$

$= 1000000 - 10000 + 25$

$= 1010025$

Practice Set 51

1. Use the formula to multiply the following.

(i) $(x + y)\,(x - y)$

Solution:

$(x + y)\,(x - y) = (x)^2 - (y)^2$

$= x^2 - y^2$

(ii) $(3x - 5)(3x + 5)$

Solution:

$(3x - 5)(3x + 5) = (3x)^2 - (5)^2$

$= 9x^2 - 25$

(iii) $(a + 6)(a - 6)$

Solution:

$(a + 6)(a - 6) = (a)^2 - (6)^2$

$= a^2 - 36$

(iv) $\left(\frac{x}{5} + 6\right)\left(\frac{x}{5} - 6\right)$

Solution:

$\left(\frac{x}{5} + 6\right)\left(\frac{x}{5} - 6\right) = \left(\frac{x}{5}\right)^2 - (6)^2$

$= \frac{x^2}{25} - 36$

2. Use the formula to find the values.

(i) 502×498

Solution:

$502 = 500 + 2, \quad 498 = 500 - 2$

$\therefore 502 \times 498 = (500 + 2) \times (500 - 2)$

$= (500)^2 - (2)^2$

$= 250000 - 4$

$= 249996$

(ii) 97×103

Solution:

$97 = 100 - 3, 103 = 100 + 3$

$\therefore 97 \times 103 = (100 - 3)(100 + 3)$

$= (100)^2 - (3)^2$

$= 10000 - 9$

$= 9991$

(iii) 54×46

Solution:

$54 = 50 + 4, 46 = 50 - 4$

$\therefore 54 \times 46 = (50 + 4)(500 - 4)$

$= (50)^2 - (4)^2$

$= 2500 - 16$

$= 2484$

(iv) 98×102

Solution:

$98 = 100 - 2, 102 = 100 + 2$

$\therefore 98 \times 102 = (100 - 2)(100 + 2)$

$= (100)^2 - (2)^2$

$= 10000 - 4$

$= 9996$

Practice Set 52

◘ Factorise the following expressions and write them in the product form.

(i) $201a^3b^2$

Solution: $201 = 3 \times 67$

$\therefore 201a^3b^2 = 3 \times 67 \times a \times a \times a \times b \times b$

(ii) $91xyt^2$

Solution: $91 = 7 \times 13$

$\therefore 91xyt^2 = 7 \times 13 \times x \times y \times t \times t$

(iii) $24a^2b^2$

Solution: $24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$

$\therefore 24a^2b^2 = 2 \times 2 \times 2 \times 3 \times a \times a \times b \times b$

Practice Set 53

◘ Factorise the following expressions.

(i) $p^2 - q^2$

Solution:

$p^2 - q^2 = (p)^2 - (q)^2 = (p + q)(p - q)$

$\therefore p^2 - q^2 = (p + q)(p - q)$

(ii) $4x^2 - 25y^2$

Solution:

$4x^2 - 25y^2 = (2x)^2 - (5y)^2 = (2x + 5y)(2x - 5y)$

$\therefore 4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$

(iii) $y^2 - 4$

Solution:

$y^2 - 4 = (y)^2 - (2)^2 = (y + 2)(y - 2)$

$\therefore y^2 - 4 = (y + 2)(y - 2)$

(iv) $p^2 - \frac{1}{25}$

Solution:

$$p^2 - \frac{1}{25} = (p)^2 - \left(\frac{1}{5}\right)^2$$

$$= \left(p + \frac{1}{5}\right)\left(p - \frac{1}{5}\right)$$

$$\therefore p^2 - \frac{1}{25} = \left(p + \frac{1}{5}\right)\left(p - \frac{1}{5}\right)$$

(v) $9x^2 - \frac{1}{16}y^2$

Solution:

$$9x^2 - \frac{1}{16}y^2 = (3x)^2 - \left(\frac{1}{4}y\right)^2$$

$$= \left(3x + \frac{1}{4}y\right)\left(3x - \frac{1}{4}y\right)$$

$$\therefore 9x^2 - \frac{1}{16}y^2 = \left(3x + \frac{1}{4}y\right)\left(3x - \frac{1}{4}y\right)$$

(vi) $x^2 - \frac{1}{x^2}$

Solution:

$$x^2 - \frac{1}{x^2} = (x)^2 - \left(\frac{1}{x}\right)^2$$

$$= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$\therefore x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

(vii) $a^2b - ab$

Solution:

$a^2b - ab = ab(a - 1)$

$\therefore\ a^2b - ab = ab(a - 1)$

(viii) $4x^2y - 6x^2$

Solution:

$4x^2y - 6x^2 = 2 \times 2 \times x^2y - 2 \times 3 \times x^2$

$= 2x^2(2y - 3)$

$\therefore\ 4x^2y - 6x^2 = 2x^2(2y - 3)$

(ix) $\frac{1}{2}y^2 - 8z^2$

Solution:

$\frac{1}{2}y^2 - 8z^2 = \frac{1}{2}y^2 - \frac{1}{2} \times 16z^2$ $\quad \ldots\ldots\left(8 = 16 \times \frac{1}{2}\right)$

$= \frac{1}{2}(y^2 - 16z^2)$

$= \frac{1}{2}[(y)^2 - (4z)^2]$

$= \frac{1}{2}(y + 4z)(y - 4z)$

$\therefore\ \frac{1}{2}y^2\ 8z^2 = \frac{1}{2}(y + 4z)(y - 4z)$

(x) $2x^2 - 8y^2$

Solution:

$2x^2 - 8y^2 = 2\,(x^2 - 4y^2)$

$= 2\,[(x)^2 - (2y)^2]$

$= 2\,(x + 2y)\,(x - 2y)$

$\therefore\ 2x^2 - 8y^2 = 2\,(x + 2y)\,(x - 2y)$

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