CHAPTER 2

ARITHMETIC PROGRESSION

LONG QUESTIONS

Q. 1

Find out if the following sequence is an A.P.? If the sequence is AP, find the common difference.

-12, - 8, -4, 2,…

SOLUTION:

Here the first term $t_1 = -12$

$t_2 - t_1 = - 8 (-12) = -8 + 12 = 4$

$t_3 - t_2 = - 4 (- 8) = - 4 + 8 = 4$

The difference between any two consecutive terms is constant.

ANS: The given sequence is an A.P. The common difference = d = 4.

Q. 2

Find out if the following sequence is an A.P.? If the sequence is AP , find the common difference.

$\mathbf{2}, \quad \mathbf{2+\sqrt{2}}, \quad \mathbf{2+2\sqrt{2}}, \quad \mathbf{2+3\sqrt{2}}$

SOLUTION:

Here the first term $t_1 = 3$

$$t_2 - t_1 = 2 + \sqrt{2} - 2 = \sqrt{2}$$

$$t_3 - t_2 = 2 + 2\sqrt{2} - \left(2 + \sqrt{2}\right) = \sqrt{2}$$

$$t_4 - t_3 = 2 + 3\sqrt{2} - \left(2 + 2\sqrt{2}\right) = \sqrt{2}$$

The difference between any two consecutive terms is constant.

Ans: The given sequence is an A.P. The common difference is $d = \sqrt{2}$

Q. 3

First term of an AP is a & common difference is d, a = - 2.25, d = 4. Write the A.P.

SOLUTION:

$a = t_1 = - 2.25$,

$t_2 = t_1 + d = -2.25 + 4 = 1.75$

$t_3 = t_2 + d = 1.75 + 4 = 5.75$,

$t_4 = t_3 + d = 5.75 + 4 = 9.75$

Ans.: - - 2.25, 1.75, 5.75, 9.75 is the required A.P.

Q. 4

Find the first term and common difference for the A.P. 0.9, 1.2, 1.5, 1.8…

SOLUTION:

Here, $t_1 = 0.9$, $t_2 = 1.2$, $t_3 = 1.5$, $t_4 = 1.8$, …

$d = t_2 - t_1 = 1.2 - 0.9 = 0.3$,

$d = t_3 - t_2 = 1.5 - 1.2 = 0.3$,

$d = t_4 - t_3 = 1.8 - 1.5 = 0.3$.

Ans: The first term $a = 0.9$ and $d = 0.3$

Q. 5

Find the first term and common difference for the A.P. $\frac{1}{11}, \frac{3}{11}, \frac{5}{11}, \frac{7}{11}$

SOLUTION:

$$t_1 = \frac{1}{11}, t_2 = \frac{3}{11}, t_3 = \frac{5}{11}, t_4 = \frac{7}{11}, \ldots$$

$$d = t_2 - t_1 = \frac{3}{11} - \frac{1}{11} = \frac{3-1}{11} = \frac{2}{11}$$

$$d = t_3 - t_2 = \frac{5}{11} - \frac{3}{11} = \frac{5-3}{11} = \frac{2}{11}$$

$$d = t_4 - t_3 = \frac{7}{11} - \frac{5}{11} = \frac{7-5}{11} = \frac{2}{11}$$

Ans: The first term a = $\frac{1}{11}$ **and d =** $\frac{2}{11}$

Q. 6

Decide whether the following sequence is an A.P. If so, find the 12th term of the progression:

-12, -5, 2, 9, 16, 23, 30,…

SOLUTION:

Here, $a = t_1 = -12$, $t_2 = -5$, $t_3 = 2$, $t_4 = 9$, $t_5 = 16$, …

$t_2 - t_1 = -5 - (-12) = -5 + 12 = 7$

$t_3 - t_2 = -2 - (-5) = -2 + 5 = 7$

$t_4 - t_3 = 9 - 2 = 7$

$t_5 - t_4 = 16 - 9 = 7$

The common difference $d = 7$ … (a constant number)

therefore, the given sequence in an A.P.

$t_n = a + (n - 1)\, d$ **for 12 th term $n = 12$**

$t_{12} = -12 + (12 - 1) \times 7$ **…. (Substituting the values)**

$= -12 + 11 \times 7$

$= -12 + 77$

$T_{12} = 65$

Ans: The given sequence is an A.P.

The 12^{th} term of the A.P is 65.

Q. 7

How many two-digit natural numbers are divisible by 2?

SOLUTION:

Two -digit natural numbers divisible by 2 are 12, 14, 16, …, 98

The smallest and the biggest two-digit natural numbers divisible by 2 are 12 and 98.

Here, a = 12, d = 2, t_n = 98.

$t_n = a + (n - 1)\, d$

$98 = 12 + (n - 1) \times 2$

$(n - 1)\, 2 = 98 - 12$

$(n - 1)\, 2 = 86$

$n - 1 = \frac{86}{2}$

$n - 1 = 43$

$n = 43 + 1$

n = 44

Ans: There are 44 two-digit numbers divisible by 2

Q. 8

Mr. Rahul borrows ₹ 3440 and agrees to pay to repay with a total interest of ₹ 1360 in 12 monthly installments. Each installment being less than the preceding one by ₹ 40, find the amount of the first and last installment.

SOLUTION:

The amount repaid = ₹ 3440 + ₹ 1360 = ₹ 4800

The number of installments = 12

therefore n = 12, $S_n = S_{12} = 4800$

Each installment is Rs.40 less than the preceding one.

d = - 40, this is an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = 4800 = \frac{12}{2}[2a + (12-1) \times (-40)]$$

(Substituting the values)

$$4800 = 6[2a + 11 \times (-40)]$$

$$6(2a - 440) = 4800$$

$$2a - 440 = \frac{4800}{6}$$

$$2a - 440 = 800$$

$$2a = 800 + 440,$$

$$2a = 1240,$$

$$a = 620$$

The last installment = t_n

$t_n = a + (n - 1)\,d$

$t_{12} = 620 + (12 - 1) \times (-40)$

$= 620 + 11 \times (-40)$

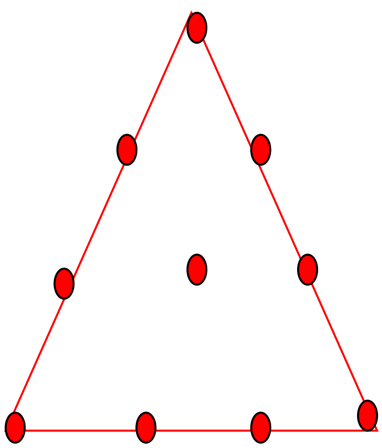
$= 620 - 440$

$t_n = 180$

Ans: The first installment is Rs. 620 and the last installment is Rs. 180.

Q. 9

On a functional day, tree plantation programme was arranged on a land in a triangular shape. Trees are planted such that in a first row there is one tree, in the second row, there are two trees, in the third row three trees and so on. Find the total number of trees in 19 rows.



SOLUTION:

The number of trees increases by 1 in consecutive rows d = 1

There is 1 tree in first row. a = 1

this is an A.P.

There are 19 rows. n = 19.

So, we have to find the total number of trees in 19 rows. It means, we have to find S_{25}.

$$S_n = \frac{n}{2}[2a + (n - 1) \times d]$$

$$S_{19} = \frac{19}{2}[2 \times 1 + (19 - 1) \times 1]$$

$$= \frac{19}{2}[2 + 18]$$

$$= \frac{19}{2} \times 20$$

$$= 19 \times 10$$

$$S_{19} = 190$$

Ans: Total number of trees is 190.

Q. 10

How many natural numbers from 24 to 150 are divisible by 3?

SOLUTION:

The numbers from 24 to 150 which are divisible by 3 are 24, 27, 30,…., 147, 150.

This is an A.P. with first term a = 24 and d = 3.

Let there are n terms in this A.P.

Then $t_n = 150$

$t_n = a + (n – 1) d$

$150 = 24 + (n – 1) \times 3$

$(n – 1) \times 3 = 150 – 24$

$(n – 1) \times 3 = 126$

$n – 1 = 126/3,$

$n – 1 = 42$

$n = 42 + 1$

n = 43

Ans: There are 43 terms divisible by 3 in this A.P.

Q. 11

Find the 23[th] term of the A.P. 9, 4, -1, -6, -11,…

SOLUTION:

Here $a = t_1 = 9, t_2 = 4, t_3 = -1, t_4 = -6, t_5 = -11,\ldots$

$d = t_2 - t_1 = 4 - 9 = -5$

$t_n = a + (n - 1)\, d$

$t_{23} = 9 + (23 - 1) \times (-5)$

$= 9 + 22 \times (-5)$

$= 9 - 110$

$t_{23} = -101$

Ans: The 23[th] term is -101

Q. 12

Find four consecutive terms in A.P. whose sum is 24 and the sum of third and fourth terms is 28.

SOLUTION:

Let the four consecutive terms in an A.P. be $a - d$, a, $a + d$ and $a + 2d$.

From the first condition,

$$(a - d) + a + (a + d) + (a + 2d) = 24$$

$$a - d + a + a + d + a + 2d = 24$$

$$4a + 2d = 24$$

Dividing both sides by 2,

$$2a + d = 12 \qquad (1)$$

From the second condition,

$$(a + d) + (a + 2d) = 28$$

$$a + d + a + 2d = 28$$

$$2a + 3d = 28 \qquad (2)$$

Subtracting eqn (1) from (2)

$2a + 3d = 28$

-

$2a + d = 12$

$2d = 16$, $d = 8$

Substituting $d = 8$ in eqn 1

$2a + 8 = 12$

$2a = 12 - 8$

$2a = 4$, $a = 2$

Taking $a = 2$, $d = 8$

First term = $a - d = 2 - 8 = -6$

Second term = $a = 2$

Third term = $a + d = 2 + 8 = 10$

Fourth term = $a + 2d = 2 + 2(8) = 2 + 16 = 18$

Ans: The four consecutive terms are - 6, 2, 10 and 18.

Q. 13

In an arithmetic series, out of four consecutive terms sum of second and third term is 18. Multiplication of first and fourth terms is 45. Find the four consecutive terms.

SOLUTION:

Let the four consecutive terms be

$a - 3d, a - d, a + d, a + 3d$.

By first condition, $(a - d) + (a + d) = 18$

$2a = 18$

$a = 9$

By second condition,

$(a - 3d)(a + 3d) = 45$

$a^2 - 9d^2 = 45$

$81 - 9d^2 = 45$ $\quad\quad$ $(a=9)$

$81 - 45 = 9d^2$

$9d^2 = 36$

$d^2 = 4$

$d = \pm 2$

For the four consecutive terms

$a - 3d, a - d, a + d, a + 3d.$

Ans:

(i) When $a = 9$, $d = 2$, four consecutive terms are

3, 7, 11, 15

(ii) When $a = 9$, $d = -2$, four consecutive terms are

15, 11, 7, 3.

Q.14

Find S_{18} in following arithmetic series.

1, 7, 13, 19, …

SOLUTION:

Let a = 1

Difference = d = 7 – 1 = 13 – 7 = 19 – 13 = 6

$t_n = a + (n - 1)\, d$

$t_{18} = 1 + (18 - 1) \times 6$

$= 1 + 17 \times 6$

$= 1 + 102$

$t_{18} = 103$

Ans: 18^{th} term = 103

Q. 15

There were 4010 literate people in a town in the year 2010. If this rate increases by 400, what will be the number of literate people in the year 2020? Find the formula to search number of literate people after n number of years.

SOLUTION:

Rise in population each year = d = 400

d is stable. So, this is Arithmetic progression.

$t_n = a + (n - 1)\,d$

$t_{10} = 4010 + (10 - 1) \times 400$

$= 4010 + 9 \times 400$

$= 4010 + 3600$

$t_{10} = 7610$

After n years the literate population:

$t_n = a + (n - 1) d$

$= 4010 + (n - 1) \times 400$

$= 4010 + 400n - 400$

$= 3610 + 400n$

Ans: In the year 2020 the literate population = 7610

After n years literate population = 3610 + 400n

Q. 16

An auditorium consists of 20 chairs in first row, 24 chairs in second row and 28 chairs in third row. Find the total number of chairs in the auditorium if the total number of rows is 30.

SOLUTION:

Difference between number of chairs in the consecutive rows is d = 4

So this Arithmetic Progression.

a = 20, d = 4, n = 30

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{30} = \frac{30}{2}[2(20) + (30-1)4]$$

$$= 15(40 + 29 \times 4)$$

$$= 15(40 + 116)$$

$$= \mathbf{15 \times 156}$$

$$= \mathbf{2340}$$

Ans : Total number of chairs = 2340

Q. 17

Which term of A.P. 11, 8, 5, 2… is -154?

SOLUTION:

Here, $a = t_1 = 11$, $t_2 = 8$, $t_3 = 5$, $t_4 = 2$…

$d = t_2 – t_1 = 8 – 11 = -3$

Let n^{th} term of the given A.P. be -154

$t_n = a + (n – 1) d$

$-154 = 11 + (n – 1) (-3)$

$(n – 1) (-3) = -154 - 11$

$(n – 1) (-3) = -165$

$(n – 1) \times (-3) = -165$

$n - 1 = \frac{-165}{-3}$

$n – 1 = 55$

$n = 55 + 1$

$n = 56$

Ans: The 56th term of the A.P. is 151.

Q. 18

The 15th term of an A.P is 7 more than the 8th term. Find the common difference.

SOLUTION:

$t_n = a + (n - 1) d$

$t_{15} = a + (15 - 1) d$

$t_{15} = a + 14d$ (i)

$t_8 = a + (8 - 1) d$

$t_8 = a + 7d$ (ii)

From the given condition $t_{15} = t_8 + 7$

and from (i) and (ii)

$a + 14d = a + 7d + 7$

$14d - 7d = 7$

$7d = 7$

$d = 1$

Ans: The common difference is 1.

Q. 19

In an A.P. 19^{th} term is 52 and 38^{th} term is 128. Find the sum of first 24 terms.

SOLUTION:

Let the first term of the A.P. be a and common difference d.

$t_{19} = 52$ and $t_{38} = 128$

$t_n = a + (n - 1)d$

$t_{19} = a + (19 - 1)d$

$t_{19} = a + 18d = 52 \ldots (i)$ Putting value of $t_{19} = 52$

$t_{38} = a + (38 - 1)d$

$= a + 37d$

= 128 … (ii) Putting value of t_{38} = 128

Subtracting eqns (i) and (ii)

a + 18d = 52

a + 37d = 128

-19d = -76

d = 4

Putting in

a + 18d = 52

a + 18x4 = 52

a + 72 = 52

a = -20

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{24} = \frac{24}{2}[2(-20) + (24-1)4]$

$= 12(-40 + 23x4)$

$= 12 \times (92 - 40)$

= 28 x (52)

$S_{24} = 1456$

Ans: The sum of the first 24 terms is 1456.

Q. 20

Rs. 800 is invested at 8 percent simple interest. Check if the interest amount at the end of every year is in A.P. If this is an A.P. find amount of interest after 10 years. For this complete the following activity.

SOLUTION:

Simple interest = $\frac{P \times R \times N}{100}$

Simple interest after one year = $\frac{800 \times 8 \times 1}{100}$ **= 64**

Simple interest after 2 years = $\frac{800 \times 8 \times 2}{100}$ **= 128**

Simple interest after 3 years = $\frac{800 \times 8 \times 3}{100}$ **= 192**

According to this the simple interest for 4, 5, 6 years will be 256, 320, 384 respectively.

From this d = 64 and a = 64

Amount of simple interest after 10 years

$t_n = a + (n - 1)\ d$

$t_{10} = 64 + (10 - 1) \times 64$

$= 64 + 9 \times 64$

$= 64 + 576$

$t_{10} = 640$

Ans: Amount of simple interest after 10 years is ₹ 640

Q. 21

Show that 29th term is twice the 19th term.

If the 9th term of an A.P. is zero, then

SOLUTION:

Let the first term of the A.P. be a and the common difference d. $t_9 = 0$

$t_n = a + (n - 1) d$

$t_9 = a + (9 - 1) d$

$0 = a + 8d$, $a + 8d = 0$, $a = -8d$ … (i)

For 19th term,

$t_{19} = a + (19 - 1) d$

$= a + 18d$

$= - 8d + 18d$…. From (i)

$t_{19} = 10d$ … (ii)

For 29th term,

$t_{29} = a + (29 - 1)\ d$

$= a + 28d$

$= -8d + 28d$

$t_{29} = 20d\ldots$ (iii)

From (iii) & (ii),

Ans: $t_{29} = 20d = 2 \times 10d = 2 \times t_{19}$

29th term is twice the 19th term.

Q. 22

Write an A.P. whose a = 15 and d = 1.5

SOLUTION:

$a = t_1 = 15$

$t_2 = t_1 + d = 15 + 1.5 = 16.5$

$t_3 = t_2 + d = 16.5 + 1.5 = 18$

$t_4 = t_3 + d = 18 + 1.5 = 19.5$

Ans: 15,16.5, 18, 19.5 is the required A.P.

Q. 23

Darjeeling's temperature was recorded from Monday to Saturday ,in a week. All readings were in A.P. The sum of temperatures of Saturday and Monday was 5°C more than sum of temperatures of Saturday and Tuesday. If temperature of Wednesday was -30°C then find the temperature on the other five days.

SOLUTION:

The temperatures are in A.P….. (given)

Let the temperatures in °C from Monday to Saturday be $a - 3d$, $a - 2d$, $a - d$, a, $a + d$ and $a + 2d$ respectively.

The temperature on Wednesday is -30°C i.e.

$$a - d = -30 \qquad \text{(i)}$$

The sum of the temperatures on Monday and Saturday = the sum of temperatures on Tuesday and Saturday + 5°C

$(a - 3d) + (a + 2d) = (a - 2d) + (a + 2d) + 5$

$a - 3d = a - 2d + 5$

$2d - 3d = 5$

$d = -5$

Substituting d in eqn (i)

$a - (-5) = -30$

$a + 5 = -30$

$a = -35$

Substituting values of a and d,

$a - 3d = -35 - 3(-5) = -35 + 15 = -20$

$a - 2d = -35 - 2(-5) = -35 + 10 = -25$

$a - d = -35 - (-5) = -30$

$a = -35$

$a + d = -35 + 2(-5) = -35 – 5 = -40$

$a + 2d = -35 + 2(-5) = -35 – 10 = -45$

Ans: Temperatures are as follows: Monday -20°C, Tuesday -25°C, Wednesday -30°C, Thursday -35°C, Friday -40°C, Saturday -45°C.

Q. 24

Write an A.P. whose a = -7 and d = 0.5

SOLUTION:

$a = t_1 = -7$

$t_2 = t_1 + d = -7 + 0.5 = -6.5,$

$t_3 = t_2 + d = - 6.5 + 0.5 = -6,$

$t_4 = t_3 + d = -6 + 0.5 = -5.5.$

Ans: -7, -6.5, -6, -5.5 is the required A.P.

Q. 25

A man pays the installment of ₹ 305 in first month against the loan of ₹ 3250. Each monthly installment being less than the preceding one by ₹ 15, find the total number of months required to repay the loan.

SOLUTION:

Let the total number of months required be n

Since each installment is ₹ 15 less than preceding one. This is an A.P.

d = -15, a = 305, S_n = 3250

$$Sn = \frac{n}{2}[2a + (n - 1)d]$$

$$3250 = \frac{n}{2}[2x305 + (n - 1)(-15)]$$

$$6500 = n[610 - 15n + 15]$$

$$6500 = 625n - 15n^2$$

$$15n^2 - 625n + 6500 = 0$$

$$3n^2 - 125n + 1300 = 0$$

$$3n^2 - 60n - 65n + 1300 = 0$$

$$3n\,(n - 20) - 65(n - 20) = 0$$

$$(n - 20)(3n - 65) = 0$$

$$n = 20 \quad \text{or} \quad 3n - 65 = 0$$

$$n = 20 \quad \text{or} \quad n = \frac{65}{3}$$

Here n is a natural number hence n = 20

Ans: To repay the loan it will take 20 months.

Q. 26 (n73)

Find the sum of first 54 even natural numbers.

SOLUTION: 2, 4, 6, .., 2n are the even natural numbers.

Here, a = t_1 = 2, t_2 = 4, t_3 = 6,…

d = t_2 - t_1 = 4 – 2 = 2

n = 54

$$S_{54} = \frac{54}{2}[2x2 + (54 - 1)d]$$

$$= \frac{54}{2}[4 + (53)2]$$

$$= \frac{54}{2}[4 + 106]$$

$$= \frac{54}{2} \times 110$$

$$= 27 \times 124$$

$$S_{54} = 3384$$

Ans: The sum of the first 54 even natural numbers is 3384.

Q. 27

1,4,7,10, … In this A.P., find out the sum of first n terms. Also find out S_{40}

SOLUTION:

First term = a = 1

d = 4 – 1 = 7 – 4 = 10 – 7 = 3

Sum of first n terms:

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1)3]$$

$$= \frac{n}{2}[2 + (3n-3)]$$

$$= \frac{n}{2}[3n-1]$$

Now, n = 40

$$S_{40} = \frac{40}{2}[3 \times 40 - 1]$$

$$= \frac{40}{2}[120 - 1]$$

$$= 20 \times 119 = 2380$$

$$S_{40} = 2380$$

Ans: The sum of first n terms $S_n = \frac{n}{2}[3n-1]$ **and**

$$S_{40} = 2380$$

Q. 28

The sum of first 10 terms is 135; sum of next 9 terms is 36 in an A.P. Find out S_{70}.

SOLUTION:

Let first term be = a, difference = d

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = 135 = \frac{10}{2}[2a + (10 - 1)d]$$

$$135 = 5[2a + 9d]$$

$$27 = [2a + 9d]$$

$$2a + 9d = 27 \text{ ....} \quad \text{(i)}$$

Sum of Next 9 terms = $S_{(11 \text{ to } 19)} = S_{19} - S_{10}$

$$36 = \frac{19}{2}[2a + 18d] - 135$$

$$\frac{19}{2}[2a + 18d] = 135 + 36$$

$$\frac{19}{2}[2a + 18d] = 171$$

$$19[a + 9d] = 171$$

$$a + 9d = \frac{171}{19} = 9 \text{ ....} \text{ (ii)}$$

Substracting (ii) from (i)

$$2a + 9d = 27$$

$$a + 9d = 9$$

$$- \; - \quad -$$

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a = 18

Putting value of a in (ii)

$18 + 9d = 9$

$9d = 9 - 18 = -9$

$d = -1$

Now n = 70

$t_n = a + (n - 1)d$

$t_{70} = 18 + (70 - 1)(-1)$

$t_{70} = 18 - 69 = -51$

Ans: 70th term of the A.P. is -51

Q. 29

In this A.P. 4, 9, 14, ….find out t_{11}

SOLUTION:

$a = 4$, $d = 9 - 4 = 14 - 9 = 5$

$n = 11$

$t_n = a + (n - 1)\, d$

$t_{11} = 4 + (11 - 1)\, 5$

$\quad = 4 + (11 - 1)\, 5$

$t_{11} = 54$

Ans: value of 11^{th} term = 54

Q. 30

The second and fourth term of an A.P. is 12 and 20 respectively. Find the sum of first 25 terms.

SOLUTION:

$t_2 = 12$, $t_4 = 20$

$t_n = a + (n - 1)d$

$t_2 = a + (2 - 1)d$

$12 = a + d$ ……(i)

$t_4 = a + (4 - 1)d$

$20 = a + 3d$ …….(ii)

Subtracting eqn(i) from eqn (ii)

$12 = a + d$

$20 = a + 3d$

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$8 = 2d$

$$d = 4$$

Putting in value of d in eqn (i)

$$12 = a + 4 \qquad a = 8$$

So, a = 8, d = 4 and n = 25

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{25} = \frac{25}{2}[2 \times 8 + (25 - 1) \times 4]$$

$$= \frac{25}{2} \times [16 + 96]$$

$$= \frac{25}{2} \times \frac{112}{1}$$

$$= 25 \times 56 = 1400$$

Ans. Sum of first 25 terms is 1400

Q. 31

Find the sum of all even numbers between 1 to 150

SOLUTION:

The even numbers from 1 to 150 are 2, 4, 6, 8, …, 148

Here, $a = t_1 = 2$, $d = t_2 - t_1 = 4 - 2 = 2$, $t_n = 148$

First we find n

$$t_n = a + (n - 1)d$$

$$148 = 2 + (n - 1) \times 2$$

$$(n - 1) \times 2 = \frac{148}{2}$$

$$(n - 1) = 74$$

$$n = 74 + 1 \qquad n = 75$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_n = \frac{75}{2}(2 + 148)$$

$$S_n = \frac{75}{2} \times 150 = 5625$$

Ans: The sum of all even numbers from 1 to 150 is 5625

Q. 32

Find the fifth term from the end of the A.P. -11, -8, -5, …, 49.

SOLUTION:

here, a = -11, d = - 8 - (-11) = - 8 + 11 and t_n = 49

$$\mathbf{t_n = a + (n - 1)d}$$

$$\mathbf{49 = -11 + (n - 1) \times 3}$$

$$\mathbf{49 + 11 = (n - 1) \times 3}$$

$$\mathbf{(n - 1) \times 3 = 49 + 11}$$

$$\mathbf{(n - 1) \times 3 = 60}$$

$$\mathbf{(n - 1) = \frac{60}{3}}$$

$$\mathbf{(n - 1) = 20}$$

$$\mathbf{n = 20 + 1 = 21}$$

There are 21 terms in this A.P.

The fifth term from end means (21 – 4) = 17^{th} term

$$\mathbf{t_n = a + (n - 1)d}$$

$$\mathbf{t_{17} = -11 + (17 - 1) \times 3}$$

$$\mathbf{t_{17} = -11 + 16 \times 3 = -11 + 48 = 37}$$

Ans: The fifth term from the end of the given A.P. is $\mathbf{37}$

Q. 33

Leena applied for a job and got selected. She has been offered a job with a starting monthly salary of Rs.8000, with an annual increment of Rs. 500 in her salary. Her salary (in Rs.) for the 1st, 2nd, 3rd,... years will be, respectively 8000, 8500, 9000, what would be her monthly salary for the fifth year?

SOLUTION:

Let us first see what her monthly salary for the second year would be.

It would be Rs. (8000 + 500) = Rs. 8500.

In the same way, we can find the monthly salary for the 3rd, 4th and 5th year by adding Rs. 500 to the salary of the previous year.

So, the salary for the 3rd year

= Rs. (8500 + 500)

= Rs. (8000 + 500 + 500)

= Rs. (8000 + 2 × 500)

= Rs. [8000 + (3 – 1) × 500] (for the 3rd year)

= Rs. 9000

Salary for the 4th year

= Rs. (9000 + 500)

= Rs. (8000 + 500 + 500 + 500)

= Rs. (8000 + 3 × 500)

= Rs. [8000 + (4 – 1) × 500] (for the 4th year)

= Rs. 9500

Salary for the 5th year = Rs. (9500 + 500)

= Rs. (8000 + 500 + 500 + 500 + 500)

= Rs. (8000 + 4 × 500)

= Rs. [8000 + (5 – 1) × 500] (for the 5th year)

= Rs. 10000

Ans: Leena's monthly salary for the fifth year is Rs. 10000

Q. 34

Which term of the AP: 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

SOLUTION:

Here, $a = 21$, $d = 18 - 21 = -3$ $a_n = -81$ and we have to find n.

As $a_n = a + (n - 1) d$,

we have $-81 = 21 + (n - 1)(-3)$

$-81 = 24 - 3n$

$-105 = -3n$

So, $n = 35$

Therefore, the 35th term of the given AP is – 81.

Next, we want to know if there is any n for which $a_n = 0$. If such an n is there, then

$21 + (n - 1)(-3) = 0$,

i.e., $3(n - 1) = 21$

$3n - 3 = 21$

i.e., $3n = 24$

$n = 8$

Ans: 35th term of the given AP is – 81 and, the eighth term is 0.

Q. 35

In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

SOLUTION:

The number of rose plants in the 1st, 2nd, 3rd, . . ., rows are : 23, 21, 19, . . ., 5

It forms an AP. Let the number of rows in the flower bed be n.

Then $a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

As, $a_n = a + (n - 1)\, d$

$$5 = 23 + (n - 1)(-2)$$

$$-18 = (n - 1)(-2)$$

$$n = 10$$

Ans: there are 10 rows in the flower bed.

Q. 36

Kusum puts Rs.100 into her daughter's money box when she was one year old and increased the amount by Rs.50 every year. The amounts of money (in Rs.) in the box on the 1st, 2nd, 3rd, 4th, . . . birthday were

100, 150, 200, 250, . . ., respectively. How much money will be collected in the money box by the time her daughter is 21 years old?

SOLUTION:

Here, a = 100, d = 50 and n = 21. Using the formula :

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{21} = \frac{21}{2}[2(100) + (21 - 1)50]$$

$$S_{21} = \frac{21}{2} \times (200 + 1000)$$

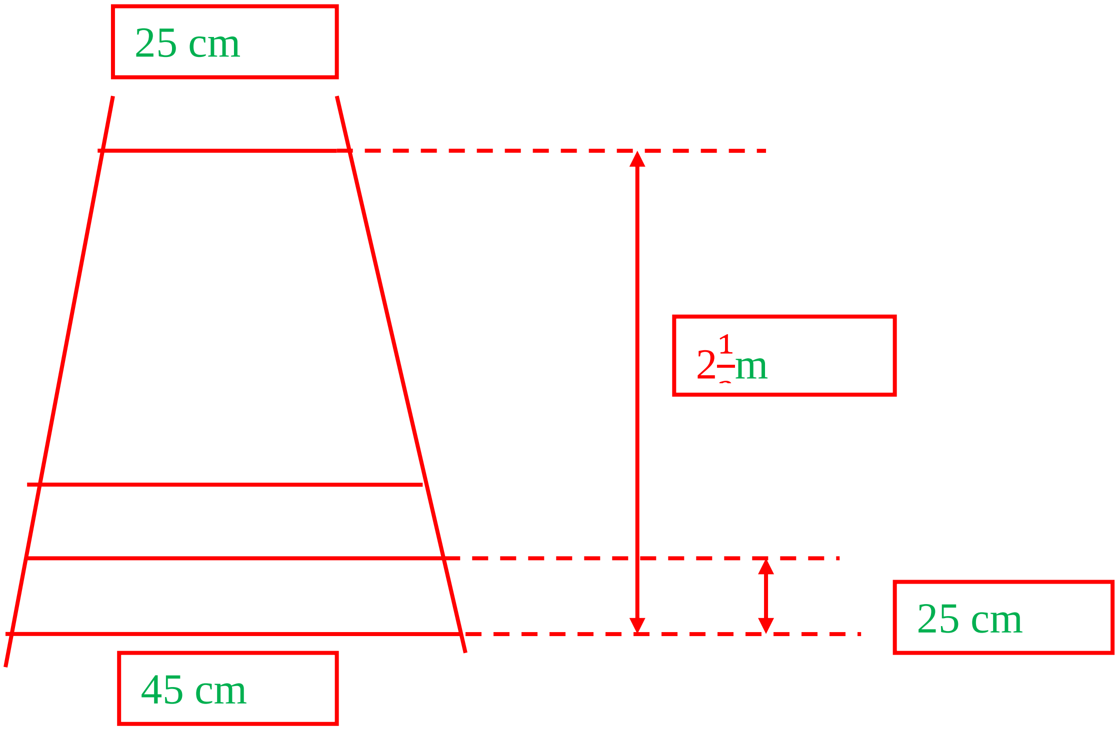
$$S_{21} = \frac{21}{2} \times (1200)$$

$$S_{21} = 12600$$

Ans: The amount of money collected on her 21st birthday is Rs. 12600.

Q. 37

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



SOLUTION:

Total number of rungs are $= \dfrac{2\frac{1}{2}\times 100}{25} + 1 = \dfrac{250}{25} + 1$

$$= 11$$

Now as length of the rungs decrease uniformly, they will be A.P. first term = a = 45,

Last term = L = 25, n = 11

$$S_n = \frac{n}{2}[a + L]$$

$$S_{11} = \frac{11}{2}[45 + 25]$$

$$= \frac{11}{2} \times 70 = 11 \times 35$$

$$S_{11} = 385 \text{ cm}$$

Ans: The length of the wood required for the rungs is 385 cm.

Q. 38

How many terms of AP: 9, 17, 25, … must be taken to give a sum of 636?

SOLUTION:

Here, a = 9, d = 17 – 9 = 25 – 17 = 8 and $S_n = 636$.

Using the formula:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$636 = \frac{n}{2}[2 \times 9 + (n-1)8]$$

$$636 \times 2 = 18n + 8n^2 - 8n$$

$$636 \times 2 = 10n + 8n^2$$

$$636 = \frac{10n+8n^2}{2}$$

$$636 = 5n + 4n^2$$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n+53) - 12(4n+53) = 0$$

$$(4n+53)(n-12) = 0$$

$$4n + 53 = 0 \text{ or } n - 12 = 0$$

$$n = -\frac{53}{4} \text{ or } n = 12$$

Ans: 12 terms must be taken to give a sum of 636.

Q. 39

The angles of a triangle are in A.P., the least being half the greatest. Find the angles.

SOLUTION:

Let the angles be $a - d, a, a + d$; $a > 0, d > 0$

$\because$ Sum of angles = 180°

$\therefore$ a – d + a + a + d = 180°

$\Rightarrow$ 3a = 180° $\therefore$ a = 60° …(i)

By the given condition

a – d = $\frac{a+d}{2}$

$\Rightarrow$ 2a – 2d = a + d

$\Rightarrow$ 2a – a = d + 2d

$\Rightarrow$ a = 3d

$\Rightarrow$ d = $\frac{60}{3}$ = 20° … From (i)

a – d, a, a + d

Ans: Angles are: 60° – 20°, 60°, 60° + 20°

i.e., 40°, 60°, 80°

Q. 40

The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

SOLUTION:

Let 1st term = a, Common difference = d

$a_4 = 0$

$a + 3d = 0$

$a = -3d$ … (i)

To prove: $a_{25} = 3 \times a_{11}$

LHS

$\Rightarrow$: $a_{25} = a + 24d = -3d + 24d = 21d$

RHS

$\Rightarrow a_{11} = a + 10d = -3d + 10d = 7d$

From above, $a_{25} = 3(a_{11})$

Ans: $a_{25} = 3(a_{11})$ (Hence proved)

Q. 41

Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

SOLUTION:

Numbers divisible by both 2 and 5 are 110, 120, 130, …, 990.

Here a = 110,

d = 120 – 110 = 10,

$a_n = 990$

$a + (n – 1) d = a_n = 990$

110 + (n – 1) (10) = 990

(n – 1) (10) = 990 – 110 = 880

$$(n - 1) = \frac{880}{10} = 88$$

∴ n = 88 + 1 = 89

Ans: 89 natural numbers between 101 and 999 which are divisible by both 2 and 5

Q. 42

In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S, denotes the sum of its first n terms.

SOLUTION:

Given: $S_5 + S_7 = 167$

$\Rightarrow \frac{5}{2}[2a + (5 - 1)d] + \frac{7}{2}[2a + (7 - 1)d] = 167$

$\Rightarrow \frac{5}{2}[2a + 4d] + \frac{7}{2}[2a + 6d] = 167$

$\Rightarrow 5(a + 2d) + 7(a + 3d) = 167$

$\Rightarrow 5a + 10d + 7a + 21d = 167$

$\Rightarrow 12a + 31d = 167\ldots$ (i)

Now, $S_{10} = \frac{10}{2}(2a + (10 - 1)\, d = 235$

$\Rightarrow 5[2a + 9d] = 235$

$\Rightarrow 10a + 45d = 235\ldots$(ii)

Solving (i) and (ii), we get $a = 1$ and $d = 5$

$a_1 = 1$

$a_2 = a + d \Rightarrow 1 + 5 = 6$

$a_3 = a + 2d \Rightarrow 1 + 10 = 11$

1, 6, 11…

Ans : Hence A.P. is 1, 6, 11…

Q. 43

Find the value of the middle term of the following A.P.: -6, -2, 2, …, 58.

SOLUTION:

Here a = -6,

d = -2 - (-6) = 4,

$a_n = 58$

As we know, a + (n – 1) d = 58

∴ -6 + (n – 1) 4 = 58

⇒ (n – 1) 4 = 58 + 6 = 64

⇒ (n – 1) = 64/4 = 16

⇒ n = 16 + 1 = 17 (odd)

Middle term = $\frac{n+1}{2}$ term

$\frac{17+1}{2}$ term = 9^{th} term

$a_9 = a + 8d$

= -6 + 8 (4)

= -6 + 32 = 26

Ans: Middle term = 26

Q. 44

The nth term of an A.P. is given by (- 4n + 15). Find the sum of first 20 terms of this A.P.

SOLUTION:

We have, a_n = -4n + 15

Put n = 1, a_1 = -4(1) + 15 = 11

Put n = 2, a_2 = -4(2) + 15 = 7

∴ d = $a_2 - a_1$ = 7 – 11 = - 4

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(11) + (20 - 1)(-4)]$$

$$= 10\,(22 - 76)$$

$$= 10\,(-54)$$

$$S_{20} = -540$$

Ans: sum of first 20 terms is -540

Q. 45

The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

SOLUTION:

First term, $a = 5$,

Last term, $a_n = 45$

Let the number of terms = n

$S_n = 400$

$$\frac{n}{2}(a + a_n) = 400$$

$$\frac{n}{2}(5 + 45) = 400$$

$$\frac{n}{2}(50) = 400$$

n = $\frac{400}{25}$ = 16 = Number of terms

Now, $a_n = 45$

$a + (n - 1)d = a_n$

$5 + (16 - 1)d = 45$

$15d = 45 - 5$

$d = \frac{40}{15} = \frac{8}{3}$

Ans: Number of terms = 16 and the common difference is 8/3

Q. 46

The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms.

SOLUTION:

$$a_{13} = 4a_3$$

$$a + 12d = 4(a + 2d) \ldots \ [\because \ an = a + (n - 1)d]$$

$$a + 12d - 4a - 8d = 0$$

$$4d = 3a$$

$$d = \frac{3a}{4} \ \ldots (i)$$

$$a_5 = 16 \ \ldots \ [\text{Given}]$$

$$a + 4d = 16$$

$$a + 4 \times \left(\frac{3a}{4}\right) = 16 \ \ldots \ [\text{From (i)}]$$

$$a + 3a = 16$$

$$4a = 16$$

$$\Rightarrow a = 4 \,..\, (ii)$$

Putting $a = 4$ in (i), we get $d = 3$

Sum of first ten terms

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(4) + (10 - 1)(3)]$$

$$S_{10} = 5\,(8 + 27) \Rightarrow 5(35)$$

$$S_{10} = 175$$

Ans: Sum of first ten terms is 175

Q. 47

Which term of the A.P. 3, 14, 25, 36, … will be 99 more than its 25th term?

SOLUTION:

Let the required term be n^{th} term, i.e., a_n

Here, $d = 14 – 3 = 11$, $a = 3$

According to the condition,

$a_n = 99 + a_{25}$

$\therefore a + (n – 1) d = 99 + a + 24d$

$(n – 1) (11) = 99 + 24 (11)$

$(n – 1) (11) = 11 (9 + 24)$

$n – 1 = 33$

n = 33 + 1 = 34

∴ 34^{th} term is 99 more than its 25th term.

Q. 48

A sum of ₹1,600 is to be used to give ten cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

SOLUTION:

Here,

S_{10} = 1600, d = -20, n = 10

$$Sn = \frac{n}{2}(2a + (n - 1)d]$$

$$\therefore \frac{10}{2}[2a + (10 - 1)(-20)] = 1600$$

$$2a - 180 = 320$$

$2a = 320 + 180 = 500$

$a = 250$

$\therefore$ 1st prize = a = ₹ 250

2nd prize = $a_2 = a + d = 250 + (-20) =$ ₹ 230

3rd prize = $a_3 = a_2 + d = 230 - 20 =$ ₹ 210

4th prize = $a_4 = a_3 + d = 210 - 20 =$ ₹ 190

5th prize = $a_5 = a_4 + d = 190 - 20 =$ ₹ 170

6th prize = $a_6 = a_5 + d = 170 - 20 =$ ₹ 150

7th prize = $a_7 = a_6 + d = 150 - 20 =$ ₹ 130

8th prize = $a_8 = a_7 + d = 130 - 20 =$ ₹ 110

9th prize = $a_9 = a_8 + d = 110 - 20 =$ ₹ 90

10th prize = $a_{10} = a_9 + d = 90 - 20 =$ ₹ 70

Q. 49

The sum of first n terms of an AP is $3n^2 + 4n$. Find the 25^{th} term of this AP.

SOLUTION:

We have, $S_n = 3n^2 + 4n$

Put n = 25,

$$S_{25} = 3(25)^2 + 4(25)$$

$$= 3(625) + 100$$

$$= 1875 + 100 = 1975$$

Put n = 24,

$S_{24} = 3(24)^2 + 4(24)$

$= 3(576) + 96$

$= 1728 + 96 = 1824$

$\therefore$ 25^{th} term $= S_{25} - S_{24} = 1975 - 1824 = 151$

Ans: 25^{th} term is 151

Q. 50

The sum of the 5^{th} and the 9^{th} terms of an AP is 30. If its 25^{th} term is three times its 8^{th} term, find the AP.

SOLUTION:

$a_5 + a_9 = 30$ … [Given]

$a_n = a + (n - 1)d$

$a_5 = a + 4d, \quad a_9 = a + 8d$

$a + 4d + a + 8d = 30$

$2a + 12d = 30$

$a + 6d = 15$ …[Dividing by 2]

$a = 15 – 6d$ …(i)

Now, $a_{25} = 3(a_8)$

$a + 24d = 3(a + 7d)$ put value of $a= (15 – 6d)$ from(i)

$15 – 6d + 24d = 3(15 – 6d + 7d)$

$15 + 18d = 3(15 + d)$

$15 + 18d = 45 + 3d$

$18d – 3d = 45 – 15$

$15d = 30$

$\therefore d = 2$

From (i), $a = 15 – 6(2)$

$= 15 – 12 = 3$

therefore,

$a = 3$,

$a + d = 5$,

$a + 2d = 7$,

a + 3d = 9

Ans: A.P. is 3,5,7,9,…

Q. 51

The first and the last terms of an AP are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference.

SOLUTION:

Here , a_1 = 8 ; a_n = 65

Given: S_n = 730

$$Sn = \frac{n}{2}(a_1 + a_n)$$

$$\frac{n}{2}(8 + 65) = 730$$

$\frac{n}{2} \times 73 = 730$

$n = 730 \times \frac{2}{73} = 20$

Now, $a_n = a_1 + (n - 1)d = 65$

$8 + (20 - 1)d = 65$

$19d = 65 - 8 = 57$,

$d = 3$

Ans: Common difference d = 3