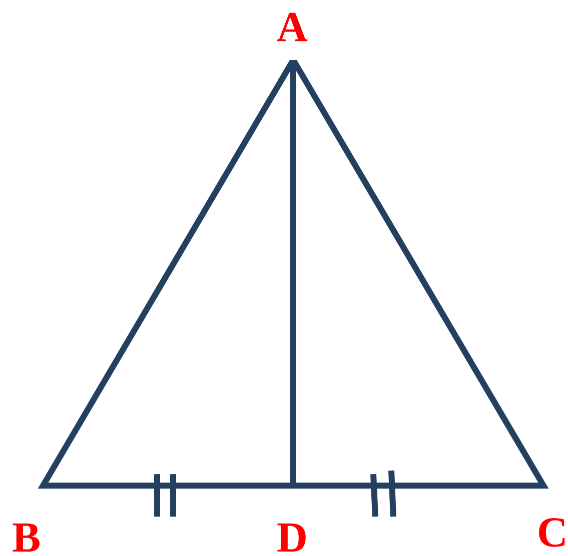
CHAPTER - 2

PYTHAGORAS THEOREM

LONG QUESTIONS

Q. 1 (jeevandeep 188)

In the given figure, line AD is the median of ΔABC. $AB^2 + AC^2 = 160$ & BC = 8. Then find the length of the median.



SOLUTION:

In Δ ABC, line AD is median, Point D is centre point of line BC

$\therefore$ **BD = DC =** $\frac{BC}{2} = \frac{8}{2} = 4$

By Apollonius theorem,

$AB^2 + AC^2 = 2BD^2 + 2AD^2$

$\therefore 160 = 2(4)^2 + 2AD^2$

$\therefore 160 = 32 + 2AD^2$

$\therefore$ 160 - 32 = $2AD^2$

$\therefore$ 128 = $2AD^2$

$\therefore$ $AD^2 = \frac{128}{2} = 64$

$\therefore$ $AD^2 = 64$

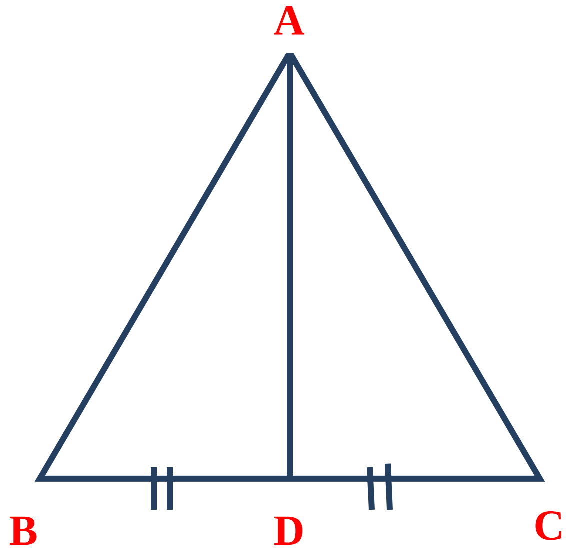
$\therefore$ AD $=\sqrt{64}$

$\therefore$ AD = 8

Ans.: Length of the median is 8 units.

Q. 2 (jeevandeep 182)

In the given figure, line AD is the median of Δ ABC. $AB^2 + AC^2$= 410 & BC = 12. Then find the length of the median.



SOLUTION:

In Δ ABC, line AD is median, point D is centre point of line BC.

BD = DC = $\frac{BC}{2} = \frac{12}{2}$ **= 6**

By Apollonius theorem,

$AB^2 + AC^2 = 2BD^2 + 2AD^2$

∴ $410 = 2(6)^2 + 2AR^2$

∴ $410 = 72 + 2AD^2$

∴ $410 - 72 = 2AD^2$

∴ $338 = 2AD^2$

∴ $AD^2 = \frac{338}{2} = 169$

∴ $AD^2 = 169$

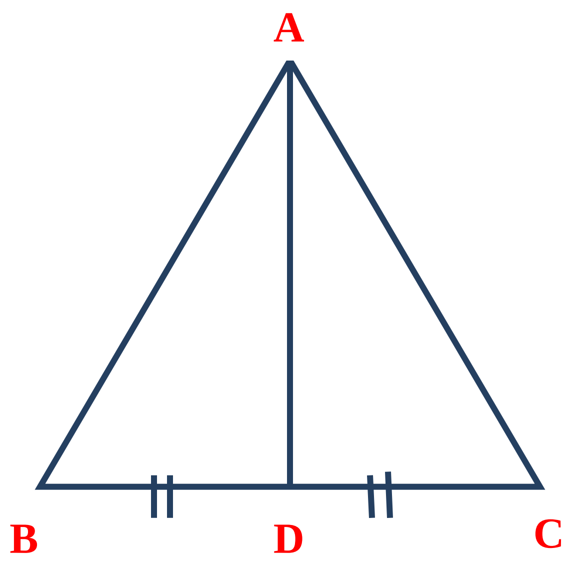
∴ $AD = \sqrt{169}$

∴ **AD =13**

Ans.: Length of the median is 8 units.

Q. 3 (jeevandeep 154)

In the given figure, line AD is the median of Δ ABC. $AB^2 + AC^2$= 292 & AD = 11. Then find the length BC.



SOLUTION:

In Δ ABC, line AD is median, point D is centre point of line BC.

BD = DC = $\frac{BC}{2}$

By Apollonius theorem,

$AB^2 + AC^2 = 2BD^2 + 2AD^2$

$\therefore$ 292 = $2BD^2 + 2(11)^2$

$\therefore$ 292 = $242 + 2BD^2$

$\therefore$ 292 - $242 = 2BD^2$

$\therefore$ 50 = $2BD^2$

$\therefore$ $BD^2 = \frac{50}{2} = 25$

$\therefore$ $BD^2 = 25$

$\therefore$ BD = $\sqrt{25}$

$\therefore$ BD = 5

$\therefore$ BC = 2 BD

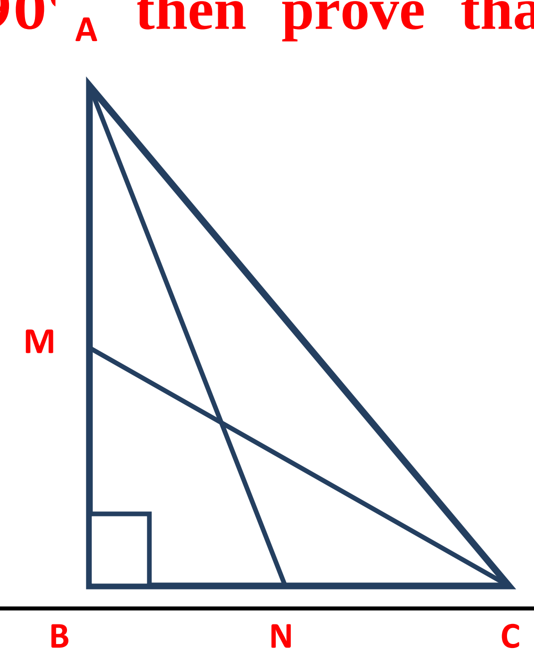
$\therefore$ BC = 2 X 5

$\therefore$ BC = 10 units

Ans.: Length of BC is 10 units.

Q. 4 (jeevandeep 130)

In the given figure, line AN & line CM are the medians of Δ ABC, $\angle$ B $= 90^{\circ}$ then prove that $4(AN^2 + CM^2) = 5AC^2$



SOLUTION:

Line AN and CM are medians of Δ ABC

AM = MB = $\frac{1}{2}$ AB & BN = CN = $\frac{1}{2}$ BC

In Δ ABC, by Pythagoras theorem,

$AN^2 = AB^2 + BN^2$

$\therefore AN^2 = AB^2 + \left(\frac{1}{2}BC\right)^2$

$\therefore AN^2 = AB^2 + \frac{1}{4}BC^2$ ….. (1)

In Δ MBC by Pythagoras theorem,

$CM^2 = BC^2 + MB^2$

$CM^2 = BC^2 + \frac{1}{4}AB^2$ …. (2)

By (1) & (2),

$AN^2 + CM^2 = AB^2 + \frac{1}{4}BC^2 + BC^2 + \frac{1}{4}AB^2$

$\therefore AN^2 + CM^2 = \frac{5}{4}AB^2 + \frac{5}{4}BC^2$

$\therefore 4\left(AN^2 + CM^2\right) = 5(AB^2 + BC^2)$ …. (3)

In Δ ABC by Pythagoras theorem,

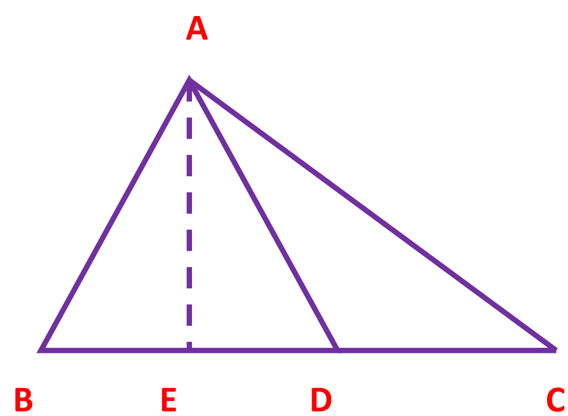
$AB^2 + BC^2 = AC^2$ …. (4)

From (3) & (4),

$4(AN^2 + CM^2) = 5AC^2$

Q. 5 (jeevandeep 103)

In Δ ABC, line AD is the median of triangle then prove that $AB^2 + AC^2 = 2AD^2 + 2BD^2$



SOLUTION:

In Δ ABC, if ∠ ADB < 90^0

By Pythagoras theorem,

$AB^2 = AD^2 + BD^2 - 2\text{ BD x DE}$ …. (1)

Now in Δ ADC ∠ ADC > 90^0

By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2 - 2\ DC \times DE$$

$$AC^2 = AD^2 + BD^2 - 2\ BD \times DE \qquad \ldots. (2)$$

(as DC = BD)

By addition of (1) & (2),

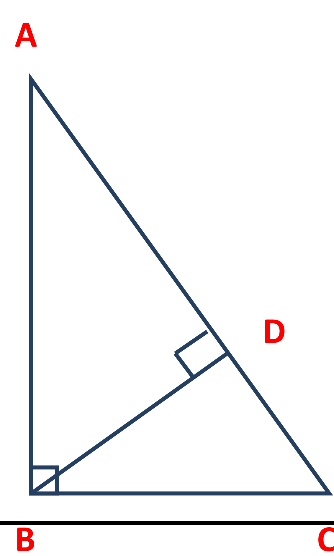
$$AB^2 + AC^2 = 2AD^2 + 2BD^2$$

Proved that in Δ ABC, if AD is median then,

$$AB^2 + AC^2 = 2AD^2 + 2BD^2$$

Q. 6

In the given figure, Δ ABC is a right angle triangle. ∠ ABC = 90^o, seg BD ⊥ hypotenuse AC such that A-D-C. Prove that BD^2 = AD x DC



SOLUTION:

In Δ ABC, $\angle$ ABC = 90^{o}

Seg BD $\perp$ hypotenuse AC

$\therefore$ Δ ADB ~ Δ BDC … (Similarity of right angled triangles)

$\therefore$ $\frac{AD}{BD} = \frac{BD}{DC}$ … (Corresponding sides of similar triangles are in proportion)

$\therefore$ BD^2 = AD x DC

Q. 7

Identify with reason, whether the following is a Pythagorean triplet: (3, 5, 4)

SOLUTION:

$3^2 = 9$, $5^2 = 25$, $4^2 = 16$

$\therefore$ $3^2 + 4^2 = 9 + 16 = 25$

$\therefore$ $3^2 + 4^2 = 5^2$

The square of the largest number is equal to the sum of the squares of the other two numbers.

Ans.: (3, 5, 4) is a Pythagorean triplet.

Q. 8

Identify with reason, whether the following is a Pythagorean triplet: (10, 24, 27)

SOLUTION:

$10^2 = 100$, $24^2 = 576$, $27^2 = 729$

$10^2 + 24^2 = 100 + 576 = 676$

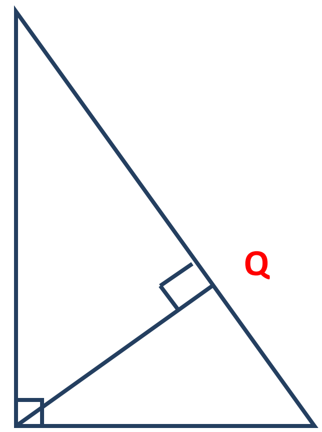
$10^2 + 24^2 \neq 27^2$

The square of the largest number is not equal to the sum of the squares of the other two numbers.

Ans.: (10, 24, 27) is not a Pythagorean triplet.

Q. 9

In the given figure, $\angle$ MNP = 90º, seg NQ $\perp$ seg MP, MQ = 9, QP = 4, find NQ.



SOLUTION:

In Δ MNP, ∠ MNP = 90º … (Given)

seg NQ ⊥ hypotenuse MP … (Given)

∴ By property of geometric mean

$NQ^2 = MQ \times PQ$

∴ $NQ^2 = 9 \times 4$

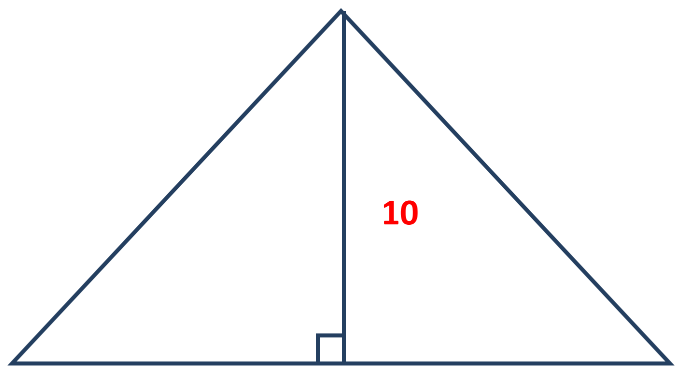
∴ $NQ^2 = 36$

∴ NQ = 6 … (Taking square roots on both the sides)

Ans.: NQ = 6

Q. 10

In figure, ∠ QPR = 90º. Seg PM ⊥ seg QR and Q – M – R. PM = 10, QM = 8; find QR.



SOLUTION:

In Δ QPR, $\angle$ QPR = 90º … (Given)

seg PM $\perp$ hypotenuse QR … (Given)

$\therefore$ By property of Geometric mean,

$PM^2 = QM \times MR$

$\therefore 10^2 = 8 \times MR$

$\therefore MR = \frac{100}{8}$

$\therefore$ MR = 12.5

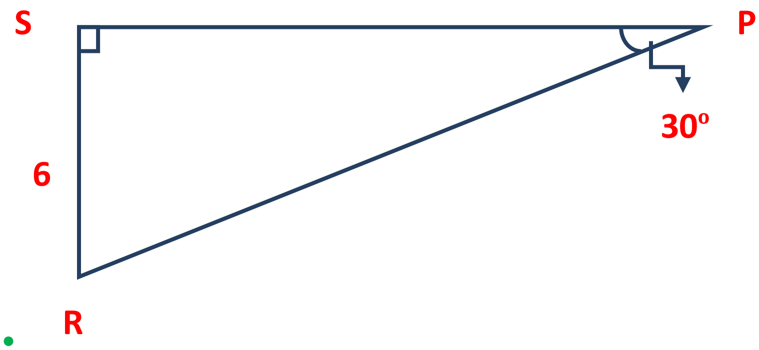
QR = QM + MR … (Q – M – R)

$\therefore$ QR = 8 + 12.5

$\therefore$ QR = 20.5

Q. 11

See figure. Find RP and PS using the information given in Δ PSR.



SOLUTION:

In Δ PSR, ∠ PSR = 90º and ∠ SPR = 30º

∴ ∠ SRP = 60º … (Remaining angle of a triangle)

∴ Δ PSR is a 30º - 60º - 90º triangle

∴ By 30º - 60º - 90º triangle theorem,

$SR = \frac{1}{2} RP$ … (Side opposite to 30º)

∴ $6 = \frac{1}{2} \times RP$

∴ RP = 6 x 2

∴ RP = 12

PS = $\frac{\sqrt{3}}{2}$ RP **… (Side opposite to 60º)**

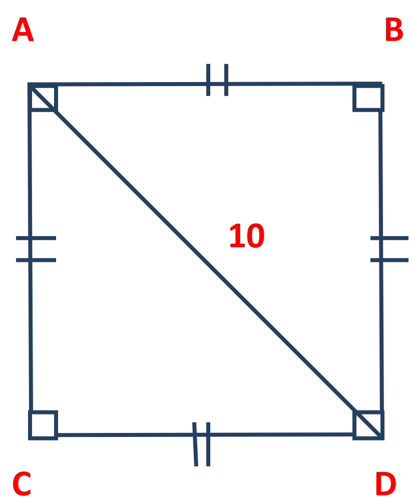
$\therefore$ PS = $\frac{\sqrt{3}}{2} \times 12$

$\therefore$ PS = $6\sqrt{3}$

Ans.: RP = 12 and PS = $6\sqrt{3}$

Q. 12

Find the side and perimeter of a square whose diagonal is 10 cm.



SOLUTION:

Let □ ABCD be the given square.

AC = 10 cm

Let the side of the square be *x* cm.

$\therefore$ AB = BC = x cm

In Δ ABC, ∠ ABC = 90º   … (Angle of a square)

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

∴ $10^2 = x^2 + x^2$

∴ $100 = 2x^2$

∴ $x^2 = \frac{100}{2}$

∴ $x^2 = 50$

∴ $x = 5\sqrt{2}$

∴ AB = $5\sqrt{2}$ cm

∴ Side of square is $5\sqrt{2}$ cm.

Perimeter of a square = 4 x side

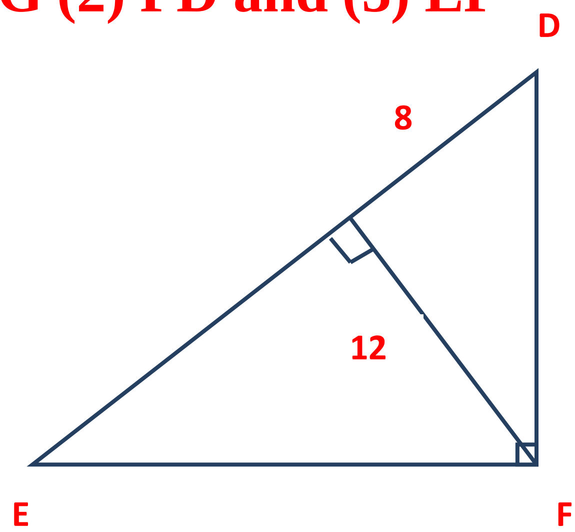
= 4 x $5\sqrt{2}$

= $20\sqrt{2}$ cm

Ans.: Side of a square is $5\sqrt{2}$ cm and its perimeter is $20\sqrt{2}$ cm

Q. 13

In figure, $\angle$ DFE is 90^o, FG $\perp$ ED. If GD = 8, FG = 12, find (1) EG (2) FD and (3) EF



SOLUTION:

(1) In Δ DFE, $\angle$ DFE = 90^o … (Given)

Seg FG $\perp$ hypotenuse DE … (Given)

$\therefore$ by property of geometric mean,

$FG^2 = DG \times EG$

$\therefore$ $12^2 = 8 \times EG$

$\therefore$ $EG = \frac{12 \times 12}{8}$

$\therefore$ EG = 18

(2) In Δ DGF, $\angle$ DGF = 90º … (Given)

$\therefore$ By Pythagoras theorem,

$FD^2 = DG^2 + GF^2$

$\therefore FD^2 = 8^2 + 12^2$

$\therefore FD^2 = 64 + 144$

$\therefore FD^2 = 208$

$\therefore FD = 4\sqrt{13}$

(3) In Δ EGF, $\angle$ EGF = 90º

$\therefore$ By Pythagoras theorem,

$EF^2 = EG^2 + GF^2$

$\therefore EF^2 = 18^2 + 12^2$

$\therefore EF^2 = 324 + 144$

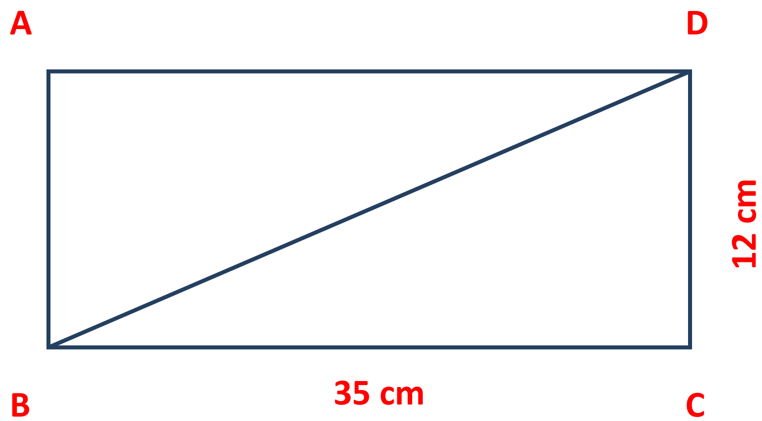
$\therefore EF^2 = 468$

$\therefore EF = 6\sqrt{13}$

Ans.: EG = 18, FD = $4\sqrt{13}$ and EF = $6\sqrt{13}$

Q. 14

Find the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.



SOLUTION:

Let □ ABCD be the given rectangle.

BC = 35 cm and CD = 12 cm

In Δ BCD, ∠ BCD = 90° … (Angle of a rectangle)

∴ By Pythagoras theorem,

$BD^2 = BC^2 + CD^2$

$\therefore BD^2 = 35^2 + 12^2$

$\therefore BD^2 = 1225 + 144$

$\therefore BD^2 = 1369$

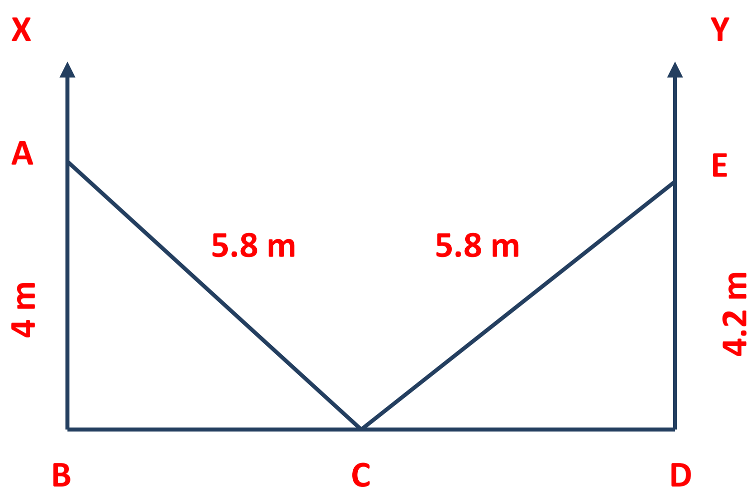
∴ BD = 37 cm

Ans.: The diagonal of rectangle is 37 cm

Q. 15

Walls of two buildings on either side of a street are parallel to each other. A ladder 5.8 m long is placed on the street such that its top just reaches the window of a building at a height of 4 m. On turning the ladder over to the other side of the street, its top touches the window of the other building at a height 4.2 m. Find the width of the street.

SOLUTION:



In the figure, seg XB and seg YD represent the walls of two buildings on either side of a street BD. Seg AC represents the first position of the ladder and seg CE represents the second position of the ladder.

AC = CE = 5.8 m, AB = 4 m and DE = 4.2 m

In Δ ABC, ∠ ABC = 90º

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

∴ $5.8^2 = 4^2 + BC^2$

∴ $33.64 = 16 + BC^2$

∴ $BC^2 = 33.44 – 16$

∴ $BC^2 = 17.64$

∴ BC = 4.2 m

In Δ EDC, ∠ EDC = 90º

∴ By Pythagoras theorem,

$CE^2 = DE^2 + CD^2$

$\therefore$ $5.8^2 = 4.2^2 + CD^2$

$\therefore$ $CD^2 = 5.8^2 – 4.2^2$

$\therefore$ $CD^2 = 33.64 – 17.64$

$\therefore$ $CD^2 = 16$

$\therefore$ CD = 4 m

BD = BC + CD

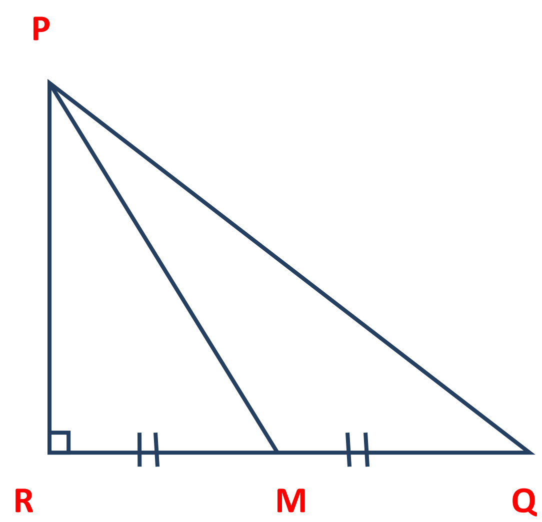
$\therefore$ BD = 4.2 + 4

$\therefore$ BD = 8.2 m

Ans.: Width of the street is 8.2 m.

Q. 16

In the figure, M is the midpoint of QR. $\angle$ PRQ = 90°, prove that $PQ^2 = 4PM^2 – 3PR^2$



SOLUTION:

In Δ PRQ, ∠ PRQ = 90°   … (Given)

∴ By Pythagoras theorem,

$PQ^2 = PR^2 + QR^2$ … (1)

In Δ PRM, ∠ PRM = 90° … (Given)

∴ By Pythagoras theorem,

$PM^2 = PR^2 + RM^2$ … (2)

$RM = \frac{1}{2} RQ$ … (M is the midpoint of seg RQ) … (3)

$\therefore PM^2 = PR^2 + \left(\frac{1}{2} RQ\right)^2$ … [From (2) and (3)]

$\therefore PM^2 = PR^2 + \frac{1}{4} RQ^2$

Multiplying each term by 4 we get,

$4PM^2 = 4PR^2 + RQ^2$

$\therefore 4PM^2 = 3PR^2 + (PR^2 + RQ^2)$

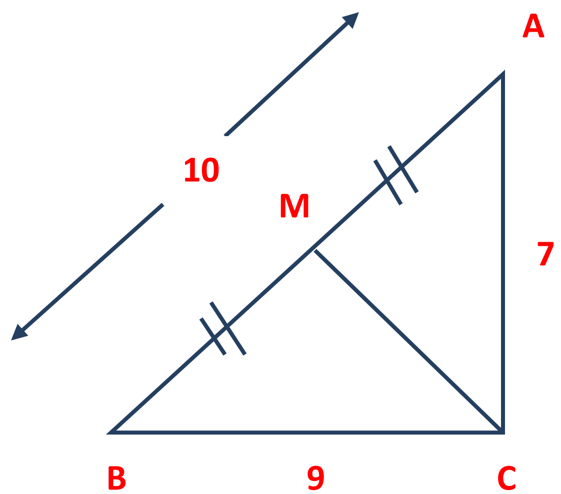
$\therefore 4PM^2 = 3PR^2 + PQ^2$

$\therefore PQ^2 = 4PM^2 - 3PR^2$

Ans : $PQ^2 = 4PM^2 - 3PR^2$

Q. 17

In Δ ABC, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB.



SOLUTION:

Let seg CM be the median drawn from the vertex C to side AB.

∴ M is the midpoint of side AB … (by definition of a median)

∴ $AM = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5$

In Δ ACB, seg CM is the median,

∴ by Appollonius theorem,

$AC^2 + BC^2 = 2CM^2 + 2AM^2$

∴ $7^2 + 9^2 = 2CM^2 + 2(5)^2$

∴ $49 + 81 = 2CM^2 + 50$

∴ $130 - 50 = 2CM^2$

∴ $CM^2 = \frac{80}{2}$

∴ $CM^2 = 40$

∴ $CM = 2\sqrt{10}$

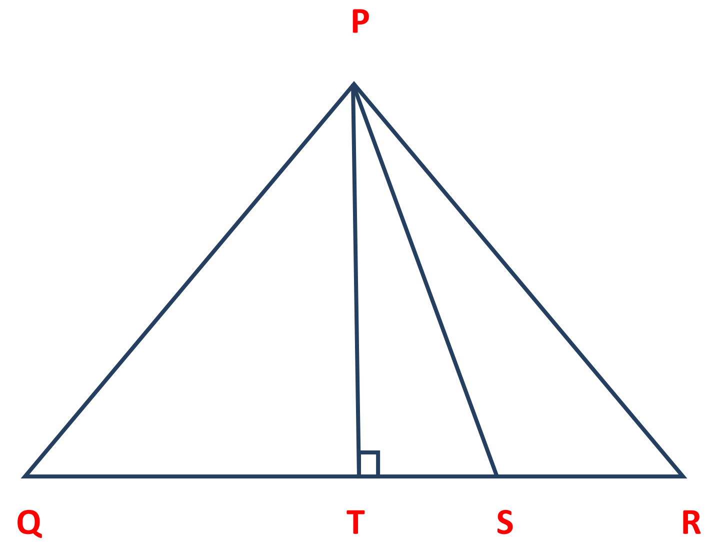
Ans.: Length of the median drawn from point C to AB is $2\sqrt{10}$

Q. 18

In the figure seg PS is the median of Δ PQR and PT ⊥ QR. Prove that:

(1) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

(2) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$



SOLUTION:

Seg PS is the median of Δ PQR … (Given)

∴ QS = SR = $\frac{1}{2}$ QR (S is the midpoint of side QR) .. (1)

In Δ PTS, ∠ PTS = 90º … (Given)

∴ by Pythagoras theorem,

$PS^2 = PT^2 + TS^2$ … (2)

(1) In Δ PTR, ∠ PTR is 90º … (Given)

∴ by Pythagoras theorem,

$PR^2 = PT^2 + TR^2$

∴ $PR^2 = PT^2 + (TS^2 + SR^2)$ … (T – S – R)

$\therefore$ $PR^2 = PT^2 + TS^2 + 2ST.SR + SR^2$ … $[(a + b)^2 = a^2 + 2ab + b^2)]$

$\therefore$ $PR^2 = (PT^2 + TS^2) + 2ST.SR + SR^2$

$\therefore$ $PR^2 = PS^2 + 2ST.\left(\frac{QR}{2}\right) + \left(\frac{QR}{2}\right)^2$ … [From (1) & (2)]

$\therefore$ $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

(2) In Δ PTQ, $\angle$ PTQ is 90^o … (Given)

$\therefore$ by Pythagoras theorem,

$PQ^2 = PT^2 + TQ^2$

$\therefore$ $PQ^2 = PT^2 + (QS^2 - TS^2)$ … (Q – T – S)

$\therefore$ $PQ^2 = PT^2 + QS^2 - 2QS.TS + TS^2$ … $[(a - b)^2 = a^2 - 2ab + b^2)]$

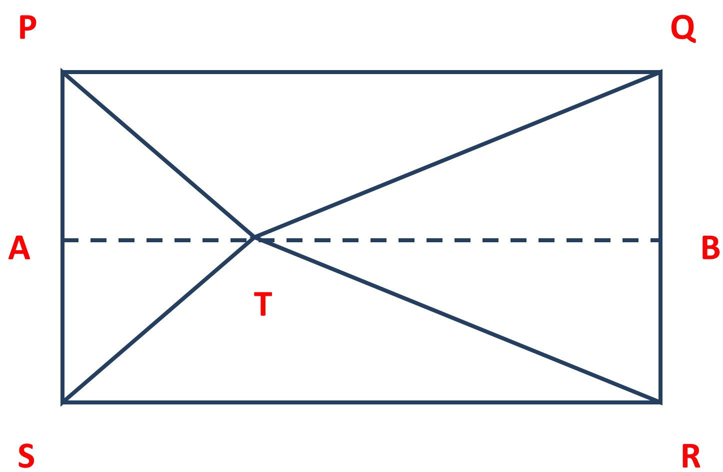
$\therefore$ $PQ^2 = (PT^2 + TS^2) - 2QS.TS + QS^2$

$\therefore$ $PQ^2 = PS^2 - 2TS.\left(\frac{QR}{2}\right) + \left(\frac{QR}{2}\right)^2$ … [From (1) & (2)]

$\therefore$ $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

Q. 19

In figure, point T is in the interior of rectangle PQRS. Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$



SOLUTION:

Seg PS ∥ seg QR … (Opposite sides of rectangle)

∴ seg AS ∥ seg BR … (P – A – S and Q – B – R)

also seg AB ∥ seg SR … (Construction)

∴ □ ASRB is a parallelogram … (by definition)

∠ S = 90º … (Angle of rectangle PSRQ)

∴ □ ASRB is a rectangle … (A parallelogram is a rectangle, if one of its angles is a right angle)

$\angle$ SAB = $\angle$ ABR = 90º … (Angles of a rectangle)

$\therefore$ seg TA $\perp$ side PS and seg TB $\perp$ side QR … (1)

AS = BR … (Opp. sides of rectangle are equal … (2)

Similarly, we can prove AP = BQ … (3)

In Δ TAS, $\angle$ TAS = 90º … [From (1)]

$\therefore$ by Pythagoras theorem,

$TS^2 = TA^2 + AS^2$ … (4)

In Δ TBQ, $\angle$ TBQ = 90º

$\therefore$ by Pythagoras theorem,

$TQ^2 = TB^2 + BQ^2$ … (5)

Adding (4) and (5) we get,

$TS^2 + TQ^2 = TA^2 + AS^2 + TB^2 + BQ^2$ … (6)

In Δ TAP, $\angle$ TAP = 90º

$\therefore$ by Pythagoras theorem,

$TP^2 = TA^2 + AP^2$ … (7)

In Δ TBR, $\angle$ TBR = 90º

∴ by Pythagoras theorem,

$TR^2 = TB^2 + BR^2$ … (8)

Adding (7) and (8) we get,

$TP^2 + TR^2 = TA^2 + AP^2 + TB^2 + BR^2$ … (6)

∴ $TP^2 + TR^2 = TA^2 + BQ^2 + TB^2 + AS^2$ … [From (2) and (3)] …. (9)

∴ From (6) and (9) we get,

$TS^2 + TQ^2 = TP^2 + TR^2$

Q. 20

Do sides 7 cm, 24 cm, 25 cm form a right angled triangle? Give reason.

SOLUTION:

$7^2 = 49$, $24^2 = 576$, $25^2 = 625$

$7^2 + 24^2 = 49 + 576 = 625$

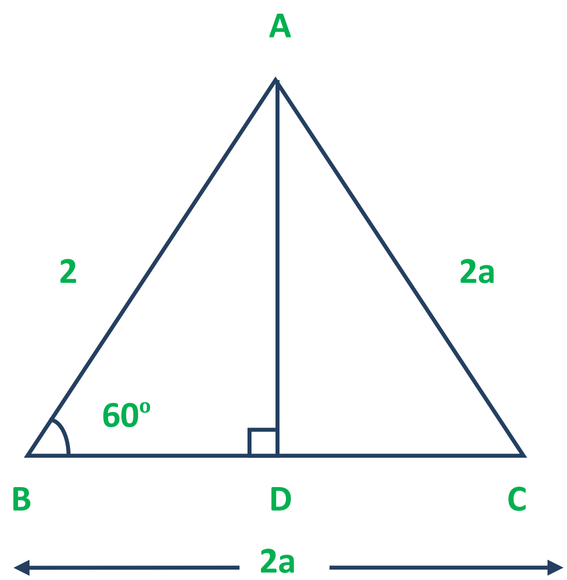
∴ $7^2 + 24^2 = 25^2$

∴ By converse Pythagoras theorem, 7 cm, 24 cm, 25 cm form a right angled triangle.

Q. 21

Find the height of an equilateral triangle having side 2*a*.

SOLUTION:



Let the given equilateral triangle be ABC and seg AD be the height.

$\angle$ B = 60º … (Angle of an equilateral triangle) … (1)

In Δ ADB,

$\angle$ ABD + $\angle$ BAD + $\angle$ ADB = 180º .. (Sum of all angles of a triangle is 180º)

∴ 60 + ∠ BAD + 90 = 180º … [From given and (1)]

∴ ∠ BAD + 150 = 180º

∴ ∠ BAD = 180º - 150º

∴ ∠ BAD = 30º

∴ Δ ADB is a 30º - 60º - 90º triangle

∴ By 30º - 60º - 90º triangle theorem,

AD = $\frac{\sqrt{3}}{2}$ x AB … (Side opposite to 60º)

∴ AD = $\frac{\sqrt{3}}{2}$ x 2a

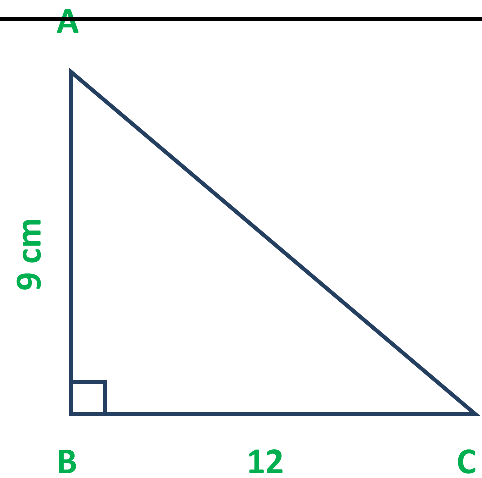
∴ AD = $\sqrt{3}a$

Ans.: Height of an equilateral triangle is $\sqrt{3}a$

Q. 22

Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.

SOLUTION:



Let Δ ABC be given right angled triangle

∠ B = 90º, AB = 9 cm and BC = 12 cm

In Δ ABC, ∠ ABC = 90º

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 9^2 + 12^2$

$\therefore AC^2 = 81 + 144$

$\therefore AC^2 = 225$

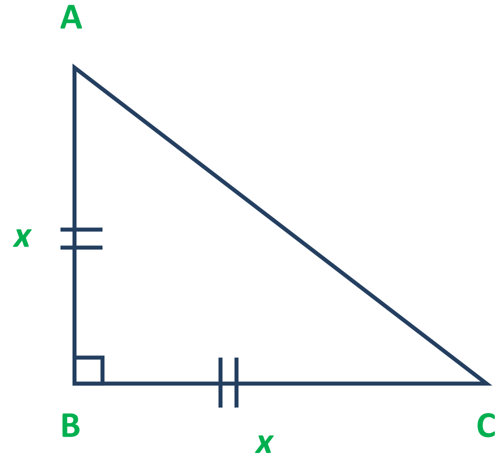
$\therefore AC = 15$ cm

Ans.: Length of hypotenuse of right angled triangle is 15 cm.

Q. 23

Side of an isosceles right angled triangle is x. Find its hypotenuse.

SOLUTION:



Let Δ ABC be given isosceles right angled triangle

AB = BC = x, ∠ B = 90°

In Δ ABC, ∠ ABC = 90° … (Given)

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = x^2 + x^2$

$\therefore AC^2 = 2x^2$

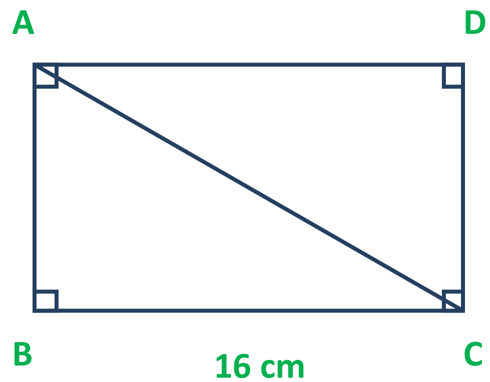
$\therefore AC = \sqrt{2}x$

Ans.: Length of hypotenuse AC is $\sqrt{2}x$

Q. 24

Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq. cm

SOLUTION:



Let □ ABCD be the given rectangle.

BC = 16 cm and A (□ ABCD) = 192 sq. cm

Area of a rectangle = length x breadth

∴ A (□ ABCD) = AB x BC

∴ 192 = AB x 16

∴ AB = $\frac{192}{16}$

∴ AB = 12 cm

In Δ ABC ∠ ABC = 90º … (Angle of a rectangle)

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 12^2 + 16^2$

$\therefore AC^2 = 144 + 256$

$\therefore AC^2 = 400$

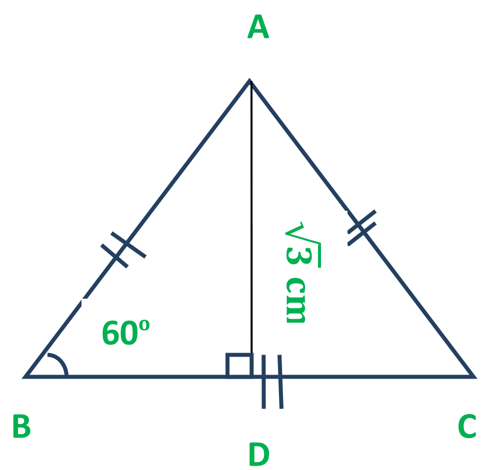
$\therefore AC = 20$ cm

Ans.: Diagonal of the rectangle is 20 cm.

Q. 25

Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

SOLUTION:



Let Δ ABC be given equilateral triangle and seg AD be its height.

AD = $\sqrt{3}$ cm

$\angle$ B = 60º … (Angle of an equilateral triangle) … (1)

In Δ ADB,

$\angle$ ADB + $\angle$ ABD + $\angle$ BAD = 180º … (Sum of all angles of a triangle is 180º)

$\therefore$ 90º + 60º + $\angle$ BAD = 180º

$\therefore$ 150 + $\angle$ BAD = 180º

$\therefore$ $\angle$ BAD = 30º

$\therefore$ Δ ADB is a 30º - 60º - 90º triangle

$\therefore$ By 30º - 60º - 90º triangle theorem,

AD = $\frac{\sqrt{3}}{2}$ AB … (Side opposite to 60º)

$\therefore$ $\sqrt{3} = \frac{\sqrt{3}}{2}$ AB

$\therefore$ AB = $\frac{2 \times \sqrt{3}}{\sqrt{3}}$

$\therefore$ AB = 2 cm

$\therefore$ Side of an equilateral triangle is 2 cm

Perimeter of Δ ABC = 3 x side

= 3 x AB

= 3 x 2

= 6 cm

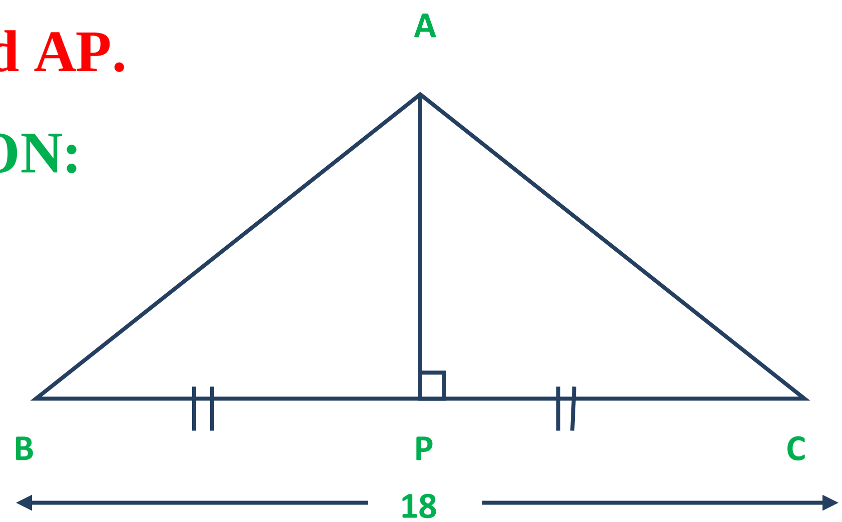
Perimeter of equilateral triangle is 6 cm.

Ans.: Side of an equilateral triangle is 2 cm and its perimeter is 6 cm.

Q. 26

In Δ ABC, seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$, find AP.

SOLUTION:



BC = 18 …. (Given)

BP = $\frac{1}{2}$ BC …. (P is the midpoint of seg BC)

$\therefore$ BP = $\frac{1}{2}$ x 18

$\therefore$ BP = 9

In Δ ABC, seg AP is the median … (Given)

$\therefore$ By Appollonius theorem,

$AB^2 + AC^2 = 2AP^2 + 2BP^2$

$\therefore 260 = 2AP^2 + 2(9)^2$

$\therefore 260 = 2AP^2 + 162$

$\therefore 2AP^2 = 260 - 162$

$\therefore 2AP^2 = 98$

$\therefore AP^2 = \frac{98}{2}$

$\therefore AP^2 = 49$

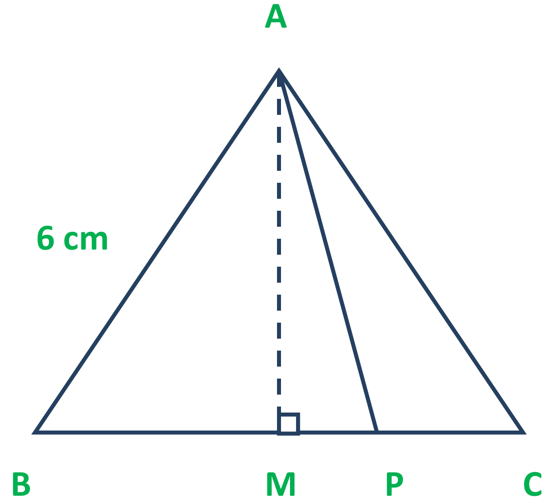
$\therefore$ AP = 7

Q. 27

Δ ABC is an equilateral triangle. Point P is on base BC such that PC = $\frac{1}{3}$ BC. If AB = 6 cm, find AP.

SOLUTION:

Draw seg AM ⊥ side BC such that B – M – C



Δ ABC is an equilateral triangle.

∴ AB = BC = AC = 6 cm … (Sides of an equilateral triangle) … (1)

∠ C = 60° … (Angle of an equilateral triangle) … (2)

In Δ AMC,

∠ AMC + ∠ ACM + ∠ MAC = 180° … (Sum of all angles of a triangle is 180°)

∴ 90° + 60° + ∠ MAC = 180° … [From (1) and (2)]

$\therefore$ 150 + $\angle$ MAC = 180º

$\therefore$ $\angle$ MAC = 180º – 150º

$\therefore$ $\angle$ MAC = 30º

$\therefore$ Δ AMC is a 30º - 60º - 90º triangle

$\therefore$ By 30º - 60º - 90º triangle theorem,

AM = $\frac{\sqrt{3}}{2}$ AC ... (Side opposite to 60º)

$\therefore$ AM = $\frac{\sqrt{3}}{2} \times 6$

$\therefore$ AM = $3\sqrt{3}$ cm

MC = $\frac{1}{2}$ BC ... (Side opposite to 30º)

$\therefore$ MC = $\frac{1}{2} \times 6$ ... [From (1)]

$\therefore$ MC = 3 cm

PC = $\frac{1}{3}$ BC ... (Given)

$\therefore$ PC = $\frac{1}{3} \times 6$ ... [From (1)]

$\therefore$ PC = 2 cm

MP + PC = MC (M – P – C)

∴ MP + 2 = 3

∴ MP = 3 – 2

∴ MP = 1 cm

In Δ AMP, ∠ AMP = 90º … (Construction)

∴ By Pythagoras theorem,

$AP^2 = AM^2 + MP^2$

$\therefore AP^2 = \left(3\sqrt{3}\right)^2 + 1^2$

$\therefore AP^2 = 27 + 1$

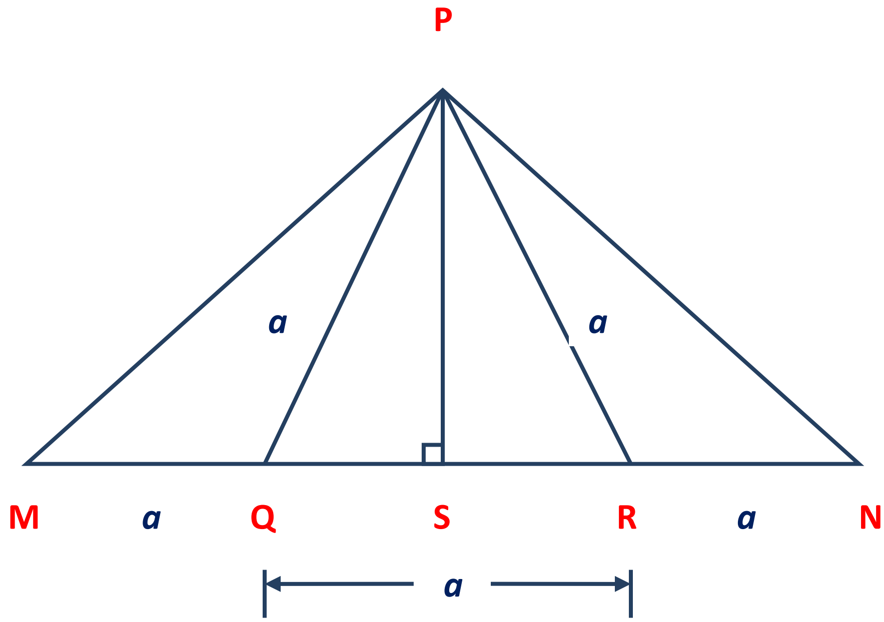
$\therefore AP^2 = 28$

$\therefore AP^2 = 4 \times 7$

$\therefore AP = 2\sqrt{7}$ cm

Q. 28

From the information given in the figure prove that $PM = PN = \sqrt{3}a$



Proof:

PQ = PR = QR = QM = RN = a … (Given) … (1)

Consider Δ PMR,

QM = QR … [From (1)]

∴ seg PQ is the median

∴ By Appollonius theorem,

$PM^2 + PR^2 = 2PQ^2 + 2QM^2$

∴ $PM^2 + a^2 = 2a^2 + 2a^2$ … [From (1)]

∴ $PM^2 = 4a^2 – a^2$

∴ $PM^2 = 3a^2$

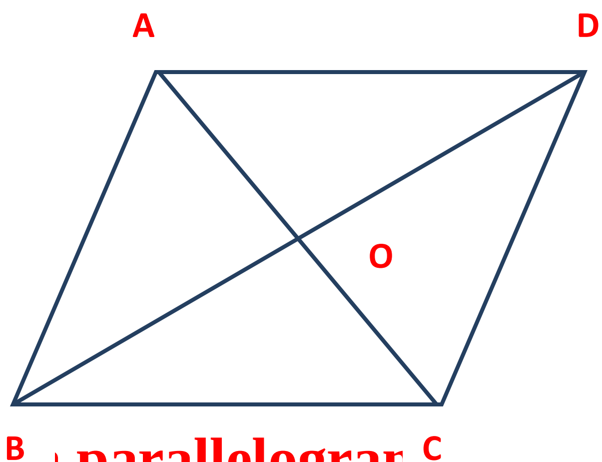
∴ $PM^2 = \sqrt{3}$ a

Similarly, we can prove PN = $\sqrt{3}$ a

$\therefore$ **PM = PN = $\sqrt{3}$ a**

Q. 29

Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.



Given:

(i) □ ABCD is a parallelogram

(ii) Diagonals AC and BD intersect at O

To Prove: $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

Proof:

□ ABCD is a parallelogram … (Given)

$\therefore$ **CD = AB … (1) and**

AD = BC … (2) (Opposite sides of a parallelogram are equal)

Also, AO = OC = $\frac{1}{2}$ AC … (3)

BO = OD = $\frac{1}{2}$ BD … (4) (Diagonals of parallelogram bisect each other)

In Δ ABC, seg BO is the median … (By definition)

∴ By Appollonius theorem,

$AB^2 + BC^2 = 2BO^2 + 2OC^2$

∴ $AB^2 + BC^2 = 2\left(\frac{1}{2}BD\right)^2 + 2\left(\frac{1}{2}AC\right)^2$ … [From (3) and (4)]

∴ $AB^2 + BC^2 = 2 \times \frac{1}{4}BD^2 + 2 \times \frac{1}{4}AC^2$

∴ $AB^2 + BC^2 = \frac{1}{2}BD^2 + \frac{1}{2}AC^2$

Multiplying each term by 2 we get,

$2AB^2 + 2BC^2 = BD^2 + AC^2$

∴ $AB^2 + AB^2 + BC^2 + BC^2 = BD^2 + AC^2$

∴ $AB^2 + CD^2 + BC^2 + AD^2 = BD^2 + AC^2$ … [From (1) and (2)]

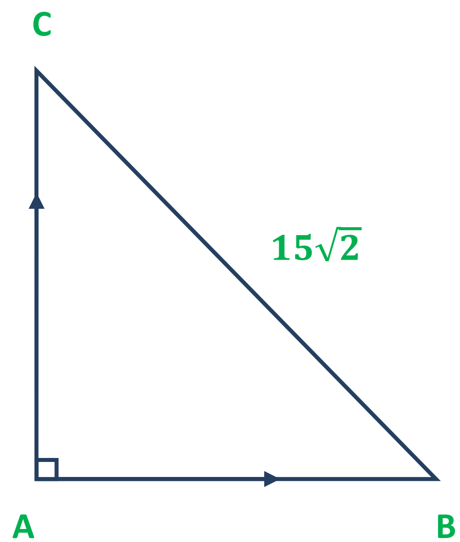
$\therefore\ AB^2 + BC^2 + CD^2 + AO^2 = BD^2 + AC^2$ OR

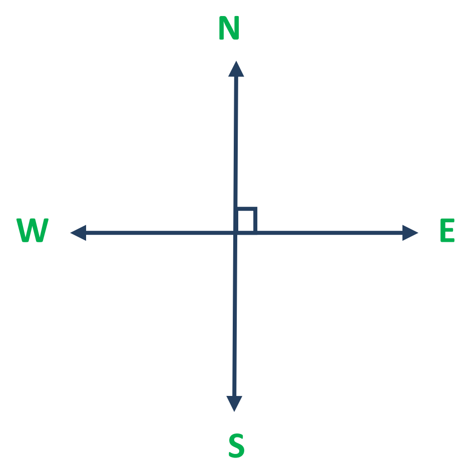
$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

Q. 30

Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After two hours, distance between them was $15\sqrt{2}$ km. Find their speed per hour.

SOLUTION:





In the figure, Point A represents the point from where Pranali and Prasad start walking. Seg AB represents the path of Pranali and seg AC represents the path of Prasad.

After 2 hours, Pranali reaches point B and Prasad reaches point C.

BC = $15\sqrt{2}$ km

Pranali and Prasad are walking at the same speed.

Let the speed be x km/hr.

We know, distance covered = speed x time

∴ distance AB = $x \times 2$

∴ AB = $2x$

also distance AC = $x \times 2$

∴ AC = $2x$ km

In right angled Δ CAB, by Pythagoras theorem,

$BC^2 = AC^2 + AB^2$

∴ $(15\sqrt{2})^2 = (2x)^2 + (2x)^2$

∴ $225 \times 2 = 4x^2 + 4x^2$

∴ $8x^2 = 225 \times 2$

∴ $x^2 = \frac{225 \times 2}{8}$

$\therefore x^2 = \frac{225}{4}$

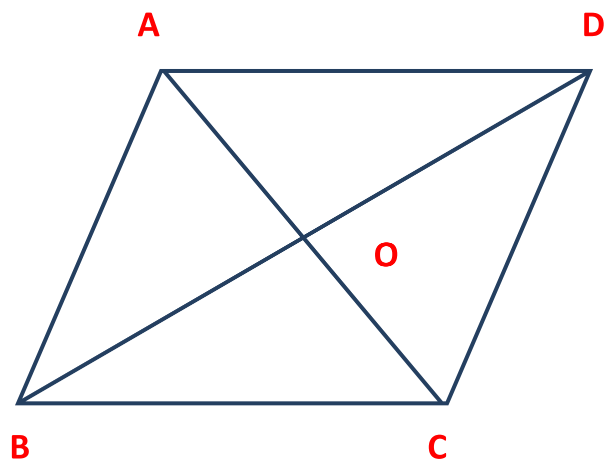
$\therefore x = \frac{15}{2}$

$\therefore x = 7.5$

Ans.: The required speed is 7.5 km/hr.

Q. 31

Sum of the squares of adjacent sides of a parallelogram is 130 sq. cm. and length of one of its diagonals is 14 cm. Find the length of the other diagonal.



SOLUTION:

Let □ ABCD be the given parallelogram. Let diagonals AC and BD intersect at point O.

$AB^2 + BC^2 = 130$ sq. cm and AC = 14 cm

AO = $\frac{1}{2}$ AC … (Diagonals of parallelogram bisect each other)

$\therefore$ AO = $\frac{1}{2} \times 14$

$\therefore$ AO = 7 cm

In Δ ABC, seg BO is the median … (By definition)

$\therefore$ By Appollonius theorem,

$AB^2 + BC^2 = 2BO^2 + 2AO^2$

$\therefore 130 = 2BO^2 + 2(7)^2$

$\therefore 130 = 2BO^2 + 2 \times 49$

$\therefore 130 = 2BO^2 + 98$

$\therefore 2BO^2 = 130 - 98$

$\therefore 2BO^2 = 32$

$\therefore BO^2 = \frac{32}{2}$

$\therefore BO^2 = 16$

∴ BO = 4 cm

BO = $\frac{1}{2}$ BD … (Diagonals of parallelogram bisect each other)

∴ 4 = $\frac{1}{2}$ BD

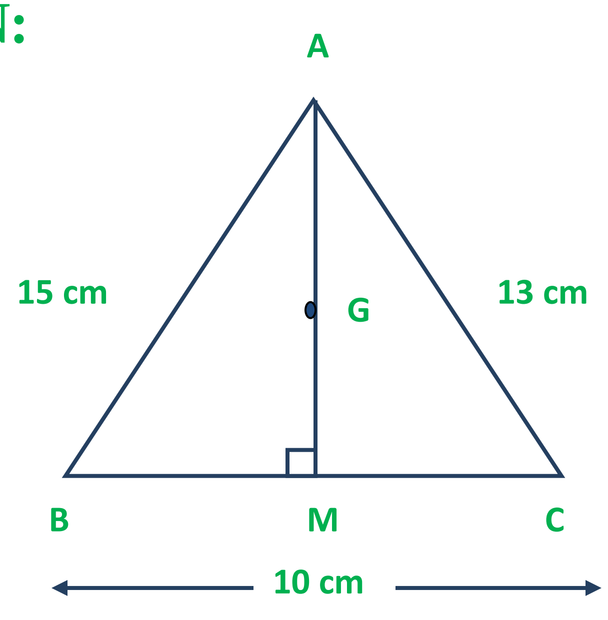
∴ BD = 8 cm

Ans.: Length of other diagonal of parallelogram is 8 cm.

Q. 32

In an isosceles triangle, length of the congruent sides is 13 cm and its base is 10 cm. Find the distance between the vertex opposite the base and the centroid.

SOLUTION:



Let Δ ABC be the given isosceles triangle.

AB = AC = 13 cm, BC = 10 cm

Draw seg AM $\perp$ side BC such that B – M – C

In Δ AMB and Δ AMC,

$\angle$ AMB = $\angle$ AMC = 90º … (Construction)

Hypotenuse AB ≅ hypotenuse AC … (Given)

Side AM ≅ side AM … (Common side)

∴ Δ AMB ≅ Δ AMC … (By hypotenuse side theorem)

∴ seg BM ≅ seg CM … (Congruent side of a congruent triangle) … (1)

∴ BM = CM = $\frac{1}{2}$ BC

∴ BM = CM = $\frac{1}{2}$ x 10

∴ BM = CM = 5 cm

In Δ AMB, $\angle$ AMB = 90º … (Construction)

∴ By Pythagoras theorem,

$AB^2 = AM^2 + BM^2$

$\therefore\ 13^2 = AM^2 + 5^2$

$\therefore\ AM^2 = 13^2 - 5^2$

$\therefore\ AM^2 = 169 - 25$

$\therefore\ AM^2 = 144$

$\therefore$ AM = 12 cm

Now, BM = MC … [From (1)]

$\therefore$ seg AM is also the median … (By definition)

$\therefore$ centroid G lies on it

By centroid property of triangle,

$AG = \frac{2}{3} AM$

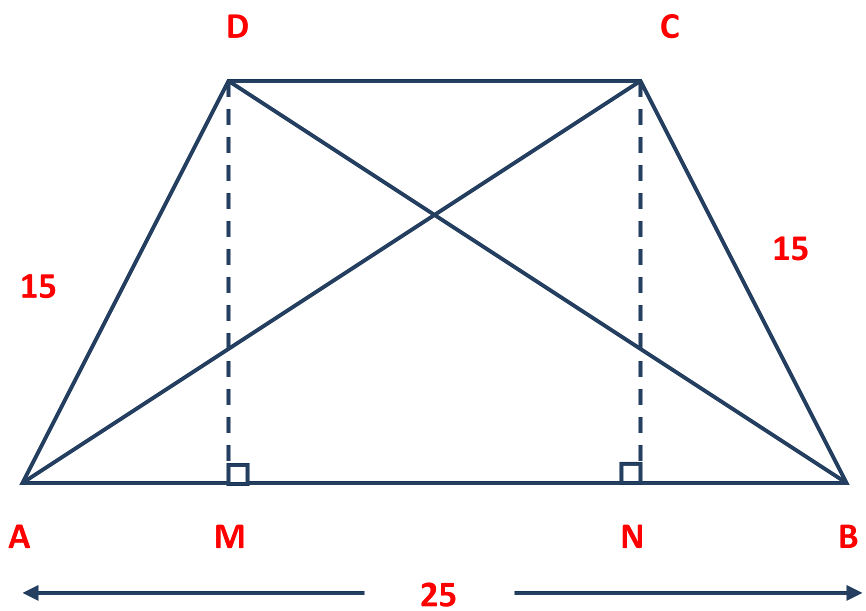
$\therefore\ AG = \frac{2}{3} \times 12$

$\therefore$ AG = 8 cm

Ans.: The distance between the vertex opposite the base and the centroid is 8 cm.

Q. 33

In a trapezium ABCD, seg AB ∥ seg DC, seg BD ⊥ seg AD, seg AC ⊥ seg BC, if AD = 15, BC = 15 and AB = 25. Find A (□ ABCD).



SOLUTION:

In Δ ADB, ∠ ADB is 90º … (Given)

∴ By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\therefore 25^2 = 15^2 + BD^2$

$\therefore BD^2 = 252 - 152$

$\therefore BD^2 = 625 - 225$

$\therefore BD^2 = 400$

$\therefore BD = 20$

Similarly, AC = 20

Draw seg DM $\perp$ side AB such that A – M – B and seg CN $\perp$ side AB such that A – N – B

$A(\Delta ADB) = \frac{1}{2} \times BD \times AD$

$\therefore A(\Delta ADB) = \frac{1}{2} \times 20 \times 15$

$\therefore A(\Delta ADB) = 150$ sq units

also, $A(\Delta ADB) = \frac{1}{2} \times AB \times DM$

$\therefore A(\Delta ADB) = \frac{1}{2} \times 25 \times DM$

$\therefore 150 = \frac{1}{2} \times 25 \times DM$

$\therefore DM = \frac{150 \times 2}{25}$

$\therefore DM = 12$

Similarly, CN = 12

In Δ DMA, ∠ DMA = 90º … (Construction)

∴ By Pythagoras theorem,

$AD^2 = DM^2 + AM^2$

∴ $15^2 = 12^2 + AM^2$

∴ $AM^2 = 225 - 144$

∴ $AM^2 = 81$

∴ AM = 9

Similarly BN = 9

AM + MN + NB = AB (A – M – N and M – N – B)

∴ 9 + MN + 9 = 25

∴ 18 + MN = 25

∴ MN = 25 – 18

∴ MN = 7

In □ DCNM, seg DC ∥ seg MN … (seg DC ∥ seg AB and A – M – N – B)

seg DM ∥ seg CN … (Perpendiculars to same line are parallel)

∴ □ DCNM is a parallelogram … (By definition)

∴ DC = MN = 7 … (Opposite sides of a parallelogram are equal)

Area of a trapezium = $\frac{1}{2}$ x sum of parallel sides x height

∴ A (□ ABCD) = $\frac{1}{2}$ x (AB + CD) x DM

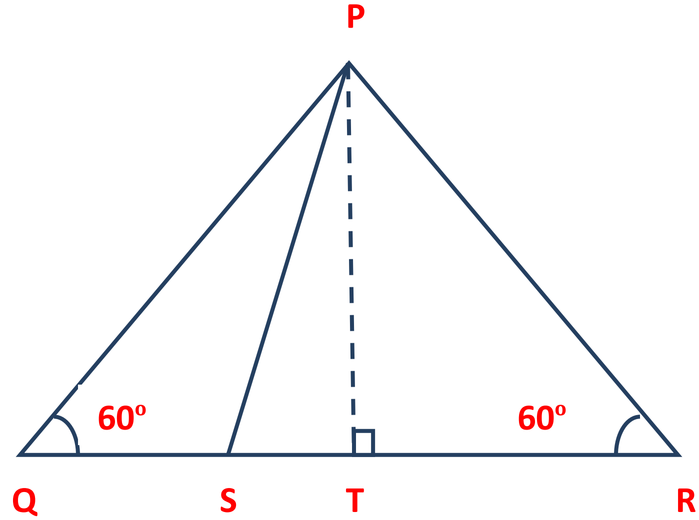
∴ A (□ ABCD) = $\frac{1}{2}$ x (25 + 7) x 12

∴ A (□ ABCD) = 32 x 6

∴ A (□ ABCD) = 192 sq units

Q. 34

In the figure, Δ PQR is an equilateral triangle. Point S is on seg QR such that QS = $\frac{1}{3}$ QR. Prove that $9PS^2 = 7PQ^2$.



Proof:

Draw seg PT ⊥ side QR such that Q – T – R

Δ PQR is an equilateral triangle

∴ PQ = QR = PR … (Sides of an equilateral triangle are equal) … (1)

∠ Q = 60° … (Angle of an equilateral triangle) … (2)

In Δ PQT,

∠ PTQ + ∠ PQT + ∠ QPT = 180° … (Sum of all angles of a triangle is 180°)

∴ 90° + 60° + ∠ QPT = 180° … [From construction and (1)]

∴ 150 + ∠ QPT = 180°

$\therefore \angle QPT = 180^\circ - 150^\circ$

$\therefore \angle QPT = 30^\circ$

$\therefore \Delta PQT$ is a $30^\circ - 60^\circ - 90^\circ$ triangle

$\therefore$ by $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

$PT = \frac{\sqrt{3}}{2} PQ$ … (Side opposite to 60°)

$QT = \frac{1}{2} PQ$ … (Side opposite to 30°)

$QS + ST = QT$ … (Q – S – T)

$\therefore \frac{1}{3} QR + ST = \frac{1}{2} PQ$

$\therefore \frac{1}{3} PQ + ST = \frac{1}{2} PQ$ … [From (1)]

$\therefore ST = \frac{1}{2} PQ - \frac{1}{3} PQ$

$\therefore ST = \frac{3PQ - 2PQ}{6}$

$\therefore ST = \frac{PQ}{6}$

In ΔPTS, $\angle PTS = 90^\circ$ … (Construction)

$\therefore$ By Pythagoras theorem,

$$PS^2 = PT^2 + ST^2$$

$$\therefore\ PS^2 = \left(\frac{\sqrt{3}}{2}PQ\right)^2 + \left(\frac{PQ}{6}\right)^2$$

$$\therefore\ PS^2 = \frac{3PQ^2}{4} + \frac{PQ^2}{36}$$

$$\therefore\ PS^2 = \frac{27PQ^2 + PQ^2}{36}$$

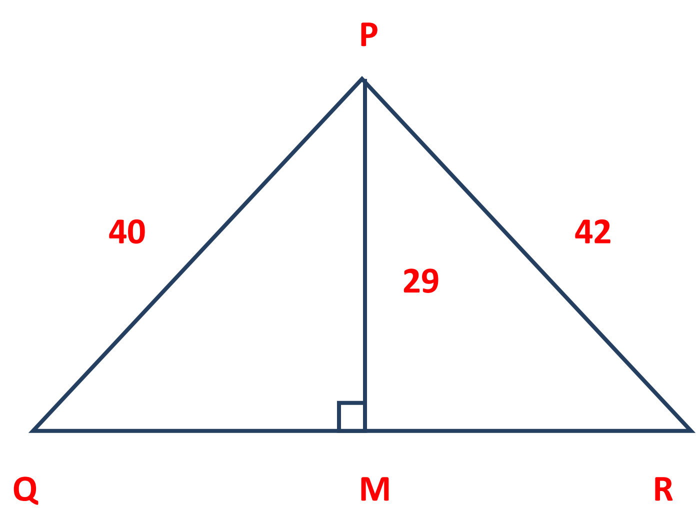
$$\therefore\ PS^2 = \frac{28PQ^2}{36}$$

$$\therefore\ PS^2 = \frac{7PQ^2}{9}$$

$$\therefore\ 9PS^2 = 7PQ^2$$

Q. 35

Seg PM is a median of Δ PQR. If PQ = 40, PR = 42 and PM = 29, find QR.



SOLUTION:

In Δ PQR, seg PM is the median … (Given)

∴ by Appollonius theorem,

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$

$\therefore\ 40^2 + 42^2 = 2\,(29)^2 + 2QM^2$

$\therefore\ 1600 + 1764 = 2 \times 841 + 2QM^2$

$\therefore\ 3364 = 1682 + 2QM^2$

$\therefore\ 2QM^2 = 3362 - 1682$

$\therefore\ 2QM^2 = 1682$

$\therefore\ QM^2 = \frac{1682}{2}$

$\therefore\ QM^2 = 841$

$\therefore\ QM = 29$

$QM = \frac{1}{2}QR$ **… (M is the midpoint of seg QR)**

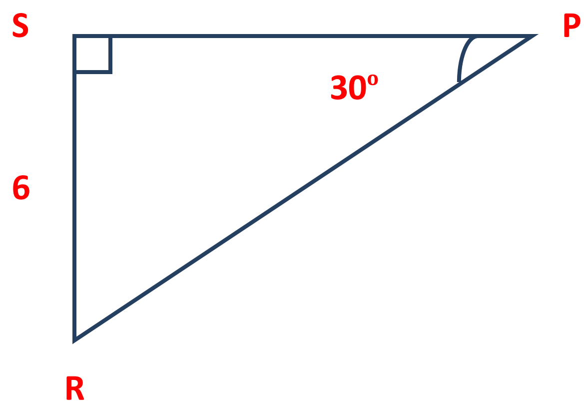
$\therefore\ 29 = \frac{1}{2}QR$

∴ QR = 29 x 2

∴ QR = 58

Q. 36

See adjoining figure. Find RP and PS using the information given in ΔPSR.



Solution:

In ΔPSR, ∠S = 90°, ∠P = 30° [Given]

∴ ∠R = 60° [Remaining angle of a triangle]

∴ ΔPSR is a 30° – 60° – 90° triangle.

RS = $\frac{1}{2}$ RP … [Side opposite to 30°]

∴ 6 = $\frac{1}{2}$ RP

∴ RP = 6 × 2 = 12 units

Also, PS = $\frac{\sqrt{3}}{2}$ RP … [Side opposite to 60°]

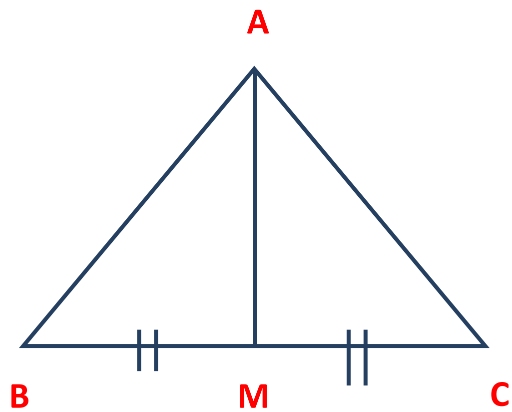
= $\frac{\sqrt{3}}{2} \times 12$

= $6\sqrt{3}$ units

∴ RP = 12 units, PS = $6\sqrt{3}$ units

Q. 37

In ΔABC, point M is the midpoint of side BC. If AB2 + AC2 = 290 cm, AM = 8 cm, find BC.



SOLUTION:

In ΔABC, point M is the midpoint of side BC. [Given]

∴ seg AM is the median.

∴ $AB^2 + AC^2 = 2\ AM^2 + 2\ MC^2$ [Apollonius theorem]

∴ $290 = 2(8)^2 + 2\,MC^2$

∴ $145 = 64 + MC^2$ … [Dividing both sides by 2]

∴ $MC^2 = 145 - 64$

∴ $MC^2 = 81$

∴ $MC = \sqrt{81}$ … [Taking square root of both sides]

MC = 9 cm

Now, BC = 2 MC … [M is the midpoint of BC]

$= 2 \times 9$

∴ BC = 18 cm

Q. 38

In △ABC, ∠C is an acute angle, seg AD ⊥ seg BC. Prove that: $AB^2 = BC^2 + A^2 - 2\,BC \times DC$.

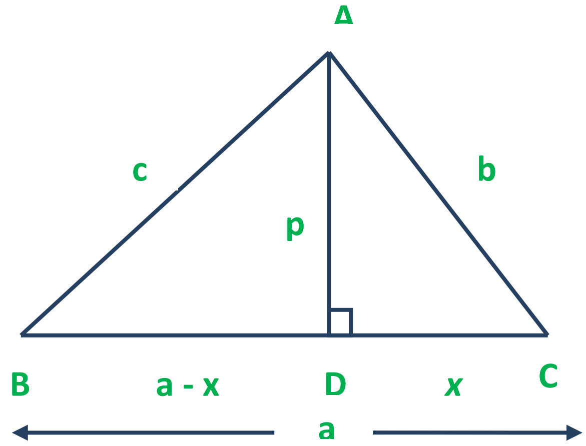
Given: ∠C is an acute angle, seg AD ⊥ seg BC.

To prove: $AB^2 = BC^2 + AC^2 - 2BC \times DC$

SOLUTION:

Proof:

∴ Let AB = c, AC = b, AD = p



$\therefore BC = a, DC = x$

$BD + DC = BC$ … [B – D – C]

$\therefore BD = BC - DC$

$\therefore BD = a - x$

In ΔABD, $\angle D = 90°$ … [Given]

$AB^2 = BD^2 + AD^2$ … [Pythagoras theorem]

$\therefore c^2 = (a - x)^2 + [P^2]$ … (i)

$\therefore c^2 = a^2 - 2ax + x^2 + [P^2]$

In ΔADC, $\angle D = 90°$ … [Given]

$AC^2 = AD^2 + CD^2$ … [Pythagoras theorem]

$\therefore b^2 = p^2 + [X^2]$

$\therefore p^2 = b^2 - [X^2]$ (ii)

$\therefore c^2 = a^2 - 2ax + x^2 + b^2 - x^2$ … [Substituting (ii) in (i)]

$\therefore c^2 = a^2 + b^2 - 2ax$

$\therefore AB^2 = BC^2 + AC^2 - 2\, BC \times DC$

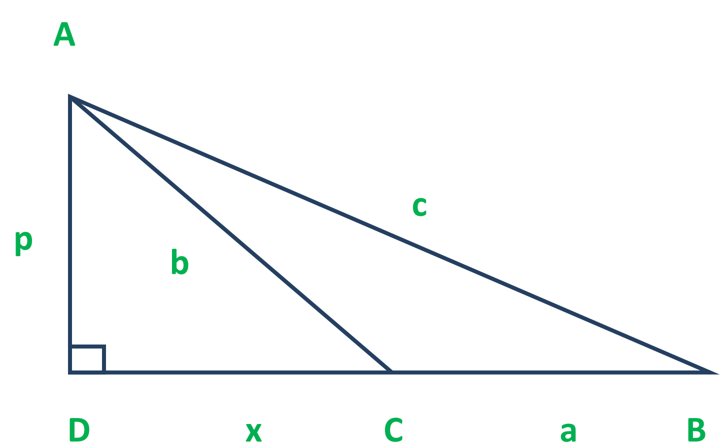
Q. 39

In $\triangle ABC$, $\angle ACB$ is an obtuse angle, seg AD $\perp$ seg BC. Prove that: $AB^2 = BC^2 + AC^2 + 2\ BC \times CD$.

Given: $\angle ACB$ is an obtuse angle, seg AD $\perp$ seg BC.

To prove: $AB^2 = BC^2 + AC^2 + 2BC \times CD$

SOLUTION:



Let AD = p, AC = b, AB = c,

BC = a, DC = x

BD = BC + DC … [B – C – D]

$\therefore$ BD = a + x

In $\triangle ADB$, $\angle D = 90°$ … [Given]

$AB^2 = BD^2 + AD^2$ … [Pythagoras theorem]

$\therefore c^2 = (a + x)^2 + p^2$ … (i)

$\therefore c^2 = a^2 + 2ax + x^2 + p^2$

Also, in ΔADC, $\angle D = 90°$ … [Given]

$AC^2 = CD^2 + AD^2$ … [Pythagoras theorem]

$\therefore b^2 = x^2 + p^2$

$\therefore p^2 = b^2 - x^2$ … (ii)

$\therefore c^2 = a^2 + 2ax + x^2 + b^2 - x^2$ … [Substituting (ii) in (i)]

$\therefore c^2 = a^2 + b^2 + 2ax$

$\therefore AB^2 = BC^2 + AC^2 + 2\,BC \times CD$

Q. 40

Do sides 7 cm, 24 cm, 25 cm form a right angled triangle? Give reason.

SOLUTION:

The sides of the triangle are 7 cm, 24 cm and 25 cm.

The longest side of the triangle is 25 cm.

$\therefore \ (25)^2 = 625$

Now, sum of the squares of the remaining sides is,

$(7)^2 + (24)^2 = 49 + 576 = 625$

∴ $(25)^2 = (7)^2 + (24)^2$

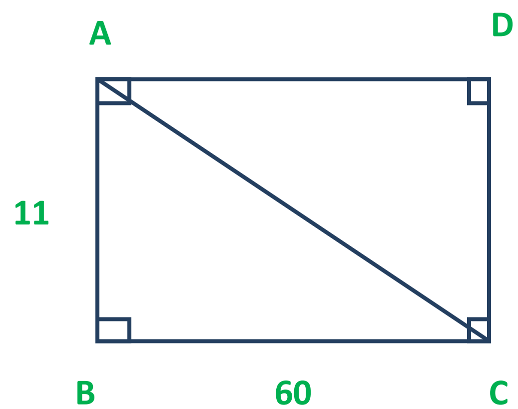
∴ Square of the longest side is equal to the sum of the squares of the remaining two sides.

∴ The given sides will form a right angled triangle [Converse of Pythagoras theorem]

Q. 41

Find the length of a diagonal of a rectangle having sides 11 cm and 60 cm.

SOLUTION:



Let □ABCD be the given rectangle.

AB = 11 cm, BC = 60 cm

In ΔABC, ∠B = 90° … [Angle of a rectangle]

∴ $AC^2 = AB^2 + BC^2$ … [Pythagoras theorem]

∴ AC^2 = 112 + 602

∴ AC^2 = 121 + 3600

∴ AC^2 = 3721

∴ AC = $\sqrt{3721}$ … [Taking square root of both sides]

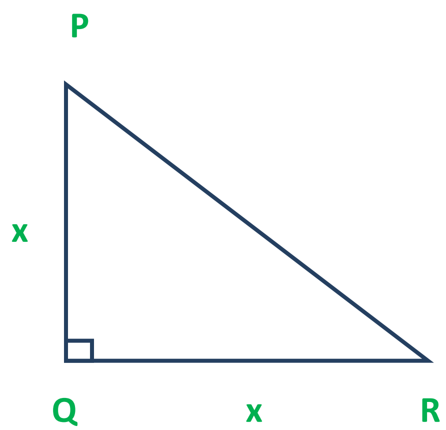
∴ AC = 61 cm

∴ The length of the diagonal of the rectangle is 61 cm.

Q. 42

A side of an isosceles right angled triangle is x. Find its hypotenuse.

SOLUTION:



Let Δ PQR be the given right angled isosceles triangle.

PQ = QR = x

In ΔPQR, ∠Q = 90° … [Pythagoras theorem]

∴ $PR^2 = PQ^2 + QR^2$

∴ $PR^2 = x^2 + x^2$

∴ $PR^2 = 2x^2$

∴ $PR = \sqrt{2x^2}$ … [Taking square root of both sides]

∴ $PR = x\sqrt{2}$ units

∴ The hypotenuse of the right angled isosceles triangle is $x\sqrt{2}$ units.

Q. 43

In ΔPQR, $PQ = \sqrt{8}$, $QR = \sqrt{5}$, $PR = \sqrt{3}$. Is Δ PQR a right angled triangle? If yes, which angle is of 90°?

SOLUTION:

Longest side of Δ PQR = $PQ = \sqrt{8}$

∴ $PQ^2 = \left(\sqrt{8}\right)^2 = 8$

Now, sum of the squares of the remaining sides is,

$QR^2 + PR^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2$

∴ $QR^2 + PR^2 = 5 + 3$

∴ $QR^2 + PR^2 = 8$

∴ $PQ^2 = QR^2 + PR^2$

∴ Square of the longest side is equal to the sum of the squares of the remaining two sides.

∴ Δ PQR is a right angled triangle. [Converse of Pythagoras theorem]

Now, PQ is the hypotenuse.

∴ ∠ PRQ = 90° [Angle opposite to hypotenuse]

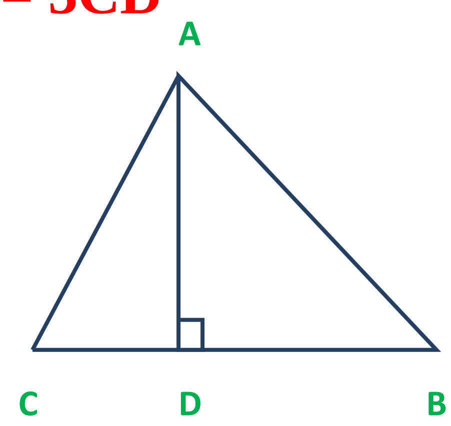
∴ Δ PQR is a right angled triangle in which ∠ PRQ is of 90°

Q. 44

In ΔABC, seg AD ⊥ seg BC and DB = 3 CD. Prove that: 2 AB2 = 2 AC2 + BC2

Given: seg AD ⊥ seg BC and DB = 3CD

To prove: $2AB^2 = 2AC^2 + BC^2$



SOLUTION:

DB = 3CD … (i) … [Given]

In ΔADB, $\angle ADB = 90°$ … [Given]

$\therefore$ $AB^2 = AD^2 + DB^2$ … [Pythagoras theorem]

$\therefore$ $AB^2 = AD^2 + (3CD)^2$ … [From (i)]

$\therefore$ $AB^2 = AD^2 + 9CD^2$ … (ii)

In ΔADC, $\angle ADC = 90°$ … [Given]

$\therefore$ $AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$\therefore$ $AD^2 = AC^2 - CD^2$ … (iii)

$AB^2 = AC^2 - CD^2 + 9CD^2$ … [From (ii) and(iii)]

$\therefore$ $AB^2 = AC^2 + 8CD^2$ … (iv)

$CD + DB = BC$ … [C – D – B]

$\therefore$ $CD + 3CD = BC$ … [From (i)]

$\therefore$ $4CD = BC$

$\therefore$ $CD = \frac{BC}{4}$ … (v)

$AB^2 = AC^2 + 8\left(\frac{BC}{4}\right)^2$ … [From (iv) and (v)]

$\therefore$ $AB^2 = AC^2 + 8 \times \frac{BC^2}{16}$

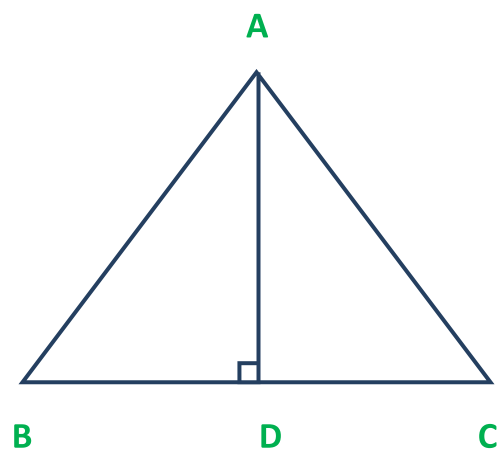
$\therefore$ $AB^2 = AC^2 + \frac{BC^2}{2}$

$\therefore$ $2AB^2 = 2AC^2 + BC^2$ ...[Multiplying both sides by 2]

Ans : $2AB^2 = 2AC^2 + BC^2$

Q. 45

In acute angled triangle ABC, seg BD $\perp$ seg BC, B – D – C, $\angle$ B < 90°; then prove that $AC^2 = AB^2 + BC^2 - BC^2 - 2\,BC.BD$



SOLUTION:

In Δ ADB, $\angle$ ADB = 90°

$\therefore$ By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ … (1)

In Δ ADC, $\angle$ ADC = 90°

∴ By Pythagoras theorem,

$AC^2 = AD^2 + DC^2$

∴ $AC^2 = AD^2 + (BC^2 – BD^2)$ … (B – D – C)

∴ $AC^2 = AD^2 + BC^2 – 2BC.BD + BD^2$

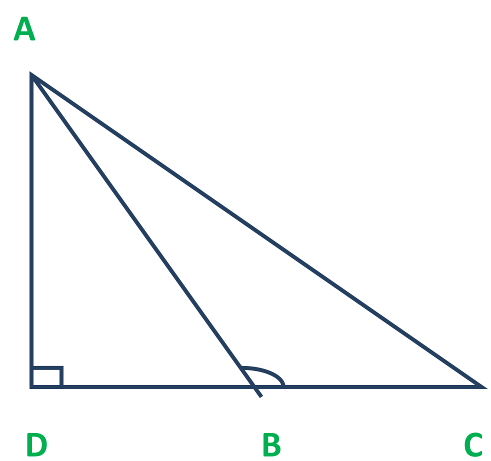
∴ $AC^2 = (AD^2 + BD^2) + BC^2 – 2BC.BD$

∴ $AC^2 = AB^2 + BC^2 – 2\ BC.BD$ … [From (1)]

Q. 46

In obtuse angled Δ ABC, ∠ B > 90º, if seg AD ⊥ side BC and D – B – C, then prove that $AC^2 = AB^2 + BC^2 + 2\ BC.DB$

SOLUTION:



In Δ ADB ∠ ADB = 90º

∴ By Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ … (1)

In Δ ADC, $\angle$ ADC = 90^o

$\therefore$ By Pythagoras theorem,

$AC^2 = AD^2 + DC^2$

$\therefore AC^2 = AD^2 + (DB^2 + BC^2)$ … (D – B – C)

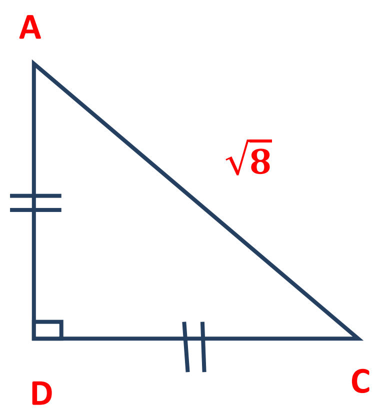
$\therefore AC^2 = AD^2 + DB^2 + 2\ DB.BC + BC^2$

$\therefore AC^2 = (AD^2 + DB^2) + BC^2 + 2DB.BC$

$\therefore AC^2 = AB^2 + BC^2 + 2\ BC.DB$ … [From (1)]

Q. 47

Find AB and BC with the help of information given in figure.



SOLUTION:

AB = BC … (Given)

$\angle$ BAC = 45°

$\therefore$ AB = BC = $\frac{1}{\sqrt{2}} \times AC$

$= \frac{1}{\sqrt{2}} \times \sqrt{8}$

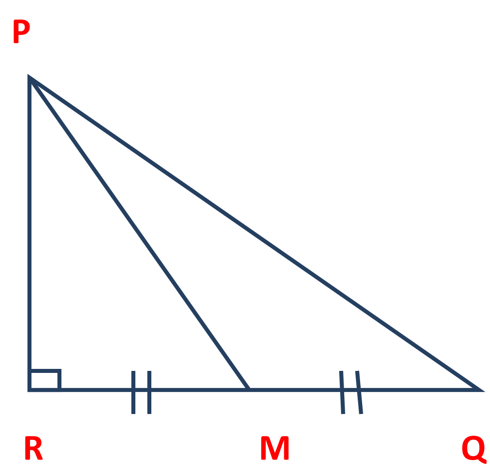
$= \frac{1}{\sqrt{2}} \times 2\sqrt{2}$

= 2

$\therefore$ AB = BC = 2

Q. 48

In the figure, M is the midpoint of QR. $\angle$ PRQ = 90°, Prove that $PQ^2 = 4PM^2 - 3PR^2$



SOLUTION:

In Δ PRQ, $\angle$ PRQ = 90° … (Given)

$\therefore$ By Pythagoras theorem,

$PQ^2 = PR^2 + QR^2$ … (1)

In Δ PRM, $\angle$ PRM = 90° … (Given)

$\therefore$ By Pythagoras theorem,

$PM^2 = PR^2 + RM^2$ … (2)

$RM = \frac{1}{2} RQ$ … (M is the midpoint of seg RQ) … (3)

$\therefore PM^2 = PR^2 + \left(\frac{1}{2} RQ\right)^2$ … [From (2) and (3)]

$\therefore PM^2 = PR^2 + \frac{1}{4} RQ^2$

Multiplying each term with 4 we get,

$4PM^2 = 4PR^2 + RQ^2$

$\therefore 4PM^2 = 3PR^2 + (PR^2 + RQ^2)$

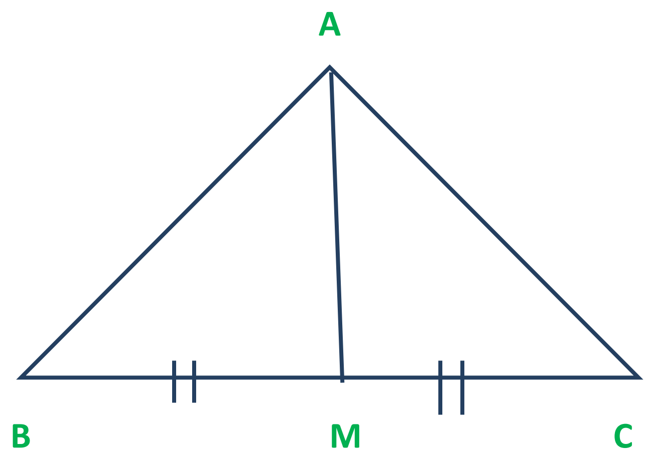
$\therefore 4PM^2 = 3PR^2 + PQ^2$ … [From (1)]

$\therefore 4PM^2 – 3PR^2 = PQ^2$ OR

$PQ^2 = 4PM^2 – 3PR^2$

Q. 49

In Δ ABC, point M is the midpoint of side BC. If $AB^2 + AC^2 = 290\ cm^2$, AM = 8 cm, find BC



SOLUTION:

In Δ ABC, seg AM is the median … (Given)

$\therefore$ By Appollonius theorem,

$$AB^2 + AC^2 = 2AM^2 + 2BM^2$$

$$\therefore 290 = 2(8)^2 + 2BM^2$$

$$\therefore 290 = 128 + 2BM^2$$

$$\therefore 290 - 128 = 2BM^2$$

$$\therefore 2BM^2 = 162$$

$$\therefore BM^2 = 81$$

$\therefore$ BM = 9 cm

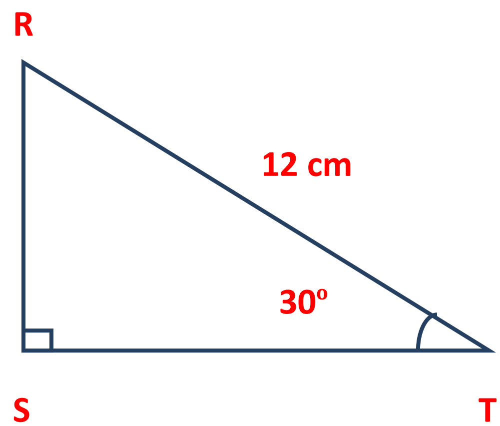
BM = $\frac{1}{2}$ BC … (M is the midpoint of side BC)

$\therefore$ 9 = $\frac{1}{2}$ BC

$\therefore$ BC = 18 cm

Q. 50

In Δ RST, ∠ S = 90º, ∠ T = 30º, RT = 12 cm, then find RS and ST



SOLUTION:

In Δ RST,

∠ RST + ∠ SRT + ∠ RTS = 180º … (Sum of all angles of a triangle is 180º)

$\therefore$ 90º + ∠ SRT + 30º = 180º

$\therefore$ ∠ SRT + 120º = 180º

$\therefore \angle$ SRT = 180º - 120º

$\therefore \angle$ SRT = 60º

$\therefore \Delta$ SRT is a 30º - 60º - 90º triangle

$\therefore$ By 30º - 60º - 90º triangle theorem,

RS = $\frac{1}{2}$ RT … (Side opposite to 30º)

$\therefore$ RS = $\frac{1}{2} \times 12$

$\therefore$ RS = 6 cm

ST = $\frac{\sqrt{3}}{2}$ RT … (Side opposite to 60º)

$\therefore$ ST = $\frac{\sqrt{3}}{2} \times 12$

$\therefore$ ST = $6\sqrt{3}$ cm

Ans.: RS = 6 cm and ST = $6\sqrt{3}$ cm