3. Triangles

Extra Question

Q. 1) The measure of angles of a triangles are in the ratio 3 : 5 : 7

Find the measure.

Solution :

Let the measure of the angles of a triangle.

be $3x, 5x, 7x$

$3x + 5x + 7x = 180^0$

$15x = 180^0$

$x = \frac{180}{15}$

$x = 12^0$

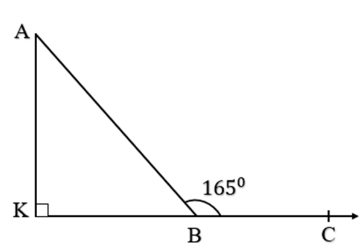
$3x = 3 \times 12 = 36$

$5x = 5 \times 12 = 60$

$7x = 7 \times 12 = 84$

∴ The measure of angles of the triangle are $36^0, 60^0$, and 84^0

Q. 2) In the adjoining figure, find the measure of $\angle A$.



Solution :

Δ AKB is the exterior $\angle$ ABC

$\therefore \angle ABC = \angle AKB + \angle KAB$

….. (Theorem of remote interior angle)

$\therefore 165^0 = 90^0 + \angle KAB$

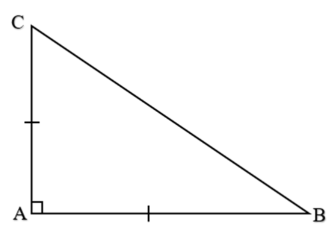
$165^0 - 90^0 = \angle KAB$

$75^0 = \angle KAB$

$\therefore \angle KAB = 75^0$

$\therefore \angle A = 75^0$

Q. 3) In the right Δ ABC and AB = BC then find $\angle B$ and $\angle C$.



Solution : In Δ ABC

AB = AC ….. (given)

Their opposite angle are equal.

$\angle ACB = \angle ABC$

Now,

$\angle A + \angle B + \angle C = 180^0$

$90^0 + \angle B + \angle C = 180^0$ …. ($\angle A = 90^0$) …. (given)

$\angle B + \angle C = 180^0 - 90^0$

$= 90^0$

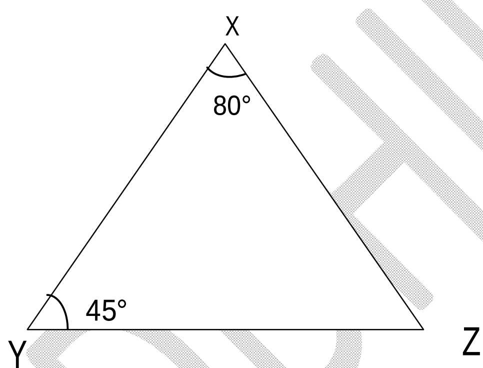
But, $\angle ABC = \angle ACB$

also, $\angle B = \angle C$

$\therefore \angle B = \angle C = \dfrac{90^0}{2} = 45^0$

$\therefore \angle B = 45^0$ and $\angle C = 45^0$

Q. 4) In $\Delta XYZ \angle X = 80^0, \angle Y = 45^0$ then. Find the measure of $\angle A$.



Solution : In ΔXYZ

$\angle X + \angle Y + \angle Z = 180^0$

….(Sum of measure of interior angle of a triangle)

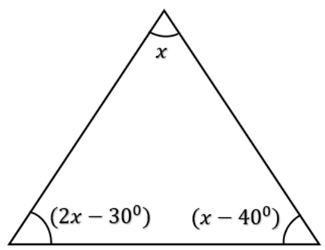
$\therefore 80^0 + 45^0 + \angle Z = 180^0$

$125^0 + \angle Z = 180^0$

$\angle Z = 180^0 - 125^0$

$\angle Z = 55^0$

Q. 5) Measure of angle of triangle is x^0, $(2x - 30^0)$, $(x - 40^0)$ then measure of each angle ?



Solution :

$x^0 + (2x - 30^0) + (x - 40^0) = 180^0$

…… (sum of measure of interior angle of triangle)

$x + 2x - 30^0 + x - 40^0 = 180^0$

$4x - 30^0 - 40^0 = 180^0$

$4x - 70^0 = 180^0$

$4x = 180^0 + 70^0$

$4x = 250$

$x = \frac{250}{4}$

$x = 62.5$

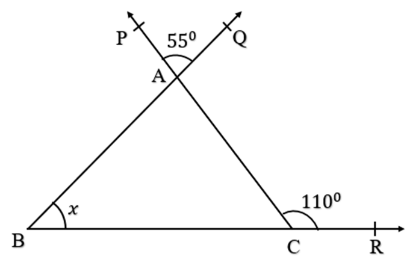
$\therefore x = 62.5$

$\therefore 2x - 30 = 2(62.5) - 30 = 125 - 30 = 95^0$

$x - 40 = 62.5 - 40 = 22.5$

∴Measure of angle of triangle is 62.5, 95, 22.5 and respectively.

Q. 6) In the adjoining figure, find the value of x.



Solution : $\angle PAQ = \angle BAC$ ….. (Vertically opposite angle)

$\because \angle PAQ = 55^0$

$\therefore \angle BAC = 55^0$

Now, $\angle ACR = \angle ABC + \angle BAC$ … (From exterior angle)

$110^0 = \angle ABC + 55^0$

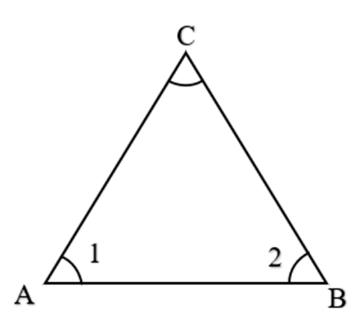
$110^0 - 55^0 = \angle ABC$

$55^0 = \angle ABC$

$\therefore \angle ABC = 55^0$

$\therefore x = 55^0$

Q. 7) In a ΔABC, $BC = CA$ and $\angle A = 40^0$ which is longer AB or BC ?



Solution :

Δ ABC in AC = BC

$\because\ \angle 1 = \angle 2$.....($\because$ AC = BC)

and $\angle A = \angle 1 = 40^0$

$\angle 2 = 40^0$

$\therefore\ \angle 1 + \angle 2 + \angle C = 180^0$ … (sum of angle of triangle is 180^0)

$40^0 + 40^0 + \angle C = 180^0$

$80^0 + \angle C = 180^0$

$\angle C = 180^0 - 80^0$

$\angle C = 100^0$

$\therefore \angle C > \angle A \Rightarrow AB > BC$

$\therefore$Thus AB is greater.

Q.8) In Δ ABC , if $\angle A + \angle B = 110^0$ and $\angle B + \angle C = 132^0$, then find $\angle A$, $\angle B$ and $\angle C$.

Solution - We have $\angle A + \angle B = 110^0$……. (1)

$\angle B + \angle C = 132^0$….. (2)

Now,

$\angle A + \angle B + \angle C = 180^0$ Sum of angle of triangle is 180^0

$110^0 + \angle C = 180^0$…… ($\because\ \angle A + \angle B = 110^0$)

$\angle C = 180^0 - 110^0$

$\angle C = 70^0$

Now, from II we have,

$\angle B + \angle C = 132^0$

$\angle B = 132^0 - 70^0$

$\angle B = 62^0$

$\angle A + \angle B = 110^0$…… (From I)

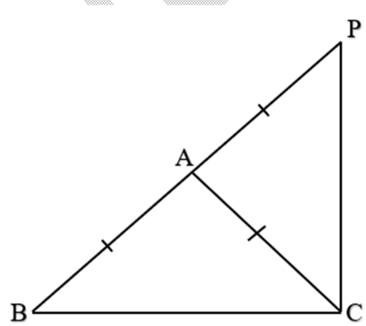
$\angle A + 62^0 = 110^0$

$\angle A = 110^0 - 62^0$

$\angle A = 48^0$

$\therefore \angle A = 48^0, \angle B = 62^0$ and $\angle C = 70^0$

Q.10) In the figure ABC is a triangle in which AB = AC. side BA is produced to p such that AB = AP. Prove that $\angle BCP = 90^0$



Solution – AB = AC --- given

side AB and side AC are equal.

$\angle ABC = \angle ACB$…… (I) …. (Angles at the base)

Also, side AB = side AC

AC = AP ------------- (Given)

∵ side AB = side AP

$\angle APC = \angle ACP$…… (II)

Adding I and II we get,

$\angle ABC + \angle APC = \angle ACB + \angle ACP$

$\angle ABC + \angle APC = \angle BCP$

$\angle PBC + \angle BPC = \angle BCP$ [∴ $\angle ABC = \angle PBC$ and $\angle APC = \angle BCP$]

Adding $\angle BPC$ to both side

$\angle PBC + \angle BCP + \angle BCP = \angle BCP + \angle BCP$

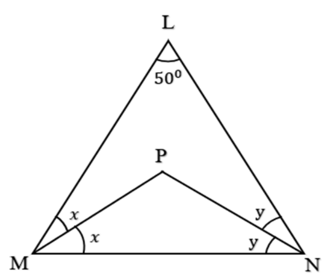
$180^0 = 2\angle BCP$

$\angle BCP = \frac{180^0}{2}$

$\angle BCP = 90^0$

∴ Thus $\angle BCP = 90^0$

Q.11) In Δ LMN bisectors of $\angle L$ and $\angle M$, intersect at the point P . If $\angle N = 50^0$ then find the measure of $\angle MPN$



Solution - $\angle LMP \cong \angle PMN$ ---- (Ray MP bisects $\angle LMN$)

$\angle LMP = \angle PMN = x^0$….. (I)

$\angle$ LNP $\cong$ $\angle$ PNM…… (Ray NP bisects $\angle$ LNM)

$\angle$ LNP = $\angle$ PNM = y^0…… (II)

$\angle$ LMN = $\angle$ LNP + $\angle$ PMN ……. (Angles addition postulate)

$\angle$ LMN = $x + x^0$……. (From I)

$\angle$ LMN = $2x^0$……. (III)

Similarly $\angle$ LNM = $2y^0$…… (IV)

In Δ LMN]

$\angle$ MLN + $\angle$ LMN + $\angle$ LNM = 180^0

… Sum of measures of all angles of a triangle is 180^0.)

$\therefore 50^0 + 2x + 2y = 180^0$…… (From (III) and (IV))

$\therefore 50^0 + 2(x + y) = 180^0$

$\therefore 2(x + y) = 180^0 - 50^0$

$\therefore 2(x + y) = 130^0$

$$x + y = \frac{130^0}{2}$$

$x + y = 65^0$….. (V)

In Δ MPN]

$\angle$ MPN + $\angle$ PMN + $\angle$ PNM = 180^0

…. Sum of measures of all angles of a triangle 180^0.)

$\therefore$ $\angle$ MPN + $x + y = 180^0$…… (From I and II)

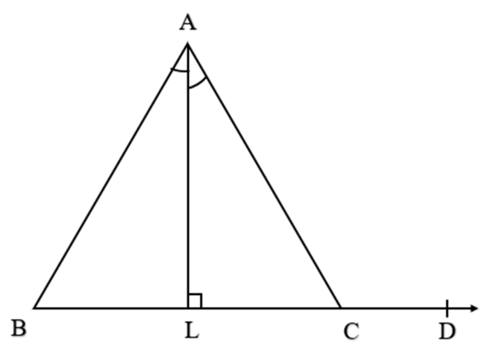
$\therefore$ $\angle$ MPN + $65^0 = 180^0$….. (From V)

$\therefore \angle MPN = 180^0 - 65^0$

$\therefore \angle MPN = 115^0$

Q.12) In the figure, side BC of Δ ABC is extended seg AL is angle bisector of $\angle$ BAC of Δ ABC and B-L-C-D, prove that,

$\angle ABC + \angle ACD = 2\angle ALC$



Solution In Δ ABC ,

seg AL is the angle bisector of $\angle$ BAC,

$\therefore \angle BAL = \angle CAL = \frac{1}{2}\angle BAC$...... (I)

$\angle ACD = \angle ABC + \angle BAC$...... (II)

----(Theorem of remote interior angle)

$\angle$ ALC is exterior angle of Δ ABL

$\therefore \angle ALC = \angle ABL + \angle BAL$----(Theorem of remote interior angle)

$\therefore \angle ALC = \angle ABC + \frac{1}{2}(\angle BAC$ ..... (B – L - C)...... (From I)

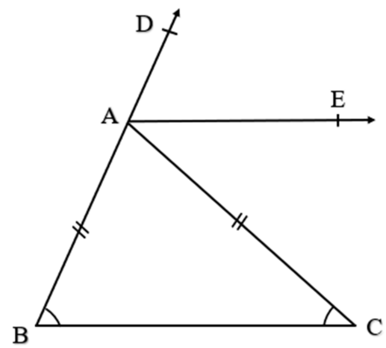
$\therefore 2\angle ALC = 2\angle ABC + \angle BAC$...... Multiplying both sides by 2

$\therefore 2\angle ALC = \angle ABC + \underline{\angle ABC + \angle BAC}$

$\therefore 2\angle ALC = \angle ABC + \angle ACD$...... (From II)

∴ ∠ ABC + ∠ ACD = 2 ∠ ALC ------- ((proved)

Q.13) In the adjoining figure, Δ ABC is isosceles triangle AB = AC and ∠ CAD is the bisector of AE then prove that AE || BC.



Solution – Opposite angle of triangle sides are equal.-

∴ AB = AC

⇒ ∠ B = ∠ C

Now, side BA is Δ ABC are extended to D.-

∴ ∠CAD = ∠B + ∠C….. (Exterior angle = sum of interior opposite angle)

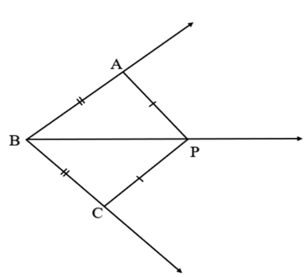
2 ∠CAE = 2 ∠C… (∵ ∠ B = ∠ C and ∠ CAD = 2 ∠ CAE)

∠ CAE = ∠ C

but, their are alternate interior angle.

∴ AE || BC

Q.14) In the adjoining figure, write the name of congruence angle.



Solution - Δ ABP ≅ Δ CBP…… (Test of S - S - S)

∠ ABP ≅ ∠ CBP

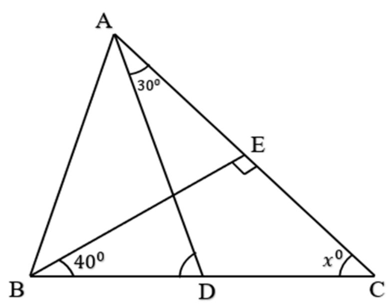
∠ BAP ≅ ∠ BCP…… } --- congruence of triangle in similar

∠ APB ≅ ∠ CPB } angle

Q.15) In Δ ABC BE ⊥ AC, ∠EBC = 40^0 and ∠CAD = 30^0.

If ∠ ACD = x^0 and ∠ ADY = y^0, x then find the

and y.



Solution –

Sum of angles of triangle is 180^0

In ∠ BCE

∠ CBE + ∠ BEC + ∠ ECB = 180^0

$40^0 + 90^0 + x^0 = 180^0$

$130^0 + x = 180^0$

$x = 180^0 - 130^0$

$x = 50^0$

Now, $\angle$ ACD side CD are extended to B -

$\angle$ BDA = $\angle$ DAC + $\angle$ ACD

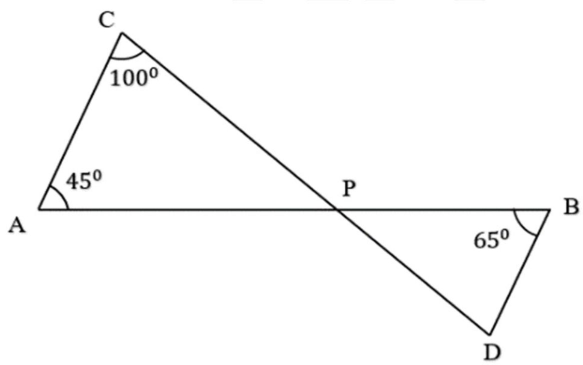
$y^0 = 30^0 + x^0$

$y^0 = 30^0 + 50^0$….. ($\because x = 50^0$)

$y^0 = 80^0$

$\therefore x = 50^0, \; y = 80^0$

Q.16) In the adjoining figure, line AB and CD are intersect to point P. Then, $\angle PAB = 45^0$, $\angle ACP = 100^0$ and $\angle$PBD = 65° find $\angle$CPA, $\angle$DPB and $\angle$BDP



Solution –

Sum of angles of triangle is 180^0

In Δ ACP

$\angle PAC + \angle ACP + \angle CPA = 180^0$

$45^0 + 100^0 + \angle CPA = 180^0$

$145^0 + \angle CPA = 180^0$

$\angle CPA = 180^0 - 145^0$

$\angle CPA = 35^0$

$\therefore \angle DPB = \angle CPA = 35^0$….. (Vertically opposite angle)

In ΔPBD

$\angle DPB + \angle PBD + \angle BDP = 180^0$……. (Sum of angles of a triangle)

$35^0 + 65^0 + \angle BDP = 180^0$

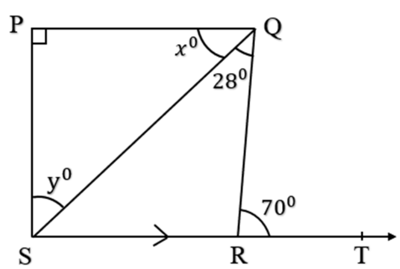
$100^0 + \angle BDP = 180^0$

$\angle BDP = 180^0 - 100^0$

$\angle BDP = 80^0$

$\therefore \angle CPA = 35^0$, $\angle DPB = 35^0$, $\angle BDP = 80^0$

Q.17) In the adjoining figure, $PQ \perp RS$, $PQ \parallel PR$, $\angle SQR = 28^0$ and $\angle QRT = 70^0$ then $\angle PQS = x^0$ and $\angle PSQ = y^0$ Then find the value of x and y.



Solution - $PQ \parallel SRT$ and QR is transversal.

$\therefore \angle PQR = \angle QRT$…… (alternate interior angle)

$\angle PQS + \angle SQR = \angle QRT$

$x + 28 = 70°$

$x = 70 - 28$

$x = 42°$

In right angled Δ PQS,

∠ SPQ + ∠ PQS + ∠ QSP = 180^0….. (sum of angles of a triangle is 180^0)

$90^0 + x^0 + y^0 = 180^0$

$90^0 + 42^0 + y = 180°$

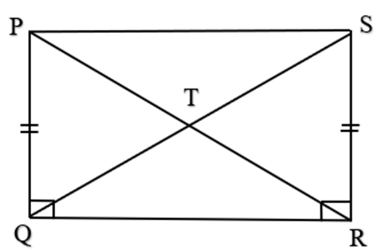
$132^0 + y = 180°$

$y = 180^0 - 132^0$

$y = 48^0$

∴ $x = 42^0$, $y = 48^0$

Q.18) In the adjoining figure, pair of triangles in test of congruence SAS ? Verify?



Solution - Δ PQR ≅ Δ SRQ…… (S – A – S test)

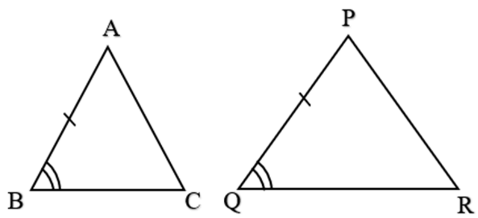
seg PR ≅ seg SQ --- congruence of triangle in similar sides

∠ QPR ≅ ∠ RSQ

and ∠ PRQ ≅ ∠ SQR….. --- congruence of triangle in similar angles

Q. 19) In the adjoining figure pair of triangles in test of

congruence ASA ? Verify.



Solution - ∆ ABC ≅ ∆ PQR….. (A – S – A test)

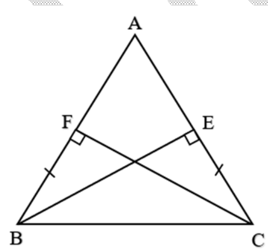
∠ ABC ≅ ∠ PQR…… (Pair of similar congruence angles)

seg BC ≅ seg QR

∠ ACB ≅ ∠ PRQ

∴ ∆ ABC ≅ ∆ PQR

Q.20) In ∆ ABC , BE and CF are the vertices and AC and AB are the equal sides then show that ∆ ABE ≅ ∆ ACF



Solution - In ∆ ABE and ∆ ACF]

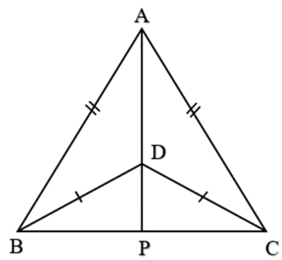
∠ AEB ≅ ∠ AFC

… Each ∠ E and ∠ F = 90^0 ∵ BE ⊥ AC and CF ⊥ AB)

∴ BE = CF…… (given)

∴ ∆ ABE ≅ ∆ ACF….. (AAS test)

Q. 21) Δ ABC and Δ DBC are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC If AD is extended to intersect BC at P then show that Δ ABD ≅ Δ ACD



Solution –

In Δ ABD and Δ ACD we have given)

AB = AC…… (given)

AD = AD ----- Common

BD = CD….. (given)

∴ Δ ABD ≅ Δ ACD….. (SSS test)

Q.22) In Δ ABC , ∠A – ∠B = 33^0 and ∠B – ∠C = 18^0.

Find the name of the angle of a triangle.

Solution-

∠ A – ∠ B = 33^0 and ∠ B – ∠ C = 18^0

∴ ∠ A = 33^0 + ∠ B and ∠ C = ∠ B – 18^0…… (2)

Sum of angle of triangle is 180^0

∴ ∠ A + ∠ B + ∠ C = 180^0

(33^0 + ∠ B) + ∠ B – 18^0 + ∠ B = 180^0

$(33^0 + \angle B + \angle B - 18^0 + \angle B = 180^0$

$3\angle B + 33^0 - 18^0 = 180^0$

$3\angle B + 15^0 = 180^0$

$3\angle B = 180^0 - 15^0$

$3\angle B = 165^0$

$\angle B = \frac{165}{3}$

$\angle B = 55$

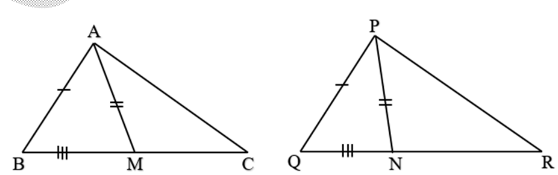
$\therefore \angle A = 33 + \angle B = 33 + 55 = 88^0$

and $\angle C = \angle B - 18^0 = 55^0 - 18^0 = 37^0$

$\therefore$ $\angle A = 88^0$, $\angle B = 55^0$ and $\angle C = 37^0$

Q.23) Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR. Show that

1) $\Delta ABM \cong \Delta PQN$ (2) $\Delta ABC \cong \Delta PQR$



Solution – Given Two sides AB and BC and medians AM of one triangle ABC are respectively equal to side PQ and QR and median PN of Δ PQR

Prove : (i) In Δ ABM and Δ PQN

AB = PQ…… (I) …… (Given)

AM = PN……. (II) ….. (Given)

BC = QR

2BM = 2QN….. (M and N are the midpoints of BC and QR)

BM = QN…… (3)

From (i), (ii) and (iii)

Δ ABM $\cong$ Δ PQN…… (Test of SSS)

(ii) Δ ABM $\cong$ Δ PQN…..(Proved in(i) above)

$\therefore$ $\angle$ ABM $\cong$ $\angle$ PQN….. (C.P.C.T)

$\angle$ ABC $\cong$ $\angle$ PQR…… (IV)

In Δ ABC and Δ PQR

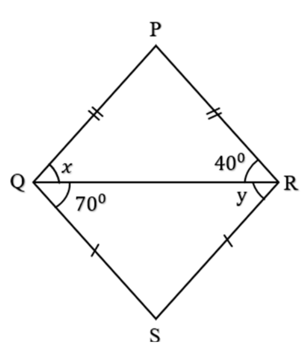
AB = PQ……. (Given)

BC = QR….. (Given)

$\angle$ ABC $\cong$ $\angle$ PQR……. From IV

$\therefore$ Δ ABC $\cong$ Δ PQR …… (Test of SAS)

Q.24) In the adjoining figure, find the value of x and y. Find the measure of $\angle$ PQS and $\angle$ PRS



Solution – In Δ PQR

seg PQ $\cong$ seg PR…… (Given)

$\angle PQR \cong \angle PRS$…… (Theorem of isosceles triangle)

$\angle PRS = 40^0$….. (Given)

$\therefore \angle PQR = 40^0$

$x = 40^0$

$\angle RQS = 70^0$

$\angle PQS = \angle PQR + \angle RQS$…… (Postulate of sum of triangle)

$\angle PQS = 40^0 + 70^0$

$\angle PQS = 110^0$

In Δ RQS

seg SQ $\cong$ seg SR ------------- given

$\therefore \angle SRQ \cong \angle SQR$….. (Theorem of isosceles triangle)

$\angle SRQ = 70^0$

$\therefore \angle SRQ = 70^0$, $y = 70^0$

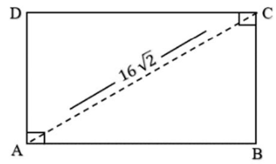
$\angle PRS = \angle PRQ + \angle SRQ$….. (Postulate of sum of triangle)

$\therefore \angle PRS = 40^0 + 70^0$

$\angle PRS = 110^0$

$\therefore x = 40^0 y = 70^0 , \angle PQS = 110^0 , \angle PRS = 110^0$

Q.25) If hypotenous $16\sqrt{2}$ cm then find the side of quadrilateral.



Solution - □ABCD is a quadrilateral.

$\therefore \angle B = 90^0$….. (Angle of quadrilateral)

And $AB = BC$….. (1) Side of quadrilateral

In right angled triangle Δ ABC

$AB^2 + BC^2 = AC^2$…… (Theorem of Pythagoras)

$AB^2 + AB^2 = (16\sqrt{2})^2$…… (AB = BC)

$2\ AB^2 = 256 \times 2$

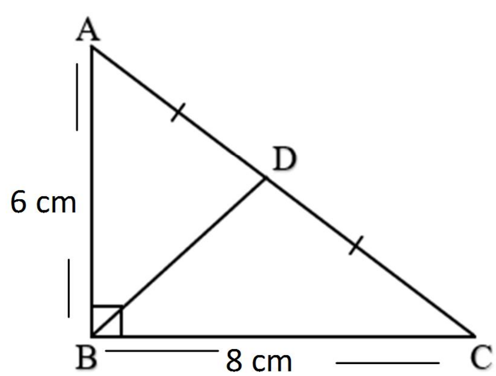
$AB^2 = \dfrac{256 \times 2}{2}$

$AB^2 = 256$ ……. (Square root of both sides)

$AB = 16$

∴Side of quadrilateral is 16 cm

Q.26) In ΔABC $\angle B = 90^0$, $AB = 6cm$ $BC = 8cm$ and seg BD is the median then find BD.



Solution – In $\Delta ABC, \angle B = 90^0$

According to Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= (6)^2 + (8)^2$

$= 36 + 64$

$AC^2 = 100$

$\therefore AC = 10$

$\therefore$ BD is the median of AC-

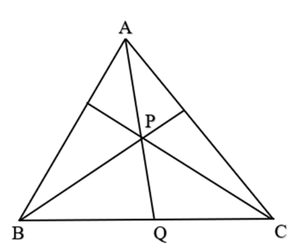
$\therefore BD = \frac{1}{2} \times AC$

$= \frac{1}{2} \times 10$

$\therefore BD = 5$

$\therefore BD = 5$ units

Q.27) In the adjoining figure, ΔABC of P is a congruence. If PQ = 3.5 cm then find the length of AP and PQ



Solution:

ΔABC of P is a concurrence divides each median in the ratio 2:1

AQ is median

PQ =3.5 cm

$$\frac{AP}{PQ} = \frac{2}{1}$$

$$\frac{AP}{3.5} = \frac{2}{1}$$

$$AP = \frac{2 \times 3.5}{1}$$

$\therefore AP = 6.5$ cm

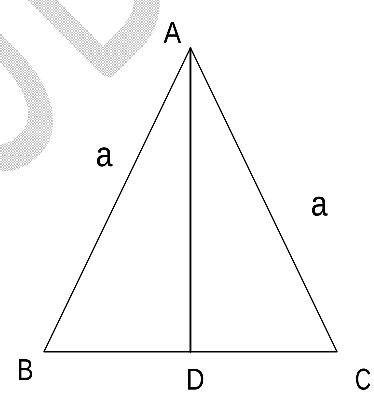
$AQ = AP + PQ$

$= 6.5 + 3.5$

$= 10$ cm

$\therefore AP = 6.5$ cm and $AQ = 10$ cm

Q.28) Length of each side of equilateral triangle is ‘a’ units. Find the height of triangle.



Solution:

In Δ ABC be an equilateral triangle-

$\therefore AB = BC = AC$

Suppose, $AB = BC = AC = a$

Seg AD ⊥ side BC

$\therefore \angle ADB = 90^0$

In Δ ADB ,

$\angle ADB = 90^0$

$\angle ABC = 60^0$……..(Angle of an equilateral triangle)

$\therefore \angle BAD = 30^0$……..(Remaining angle of triangle)

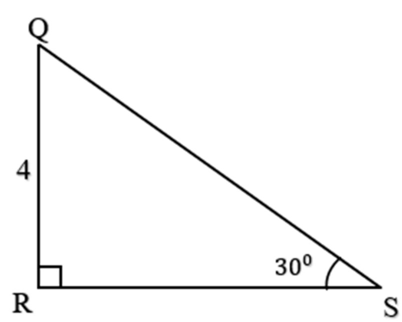
$\therefore 30^0 - 60^0 - 90^0$……..(By triangle theorem))

$AD = \frac{\sqrt{3}}{2} AB$……..

$\therefore AD = \frac{\sqrt{3}}{2} a$

$\therefore$ Height of equilateral triangle $= \frac{\sqrt{3}}{2} a$

Q.29) In .Δ QRS $\angle R = 90^0$, $\angle S = 30^0$, $QR = 4$ units then find RS and QS



Solution - Δ QRS is $30^0 - 60^0 - 90^0$ triangle-

$QR = \frac{1}{2}QS$...... (side opposite to 30^0)

$4 = \frac{1}{2}QS$

$4 \times 2 = QS$

$8 = QS$

$\therefore QS = 8$ units

$RS = \frac{\sqrt{3}}{2} \times QS$...... Side opposite to 60^0)

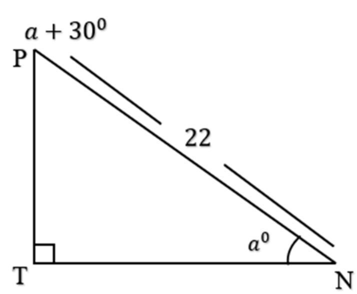
$RS = \frac{\sqrt{3}}{2} \times 8$

$RS = 4\sqrt{3}$ units

$\therefore QS = 8$ units $RS = 4\sqrt{3}$ units.

Q.30) In .ΔPTN, $\angle T = 90^0, \angle N = a^0, \angle P = (a + 30)^0$

If $PN = 22$ then find PT and TN.



Solution:

In Δ PTN

$\angle P + \angle T + \angle N = 180^0$...... (Sum of angles of triangles is 180^0)

$\therefore (a + 30)^0 + 90^0 + a^0 = 180^0$

$a + 30^0 + 90^0 + a^0 = 180^0$

$2a^0 + 120^0 = 180^0$

$2a^0 = 180^0 - 120^0$

$2a^0 = 60^0$

$a^0 = \frac{60^0}{2}$

$a^0 = 30^0$

$\angle N = a^0 = 30^0$

$\angle P = (a + 30)^0 = a^0 + 30^0 = 30^0 + 30^0 = 60^0$

Δ PTN is $30^0 - 60^0 - 90^0$ of triangle-

$PT = \frac{1}{2} \times PN$...... (side opposite to 30^0)

$= \frac{1}{2} \times 22$

$\therefore PT = 11$units

$TN = \frac{\sqrt{3}}{2} \times PN$........ (side opposite to 60^0)

$= \frac{\sqrt{3}}{2} \times 22$

$TN = 11\sqrt{3}$ units

$\therefore PT = 11$ units $TN = 11\sqrt{3}$ units

Q.31) In ΔABC AB = 5cm] BC= 8 cm] AC = 10 cm. Write all its angles in the descending order of their measure.

Solution – In Δ ABC]

AB = 5cm, BC = 8cm] AC = 10 cm

10 > 8 > 5

∴ AC > BC > AB……. (1)

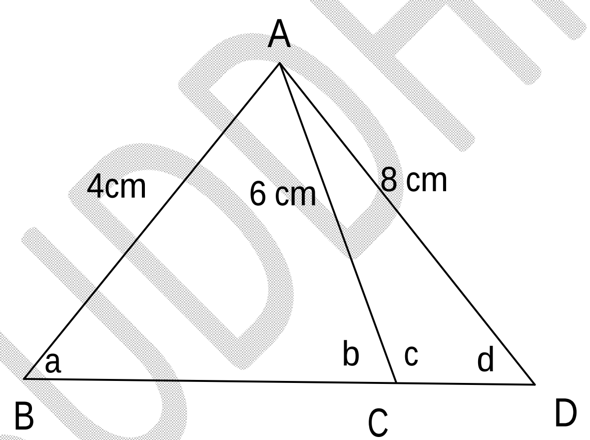
∴ ∠ B > ∠ A…… (II)(Angle opposite to greater side is greater)

BC > ∠ AB…… From II

∴ ∠ A > ∠ C--------- From III (Angle opposite to greater side)

∠ B > ∠ A > ∠ C --------- (From II and III)

Q.32) In the figure AB = 4 cm] AC = 6 cm] AD = 8 cm.
Arrange the angles a, b, c, d in the ascending order of their measure.



Solution – In %Δ ABC

AB = 4 cm ,AC = 6cm

AB < AC

∴ ∠ ACB < ∠ B…… (Angle opposite to greater side is greater)

∴ b ∠ a

∴ $a > b$....... (i)

∠ ACD is an exterior angle of Δ ABC

∴ ∠ ACD > ∠ ABC....... (Property of exterior angles)

∴ C > a ...... (ii)

But a > b......(From I) --- III

∴ c > a > b ...... (IV) .... (From I, II and III)

∠ ACB is an exterior angle of Δ ACD

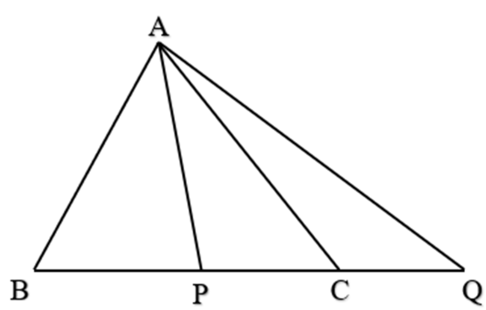
∠ ACB > ∠ ADC...... (Exterior angle property)

∴ b > d..... (V)

(From IV and V)

$c > a >$ b $> d$

Q.33) In figure AB = AC prove that PA < AG



Solution - ∠ APC is an exterior angle Δ ABP

∴∠APC > ∠ B...... (I) .... (Exterior angle property)

∠B = ∠ ACB .... (II) .... [Δ ABC AB = AC ,--- given

That means, ∠B = ∠ ACP...... (III)

$\therefore \angle APC > \angle ACP$…….. (From (I) and II)

$\therefore AC > AP$……. (IV) …. (Side opposite to greater angle is greater)

$\therefore AB > AP$….. (V) [As AB = AC --- Given]

i.e. $AP < AB$

$\angle ACB$ is an exterior angle of ΔACQ .

$\therefore \angle ACB > AQC$ …… (Property of exterior angle)

But,

$\therefore \angle ACB \cong \angle B$ …… (From II)

$\therefore \angle B > \angle AQC$

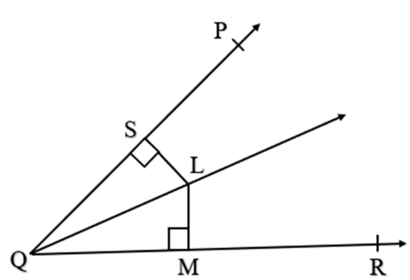
$\therefore AQ > AB$…… (Side opposite to greater angle is greater)

$\therefore AB < AQ$…….. (VI)

$\therefore AP < AB < AQ$ ….. [From (V) and (VI)]

$\therefore AP < AQ$ Proved

Q.34) In the figure point L is in interior of $\angle PQR$. $\angle S \perp$ ray QP, $LM \perp$ ray QR , $LS = LM = 5$cm, $\angle PQR = 76°$ find $\angle PQL$



Solution – $LS \perp$ ray QP and $LM \perp$ ray QR

LS = LM… (Given)

Point L is on the bisector of ∠ PQR ……(By angle bisector theorem)

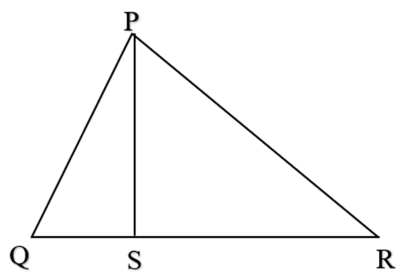
$\angle PQL = \frac{1}{2} \angle PQR$

$\angle PQL = \frac{1}{2} \times 76^0$…… ($\angle PQR = 76^0$ (Given)

$\angle PQL = 38^0$

Q.35) In the adjoining figure $\angle PQS = 40^0$, $\angle PSR = 80^0$

$\angle PRS = 35^0$, Write side PQ, side PS, side PR in descending order of their lengths. Justify the answer.



Solution-

In Δ PQR,

$\angle Q > \angle R$ ($\angle Q = 30^0, \angle R = 35$)…… (Given)

∴ $PR > PQ$……. (I) …. (Side opposite to greater angle is greater)

∴ $\angle PSQ + \angle PSR = 180^0$……. (Angles in linear pair)

∴ $\angle PSQ + 80^0 = 180^0$

∴ $\angle PSQ = 180^0 - 80^0$

∴ $\angle PSQ = 100^0$

In Δ PSQ

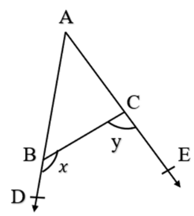
$\angle PSQ > \angle Q$……($\angle PSQ = 100^0, \angle Q = 40^0$)

$\therefore PQ > PS$…… (II) ….. (Side opposite to greater angle is greater)

$\therefore PR > PQ > PS$------- (From I and II)

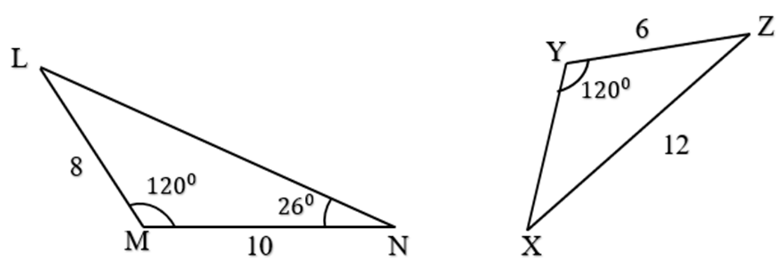
Q.36) In the given figure, the sides AB and AC of Δ ABC has been extended to D and E respectively. If x > y. Show that AB > AC.

Solution –



$x > y \Rightarrow -x < -y$
$\Rightarrow (180 - x) < (180 - y)$
$\Rightarrow \angle ABC < \angle ACB$
$\Rightarrow \angle ACB > \angle ABC$
$\Rightarrow AB > AC$…….. (Sides opposite to larger angle is larger)
$\therefore AB > AC$

Q.37) In the adjoining figure, Δ LMN ≅ Δ ZYX. Measure of the angles and the length of the sides are shown in the figure. Find ∠ N and ∠ Z



Solution - Δ LMN ≅ Δ ZYX

$\therefore \angle L = \angle Z, \quad \angle M = \angle Y, \quad \angle N = \angle X$……. (I)

And

$\frac{LM}{ZY} = \frac{MN}{YX} = \frac{LN}{ZX}$……. (II)

$\frac{LM}{ZY} = \frac{LN}{ZX}$

$\therefore \frac{8}{6} = \frac{LN}{12}$

$\therefore LN = \frac{8 \times 12}{6} = \frac{84}{6} = 16$ units

In Δ LMN

$\angle L + \angle M + \angle N = 180^0$…… (Sum of angles of triangles is 180^0)

$\angle L + 120^0 + 26^0 = 180^0$

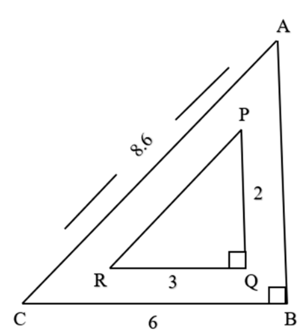
$\therefore \angle L + 146^0 = 180^0$

$\angle L = 180^0 - 146^0$

$\angle L = 34^0$

Now, $\angle L = \angle Z = 34^0$…… [From (1)]

Q.38) Observe the figure and find AB and PR.



Solution -

$\angle B = \angle Q = = 90^0$-------------------- given

$\angle C = R$, -------------------- given

$\therefore \angle A = \angle P$------------------ given

$\therefore$ ΔPQR and ΔABC are similar triangles

$\therefore$ Their sides are in the same proportion

$$\therefore \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$$

$$\therefore \frac{2}{AB} = \frac{3}{6} = \frac{PR}{8.6}$$

$$\therefore \frac{2}{AB} = \frac{3}{6}$$

$$\therefore AB = \frac{6 \times 2}{3}$$

AB = 4 units.

And

$$\therefore \frac{PR}{8.6} = \frac{3}{6}$$

$$\therefore AB = \frac{6 \times 2}{3}$$

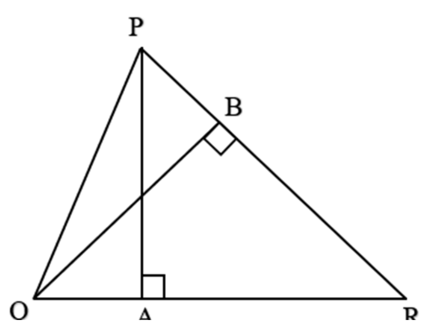
$$\therefore PR = \frac{3 \times 8.6}{6}$$

$\therefore PR = 4.3$ units

Q.39) In ΔPQR seg PA $\perp$ side QR Q – A – R

seg QB $\perp$ side PR ,P – B – R

Then prove that $\Delta PAR \sim \Delta QBR$



Solution – In Δ PAR and Δ QBR ,

$\angle$ PAR $\cong \angle$ QBR……. (Each 90^0)

$\angle$ R $\cong \angle$ R….. (Common angle)

Δ PAR ~ Δ QBR …… (A-A test)

Q.40) If Δ LMN ~ Δ PQR complete the following activity.

1) $\angle$ R $\cong$ $\square$ $\angle$ Q $\cong$ $\square$ $\angle$ P $\cong$ $\square$

2) $\frac{LM}{\square} = \frac{\quad}{\boxed{QR}} = \frac{\square}{\square}$

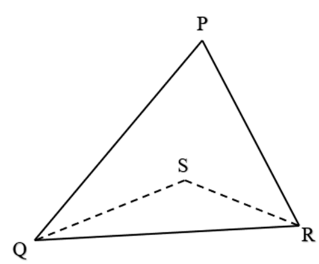
Solution -

Δ LMN ~ Δ PQR

1) $\angle$ R $\cong$ $\boxed{\angle N}$ $\angle$ Q $\cong$ $\boxed{\angle M}$ $\angle$P $\cong$ $\boxed{\angle L}$

2) $\frac{LM}{\boxed{PQ}} = \frac{\boxed{MN}}{QR} = \frac{\boxed{LN}}{\boxed{PR}}$ (corresponding angles of similar triangles)

Q.41) In Δ PQR If PQ $> PR$ and bisectors of $\angle$ Q and $\angle$ R intersect at S.Then show that SQ $> SR$. Fill in the blanks.



Proof – $\angle$ SQR = $\square$.... (i) (Ray QS bisects $\angle$ PQR)

$\angle$ SRQ = $\frac{1}{2}\angle$ PRQ….(ii) …. (Ray RS bisects $\angle$ PRQ)

In Δ PQR

PQ > PR……. (Given)

∴ ∠R > ∠Q……. $\boxed{}$

∴ $\frac{1}{2}$(∠R) > $\frac{1}{2}$(∠Q)……. (Multiplying both sides by $\frac{1}{2}$)

∴ ∠SRQ > $\boxed{}$…… (III) … (From (i) and (ii))

In Δ SQR

∠SRQ > ∠SQR……. [From (iii)]

∴ SQ > SR……. $\boxed{}$

Solution

∠SQR = $\boxed{\frac{1}{2}}$ ∠PQR…. (i) (Ray QS bisects ∠PQR)

∠SRQ = $\frac{1}{2}$∠PRQ….(ii) …. (Ray RS bisects ∠PRQ)

In Δ PQR

PQ > PR……. (Given)

∴ ∠R >
∠Q……. $\boxed{\text{Angle opposite to greater side is greater}}$

∴ $\frac{1}{2}$(∠R) > $\frac{1}{2}$(∠Q)……. (Multiplying both sides by $\frac{1}{2}$)

∴ ∠SRQ > $\boxed{\angle SQR}$…… (III) … (From (i) and (ii))

In Δ SQR

∠SRQ > ∠SQR……. [From (iii)]

$\therefore$ SQ > SR……. Side opposite to greater angle is greater

Q.42) Triangle ABC has sides of length 7.8 and 9 units while Δ PQR has perimeter of 360 units. If Δ ABC is similar to Δ PQR then find find then find the sides of Δ PQR

Solution: Sides of Δ ABC are of 7,8 and 9 units-

Perimeter of Δ PQR = 360

$\therefore x + y + z = 360$…… (i)

$\Delta ABC \sim \Delta XYZ$

$\therefore \frac{7}{x} = \frac{8}{y} = \frac{9}{z}$……. (Corresponding sides of similar triangles)

$\therefore \frac{7}{x} = \frac{8}{y} = \frac{9}{z} = \frac{7+8+9}{x+y+z}$…… (Property of equal ratios)

$$\therefore \frac{7}{x} = \frac{8}{y} = \frac{9}{z} = \frac{24}{360}$$

$$\therefore \frac{7}{x} = \frac{8}{y} = \frac{9}{z} = \frac{1}{15}$$

Now,

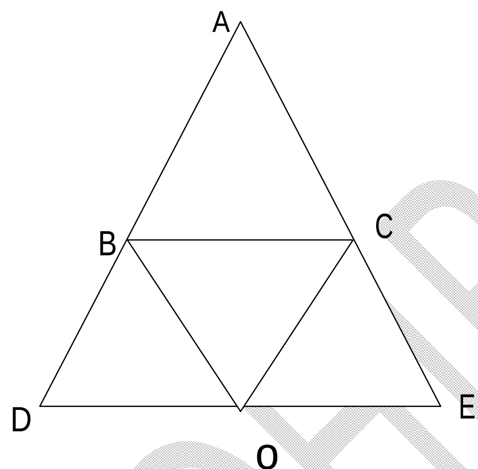
$$\frac{7}{x} = \frac{1}{15} \quad \therefore x = 7 \times 15 = 105$$

$$\frac{8}{y} = \frac{1}{15} \quad \therefore y = 8 \times 15 = 120$$

$$\frac{9}{z} = \frac{1}{15} \quad \therefore z = 9 \times 15 = 135$$

∴ Corresponding sides of Δ PQR are 105 units,120 units,135 units respectively.

Q.43) In a ΔABC, the sides AB and AC are produced to point D and E respectively. The bisector ∠DBC and ∠ECB Intersect at a point o. prove that $\angle BOC = (90° - \frac{1}{2}\angle A)$.



Solution:

Since ABD is a line, we have

$\angle B + \angle CBD = 180°$ (Linear pair)

$\frac{1}{2}\angle B + \frac{1}{2}\angle CBD = 90°$

$\angle CBO = (90° - \frac{1}{2}\angle A)$. -------- (i)

Again, ACE is a straight line

$\therefore \angle C + \angle BCE = 180°$ (Linear pair)

$\frac{1}{2}\angle C + \frac{1}{2}\angle BCE = 90°$

$\frac{1}{2}\angle C + \angle BCO = 90°$

$\angle BCO = (90° - \frac{1}{2}\angle C)$. -------- (ii)

We know that the sum of the angle of a triangle is 180°

From Δ OBC, we get

$\angle CBO + \angle BCO + \angle BOC = 180°$

$(90° - \frac{1}{2}\angle B) + (90° - \frac{1}{2}\angle C) + \angle BOC = 180°$ (using (i)and (ii))

$180°° - \frac{1}{2}(\angle B + \angle C) + \angle BOC = 180°$

$\angle BOC = \frac{1}{2}(\angle B + \angle C)$

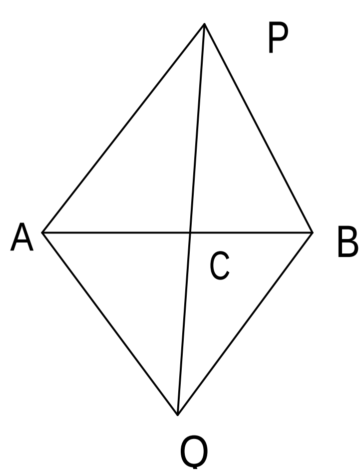
$\angle BOC = \frac{1}{2}(\angle A + \angle B + \angle C) - \frac{1}{2}\angle A$ (adding and subtracting $\frac{1}{2}\angle A$)

$\angle BOC = (\frac{1}{2} \times 180°) - \frac{1}{2}\angle A$ $(\therefore \angle A + \angle B + \angle C = 180°)$

$\angle BOC = (90° - \frac{1}{2}\angle A)$.

Hence $\angle BOC = (90° - \frac{1}{2}\angle A)$.

Q.44) In the given figure, AB is a line segment P and Q are points an opposite sides of AB such that each of them is equidistant from the point A and B show that line PQ is Perpendicular bisector of AB



Solution:

Given : A line segment AB and two points P and Q such that

PA= PB and QA = QB

Prove : let AB and PQ intersect at C. Then we have to prove

That AC =BC and ∠ACP = 90°

Proof: In ΔPAQ and Δ PBQ we have,

PA = PB

QA = QB --------(Given)

PQ = PQ ---------(Common)

∴ ΔPAQ ≅ ΔPBQ -------- (by SSS)

∠APQ = ∠BPQ ------- (i)

Now in Δ PAC and Δ PBC we have

PA = PB -------- (Given)

∠APC = ∠BPC ----- (∠APQ = ∠BPQ in (i))

PC = PC -------- (Common)

∴ ΔPAC ≅PBC (by SSS)

∴ AC = BC ---------- (ii)

And ∠ACP = ∠BCP ------ (iii)

But ∠ACP + ∠BCP = 180° ------ (linear pair)

∴ 2 ∠ACP = 180° ------ (using iii)

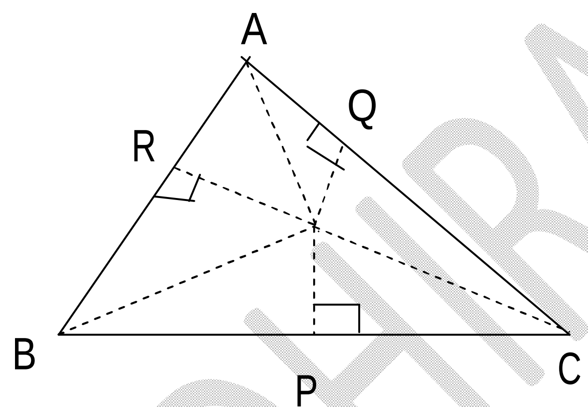
$\angle ACP = 90°$

Hence ,PQ is the perpendicular bisector of AB.

Q.45) In the given figure, the bisectors of ∠B and ∠C of

ΔABC meet at I. If IP ⊥BC, IQ ⊥CA and IR ⊥ AB

Prove that (i) IP= IQ = IR (ii) IA bisects ∠A.



Solution:

Given : A Δ ABC in which BI and CI are the bisectors of ∠B

And ∠C respectively. IP ⊥BC, IQ ⊥CA and IR ⊥ AB

To Prove: (i) IP= IQ = IR (ii) IA bisects ∠A.

Proof: i) In ΔIPC and ΔIQC, we have

$\angle IPC = \angle IQC = 90°$----- (given)

∠ICP = ∠ICQ -------(CI is the bisector of ∠C)

CI – CI --------- (common)

∴ ΔIPC ≅ ΔIQC ------- (by AAS)

∴ IP = IQ

Similarly, IQ = IR

Hence IP = IQ = IR

ii) In ΔIQA and Δ IRA, we have

IQ = IR ------- (proved in i)

∠IQA = ∠ IRA (each 90°)

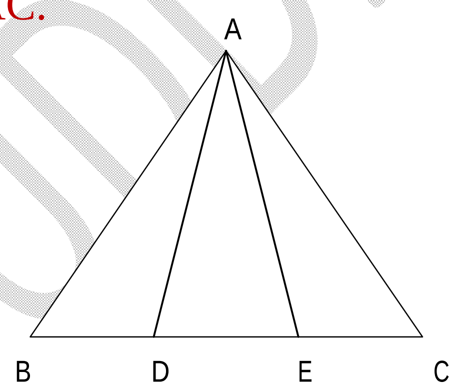
Hypotenuse IA = hypotenuse IA (common)

∴ ΔIQA ≅ ΔIRA

∴ ∠IQA ≅ ∠IRA

Hence, IA bisects ∠A

Q.46) In the figure D and E points on the base BC of a ΔABC Such that AD = AE and ∠BAD = ∠CAE. Prove that AB = AC.



Solution:

Given: D and E are points on the base BC of a ΔABC Such that AD = AE and ∠BAD = ∠CAE.

To Prove : AB = AC

Proof : In Δ ADE,

∴ AD = AE ------ (Given)

∴ ∠ADE = ∠AED --- (i) (angles opposite to equal sides of a triangle are equal)

In Δ ABD,

∠ADE = ∠BAD + ∠ ABD ------- (ii)

(An exterior angle of a triangle is equal to the sum of its interior opposite angles)

In Δ AEC,

∠AED = ∠CAE+ ∠ ACE ------- (iii)

(An exterior angle of a triangle is equal to the sum of its two interior opposite angles)

From (i), (ii) and (iii)

∠BAD + ∠ABD = ∠CAE+ ∠ACE

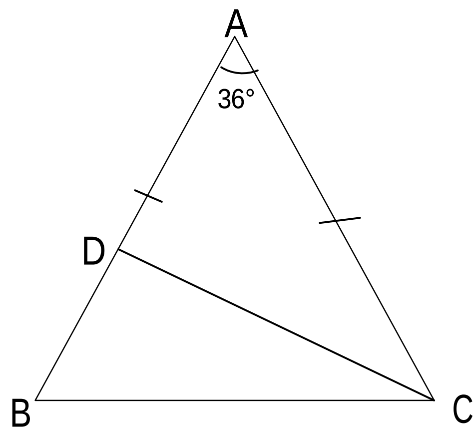
∠ABD = ∠ACE (∵ ∠BAD =∠CAE given)

∠ABC = ∠ACB

∴ AB = AC

(sides opposite to equal angles of a triangle are equal)

Q.47) In ΔABC, AB = AC, ∠A = 36°.the internal bisector of ∠C meets at D. Prove that AD = BC.



Solution:

Given : In ΔABC. AB = AC ∠A = 36°. The internal bisector of ∠C meets AB at D.

To Prove : AD = BC

Proof : In ΔABC,

∠ BAC + ∠ABC + ∠ACB= 180°

(sum of the angle of a triangle is 180°)

36° + ∠ABC + ∠ACB= 180°

∠ABC + ∠ACB= 144° ---------- (i)

In ΔABC,

∴AB = AC --------- (given)

∴ ∠ABC = ∠ACB ---------- (ii) (Angles opposite to equal sides of a triangle are equal)

From (i) and (ii)

∠ABC = ∠ACB

$=\frac{144}{2}$ = 72° --------- (iii)

CD bisects ∠ACB

∠ACD = ∠BCD

$= \frac{1}{2} = 72° = 36°$ --------- (IV)

In Δ ACD,

∵ ∠DAC = ∠DCA

AD = DC ------------ (V)

(Sides opposite to equal angles of a triangle are equal)

In Δ ADC,

∠CDB = ∠DAC + ∠DCA (Exterior angle theorem)

∠CDB = 36°+ 36° = 72° -------- (VI)

In Δ DBC,

∵ ∠BDC = ∠DBC

DC = BC ------------ (VII)

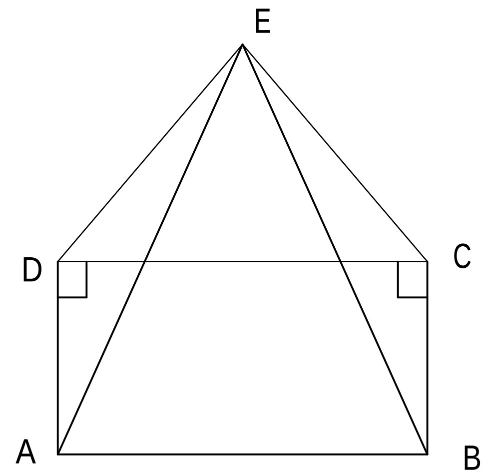
(Sides opposite to equal angles of a triangle are equal)

From (V) and (VII)

AD = BC

Q.48) In figure ABCD is square and ∠ DEC is an equilateral Triangle. Prove that

I) Δ ADE ≅ ΔBCE ii) AE = BE iii) ∠ DAE = 15°



Solution:

Given: ABCD is square and Δ DEC is an equilateral triangle.

To Prove: I) Δ ADE ≅ ΔBCE ii) AE = BE iii) ∠DAE = 15°

Proof : In Δ ADE *and* ΔBCE

AD = BC----(∵ABCD is a square ∴AB = BC = CD =DA)

DE = CE -------- (ΔEDC is equilateral ∵ ED = DC = CE)

∠ EDA = ∠ECB

∴ ΔEDC is equilateral

∴ ΔEDC= ∠ECD --------- (i)

∴ ΔABCD is a square

∴ ∠ADC = ∠BCD --------- (ii)

Adding (i) and (ii)

∴ ∠EDC + ∴ ∠ADC = ∴ ∠ECD +∴ ∠BCD

∴ ∠EDA = ∠ECB

∴ ΔADE ≅ ΔBCE -------- (by SAS)

ΔADE ≅ ΔBCE ------ (Proved in i)

∴AE = BE

In ΔDAE

DE = DA ---------- (given)

∴ ∠DAE = ∠DEA ----(Angles opposite to equal sides of a triangle are equal)

∠ADE + ∠DAE +∠DEA = 180° (Angle sum property of a triangle)

(∠ADC + ∠EDC) + ∠DAE + ∠DEA= 180°

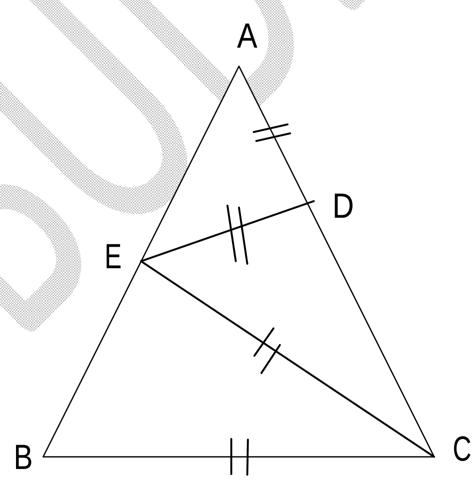
(90° + 60°) + ∠DAE + ∠DEA= 180°

∠DAE + ∠DEA= 30° ---------- (ii)

From (i) and (ii)

∠DAE = 15°= ∠DEA

Q.49) In the given figure AB = AC. D is a point on AC and E on AB such that AD = ED = EC = BC prove that∠A: ∠B = 1:3



Solution:

Let ∠A = x ° -------- (i)

In ΔAED

$AD = ED$ --------- (Given)

$\angle A = \angle DEA$ ----- (Angles opposite to equal sides of a triangle are equal)

$x° = \angle DEA$

$\angle DEA = x°$

In ΔEDC,

$\angle EDC = \angle DEA + \angle A$ (An exterior angle of a triangle is equal to the sum of its two interior opposite angle)

$= x° + x°$ (from (i) and (ii))

$= 2x°$ -------- (iii)

In ΔCED,

$\therefore EC = ED$

$\therefore \angle ECD = \angle EDC$ (Angles opposite to equal sides of a triangle are equal)

$\therefore \angle ECD = 2x°$ --------- (iv) (from iii)

In ΔAEC,

$\angle BEC = \angle ECD + \angle EAC$ (An exterior angle of a triangle is equal to the sum of its two interior opposite angle)

$\angle BEC = 2x° + x°$ (from iii and (i))

$\therefore \angle BEC = 3x°$

In ΔBCE ,

BC = CE

∠CBE = ∠BEC (Angles opposite to equal sides of a triangle are equal)

∠B= 3x° ------- (vi) (from V)

∴ ∠BEC = 3x°

∠B= 3∠A --------- (from i)

∠A : ∠B = 1 :3.

Q.50) If Δ LMN ~ Δ PQR complete the following activity.

1) ∠ R ≅ $\boxed{}$ ∠ Q ≅ $\boxed{}$ ∠ P ≅ $\boxed{}$

2) $\frac{LM}{\boxed{}} = \frac{}{\boxed{QR}} = \frac{\boxed{}}{\boxed{}}$

Solution -

Δ LMN ~ Δ PQR

1) ∠ R ≅ $\boxed{\angle N}$ ∠ Q ≅ $\boxed{\angle M}$ ∠P ≅ $\boxed{\angle L}$

2) $\frac{LM}{\boxed{PQ}} = \frac{\boxed{MN}}{QR} = \frac{\boxed{LN}}{\boxed{PR}}$ (corresponding angles of similar triangles)

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