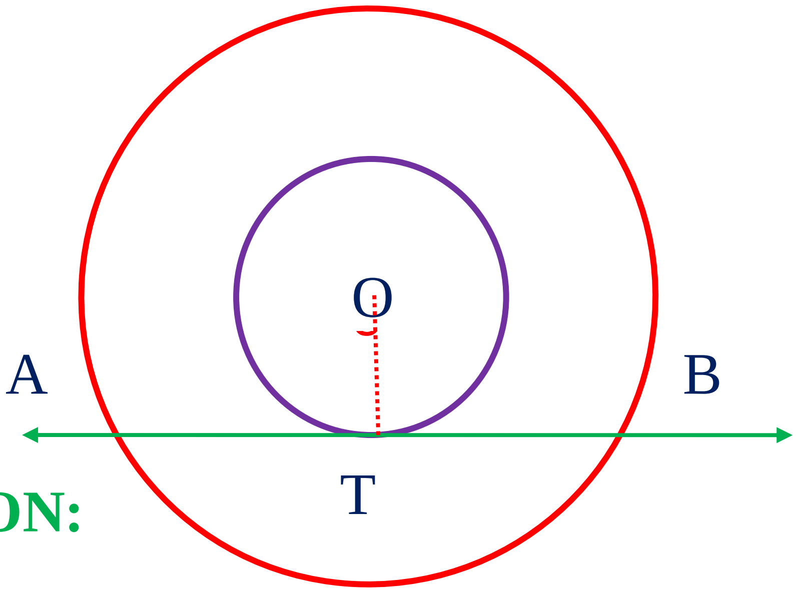
CHAPTER – 3

CIRCLE

LONG QUESTIONS AND ANSWERS

Q. 1

Two concentric circles are shown in the given figure and tangent line AB to small circle touches at point T, prove that point T is the center point of line AB



SOLUTION:

Join OT

In the given figure O is center of circle and line AB touches to the circle at point T

Now Line OT ⊥ Line AB (tangent ⊥ radius)

AT = TB

As, a perpendicular drawn from the center of the circle on the chord, bisects the chord

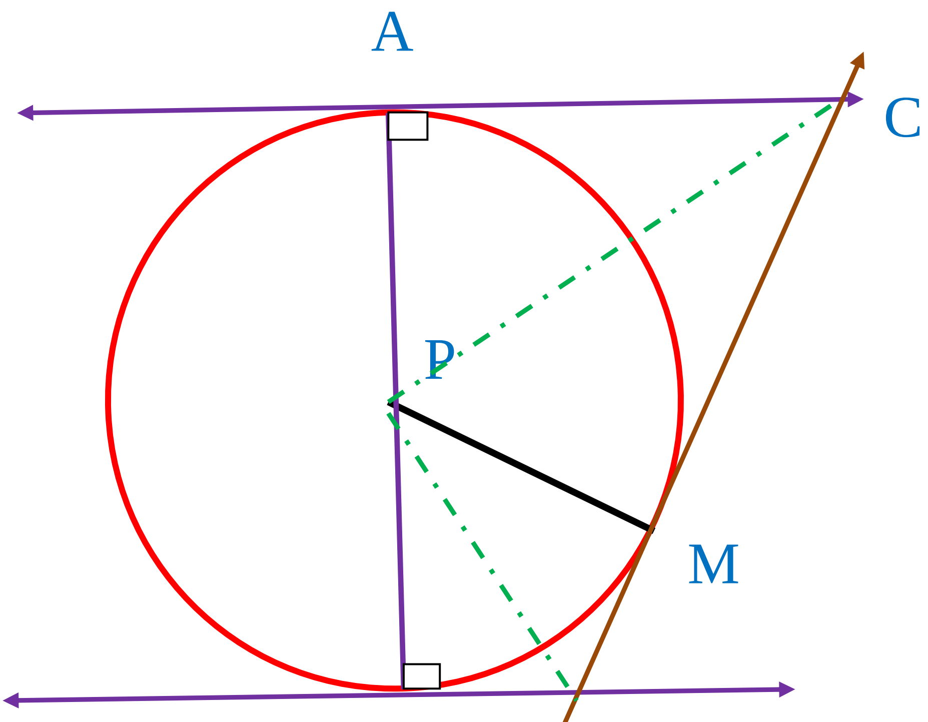
Hence T is center point of line AB

Q. 2

In the given figure AB is the diameter of circle with centre P . line L touches the circle at point M. The tangents drawn at point A and B intersects line L at point C & D respectively. Then prove that ___

(a) AC || BD

(b) $\angle$ CPD = 90^0



SOLUTION:

Join PC AND PD

$\angle$ PAC = $\angle$ BAC = 90^0

$\angle$ PBD = $\angle$ ABD = 90^0

$\angle$ BAC + $\angle$ ABD = 180^0

Line AC || Line BD

In Δ APC & Δ MPC

$\angle$CAP =$\angle$ CMP = 90^0

Hypogenous PC ≅ Hypogenous PC … (Common side)

Line AP ≅ Line MP … (Radii of same circle)

Δ APC ≅ Δ MPC

$\angle$ APC ≅ $\angle$ MPC = x … (angles of similar triangles)

$\angle$ BPD ≅$\angle$ MPD = y … (angles of similar triangles)

But,

$\angle$ APM + $\angle$ BPM = 180^0 … (Pair of angles in line)

$x + x + y + y = 180^0$

$2x + 2y = 180^0$

$x + y = 90^0$

$\angle$ CPD = 90^0

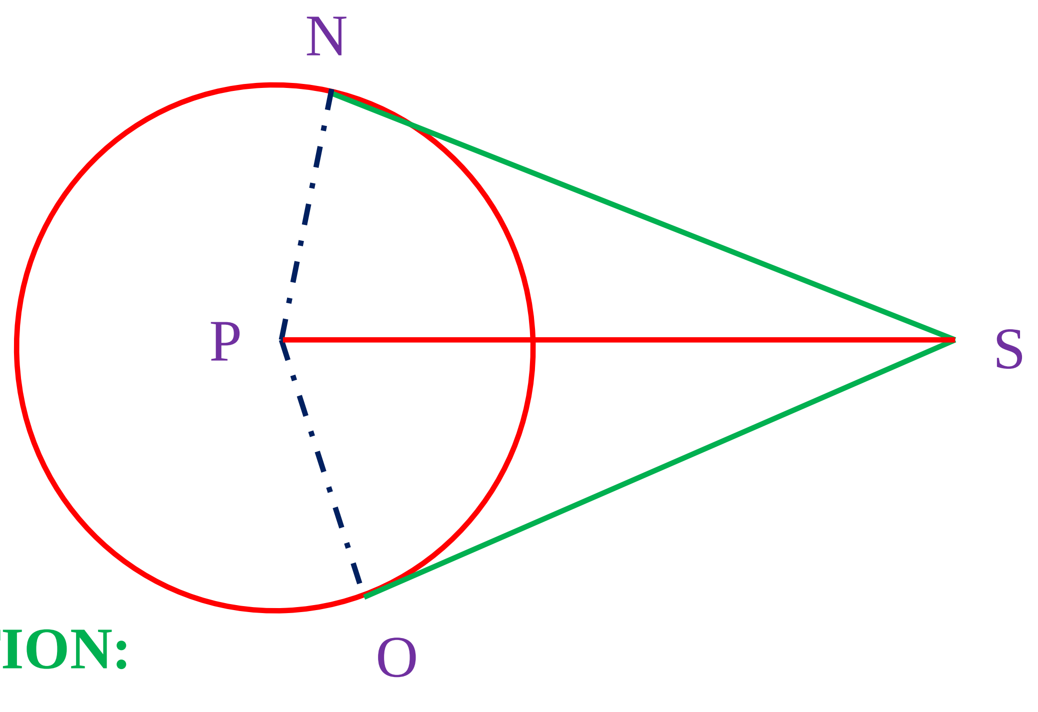
Q. 3 NAVNEET 56/2

P is the center of the circle., Seg SN and seg SO are tangent segments touching to the circle from point S at N and O. If PS = 6 cm and radius of the circle = 3 cm, then

1) What is the length of each tangent segments

2) What is the measure of ∠ NSP

3) What is the measure of ∠ NSO



SOLUTION:

Draw seg PN & seg PO

1) In ΔPNS,

By tangent theorem, $\angle$ PNS = 90^0

By Pythagoras theorem,

$PS^2 = PN^2 + NS^2$

$\therefore\ 6^2 = 3^2 + NS$

$\therefore\ NS^2 = 36 - 9$

$\therefore\ NS^2 = 27$

$\therefore\ NS = \sqrt{27}$

$\therefore\ NS = 3\sqrt{3}$ … By taking square roots of both sides

SN = SO = $3\sqrt{3}$

Length of each tangent segment is $3\sqrt{3}$ cm

2) In ΔSNP,

By tangent theorem $\angle$ PNS = 90^0

PN = 3 cm & PS = 6 cm … (Given)

Thus, PN = $\frac{1}{2}$ PS

Now by converse of $30^0 - 60^0 - 90^0$ triangle theorem,

$\angle$ NSP = 30^0

&

$\angle$ MSP = 30^0

3) By angle addition postulate,

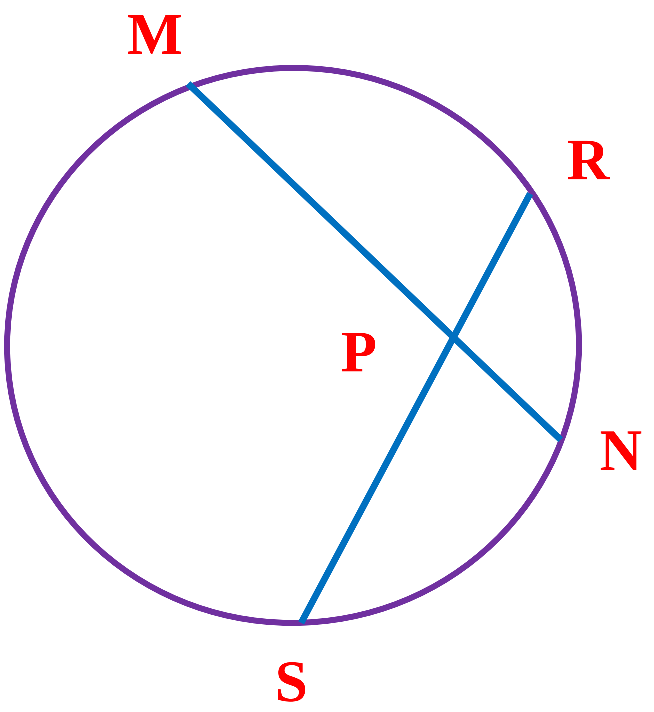
$\angle$ NSO = $\angle$ NSP + $\angle$ OSP

$\angle$ NSO = 30^0 + 30^0

$\angle$ NSO = 60^0

Q. 4

In the figure chord MN and chord RS intersect each other at point P. If PR = 6, PS = 4 and MN = 11 find PN.



SOLUTION:

By theorem of intersecting chords,

PN x PM = PR x PS … (1)

Let PN = x

PM = 11 – x

Substituting the values in (1)

$x(11 - x) = 6 \times 4$

$11x - x^2 - 24 = 0$

$x^2 - 11x - 24 = 0$

$(x - 3)(x - 8) = 0$

$x = 3$ or $x = 8$

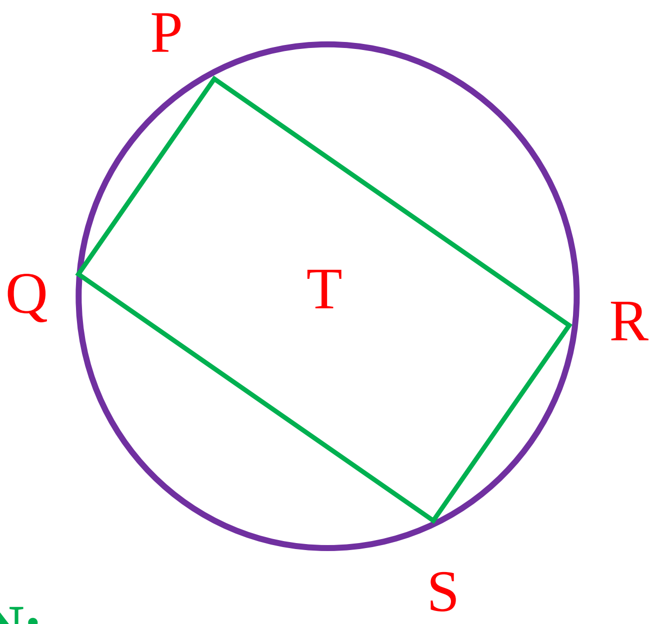
PN = 3 OR PN = 8

Q. 5

In the given figure a rectangle PQRS is inscribed in circle with centre T. Prove that

I) arc PQ ≅ arc SR

II) arc SPQ ≅ arc PQR



SOLUTION:

I) □ PQRS is a rectangle

Chord PQ ≅ chord SR

Opposite sides of a rectangle

Arc PQ ≅ arc SR

Arcs corresponding to congruent chords

II) Chord PS ≅ chord QR

Opposite sides of a rectangle

Arc SP ≅ arc QR

Arcs corresponding to congruent chords

Measures of arc SP & QR are equal

Now,

m(arc SP) + m (arc PQ) = m(arc PQ) + m (arc QR)

Hence,

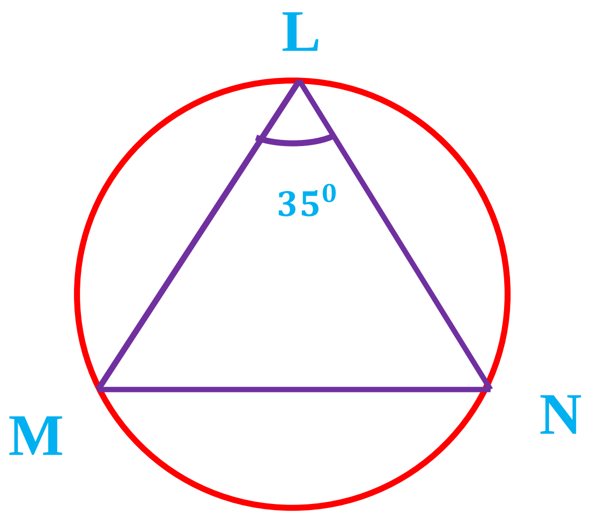
m (arc SPQ) = m (arc PQR)

Arc SPQ ≅ arc PQR

Q. 6

In given figure chord LM ≅ chord LN & ∠L = 35^0

Find (i) m (arc MN) and (ii) m (arc LN)



SOLUTION:

i) ∠L = $\frac{1}{2}$ m (arc MN) … (Inscribed angle theorem)

$\therefore$ 35 = $\frac{1}{2}$ m (arc MN)

$\therefore$ 2 x 35 = m (arc MN) = 70^0

ii) By Definition of measure of arc

m (arc MLN) = 360 - m (arc MN)

= 360 – 70 = 290

Now, chord LM ≅ chord LN

Arc LM ≅ arc LN

But by arc addition property,

m (arc LM) + m (arc LN) = m (arc MLN) = 290

m (arc LM) = m (arc LN) = $\frac{290}{2} = 145^0$

or , chord LM ≅ chord LN

∠M = ∠N … (isosceles triangle)

$2\angle M = 180^0 - 35^0 = 145^0$

$\angle M = \frac{145}{2}$

Now m (arc LN) = 2 x ∠M

$= 2 \times \frac{145}{2}$

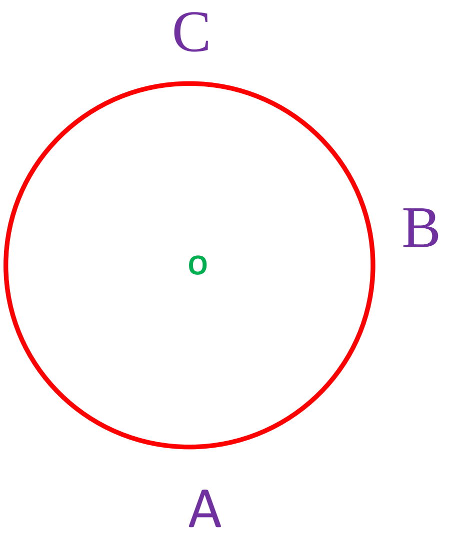
$= 145^0$

Q. 7

ABC are any points on the circle with centre O.

i) Write the names of all arcs formed due to these points.

ii) If m arc (BC) = 110^0 and m arc (AB) = 125^0 find measures of all remaining arcs.



SOLUTION:

i) Name of arcs

Arc AB, arc BC, arc AC, arc ABC, arc ACB, arc BAC

ii) m (ARC) = m (AB) + m (BC)

$= 125^0 + 110^0$

$= 235^0$

m (arc AC) = 360^0 **– m (arc ACB)**

$= 360^0 - 235^0$

$= 125^0$

Similarly,

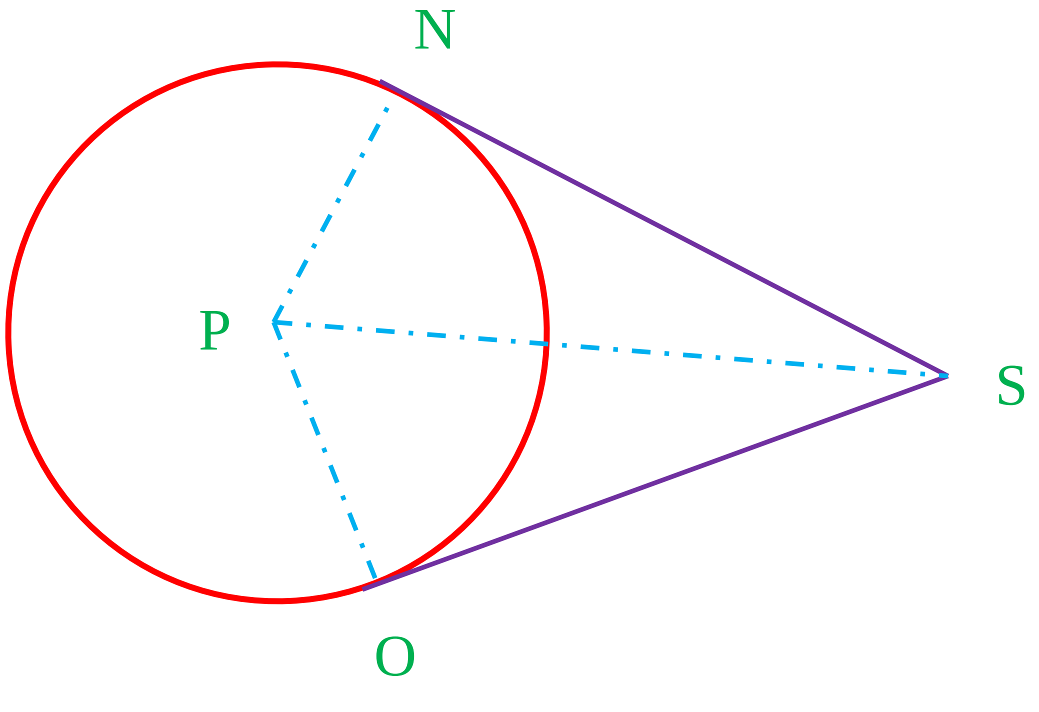
m (arc ACB) = $360^0 - 125^0$

$= 235^0$

& m (arc BAC)= $360^0 - 110^0 = 250^0$

Q. 8 NAVNEET 56/3

In adjoining figure, P is the center of the circle. From point S, seg SN and seg SO are tangent segments touching to the circle N & O. Prove that seg PS bisects ∠ NSO as well as ∠ NPO.



SOLUTION:

In Δ SNP & Δ SOP

∠ SNP = ∠ SOP = 90^0

Hypotenuse PS ≅ Hypotenuse PS

Side PN = Side PM … (radii of same circle)

ΔSNP ≅ ΔSOP … (Hypogenous side theorem)

∠ NSP = ∠ OSP … (c.a.c.t)

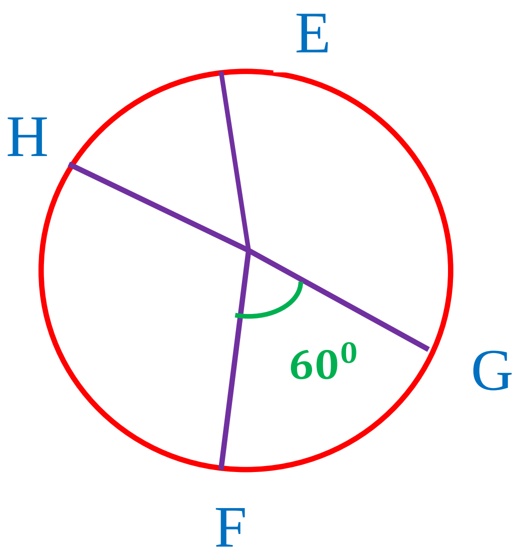
& ∠ NPS = ∠ OPS

Seg PS bisects ∠ NSO as well as ∠ NPO.

Q. 9 NAVNEET 61/1

Points H, E, F, G are concyclic points of a circle with center D. ∠FDG = 60^0 m (arc FHG) = 190^0

Find m (arc EF) & m (arc EFG)



SOLUTION:

m (arc FG) = ∠FDG … (Definition of minor arc)

∠FDG = 60^0

m (arc FG) = 60^0

m (arc EHG) + m (arc FG) + m (arc EF) = 360^0

(Measure of circle is 360^0)

$190^0 + 60^0$ + m (arc EF) = 360^0

m (arc EF) = 360^0 - 250^0

m (arc EF) = 110^0

m (arc EF) + m (arc FG) = m (arc EFG)

By addition of arc,

$90^0 + 60^0$ = m (arc EFG)

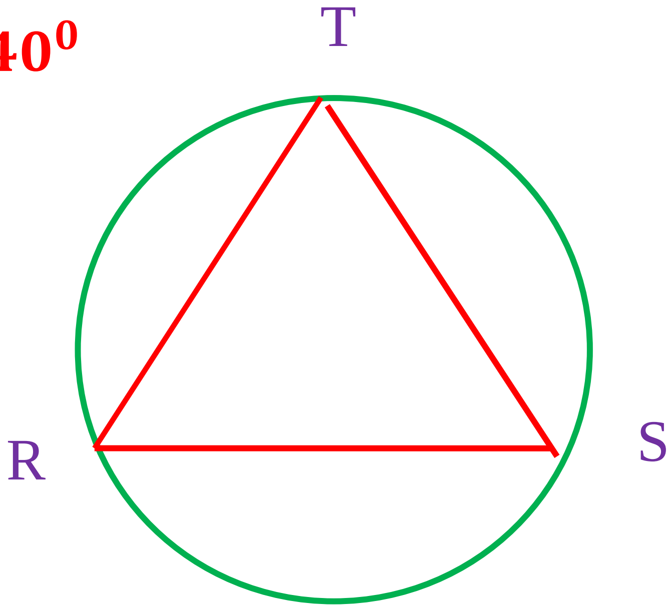
m (arc EFG) = 150^0

Q. 10 NAVNEET 62

ΔRST is an equilateral triangle. Prove that ___

(1) arc ST ≅ arc RT ≅ arc RS

(2) m (arc RST) = 240^0



SOLUTION:

Δ RST is an equilateral Δ

seg ST ≅ seg RT ≅ seg RS… (Equal Sides of an equilateral triangle)

Arcs of the same circle are equal if the related chords are congruent

arc ST ≅ arc RT ≅ arc RS

Let m (arc ST) = m (arc RT) = m (arc RS) = *p*

m (arc ST) = m (arc RT) = m (arc RS) = 360^0 **…**

(Measure of a circle is 360^0**)**

$p + p + p = 360^0$

$3\,p = 360^0$

$p = \frac{360^0}{3}$

$p = 120^0$

m (arc ST) = m (arc RT) = m (arc RS) = 120^0

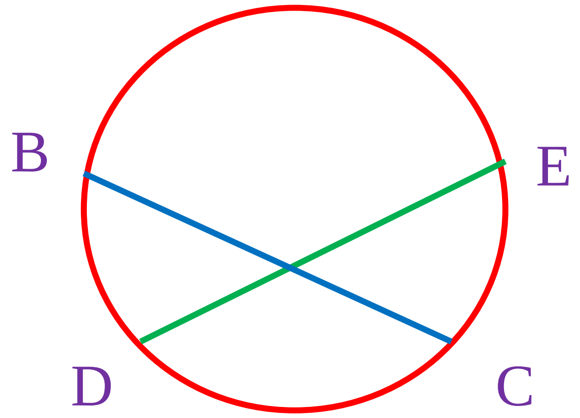
m (arc RST) = m (arc RS) + m (arc ST) … (By arc addition)

m (arc RST) = $120^0 + 120^0$

m (arc RST) = 240^0

Q. 11 NAVNEET 62 / 3

Chord BC ≅ chord DE. Prove that arc BD ≅ arc CE



SOLUTION:

Chord BC ≅ chord DE

Arc BDC ≅ arc DCE … (Arcs of congruent chords) … (1)

But m (arc BDC) = m (arc BD) + m (arc DC) … (2)

m (arc DCE) = m (arc DC) + m (arc CD) … (3)

From (1), (2) & (3) we get,

m (arc BD) + m (arc DC) = m (arc DC) + m (arc CE)

∴ m (arc BD) = m (arc CE)

∴ arc BD = arc CE

Q. 12 NAVNEET 67

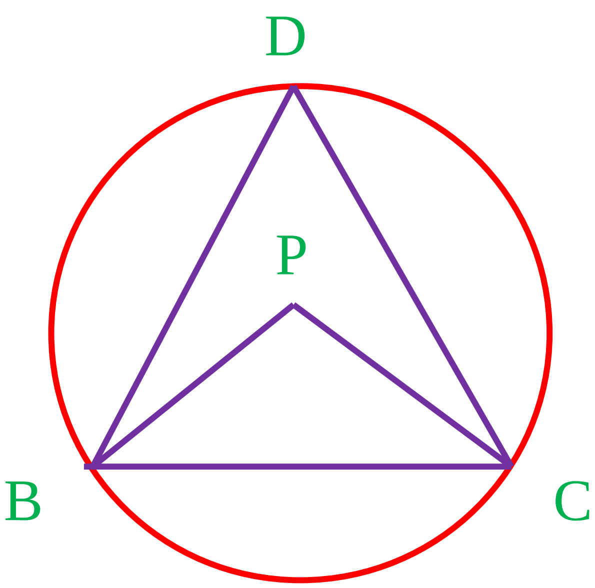
Length of chord BC is equal to the radius of the circle, with center P, what are the measures of each of following angles?

(1) ∠ BPC

(2) ∠ BDC

(3) Arc BC

(4) Arc BDC



SOLUTION:

(1) Seg PB ≅ Seg PC … (radii of same circle)

Measure of chord BC is equal to radius

Seg BC ≅ Seg PB≅ Seg PC

Sides of Δ BPC, an equilateral triangle

∴ ∠ BPC= 60^0

(2) The measure of an arc subtended by an arc at a point on the circle is half the measure of angle subtended by the arc of the center.

$\angle BPC = 2\angle BDC$

$60 = 2\angle BDC$

$60 = 2\angle BDC$

$\angle BDC = \frac{60}{2}$

$\angle BDC = 30^0$

(3) m (arc BC) = $\angle$ BPC … (Definition of measure of minor arc)

$\therefore$ m (arc BC) = 60^0

(4) m (arc BC) + m (arc BDC) = 360^0

Measure of a circle is 360^0

60^0+ m (arc BDC) = 360^0

Ans.: (1) $\angle BPC = 60^0$

(2) $\angle BDC = 30^0$

(3) m (arc BC) = 60^0 &

(4) m (arc BC) = 300^0

Q. 13 NAVNEET67

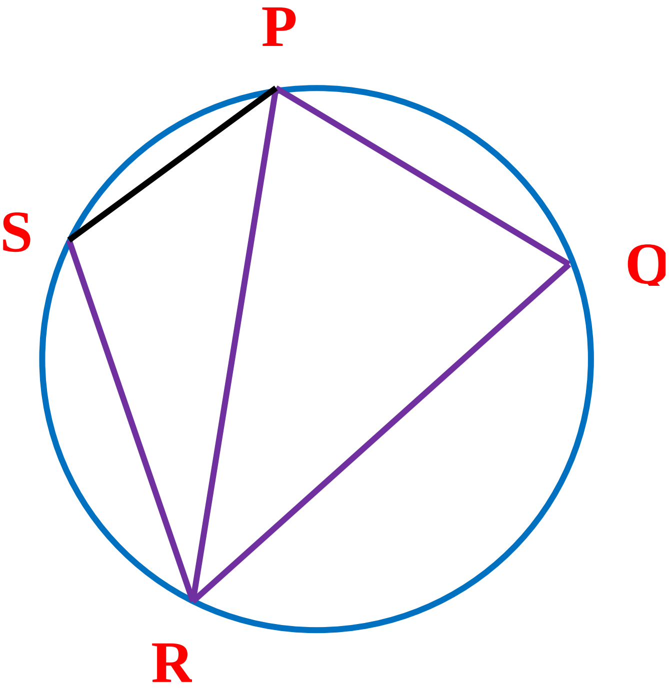
In figure □ PQRS is cyclic side PQ ≅ side RQ, ∠PSR = 100^0 Find

(1) Measure of ∠ PQR

(2) m (arc PQR)

(3) m (arc QR)

(4) Measure of ∠ PRQ



SOLUTION:

1) □ PQRS is cyclic

∠ PSR + ∠ PQR = 180^0 … (supplementary angles of a cyclic quadrilateral)

100^0 + ∠ PQR = 180^0

∠ PQR = 180^0 - 100^0

∠ PQR = 80^0

2) $\angle$ PSR = $\frac{1}{2}$ m (arc PQR) … (Inscribed angle theorem)

$100^0 = \frac{1}{2}$ m (arc PQR)

m (arc PQR) = 100^0 x 2

m (arc PQR) = 200^0

3) Chord PQ ≅ Chord QR

∴ arc PQ = arc QR ... (Corresponding congruent arcs)

Let m (arc PQ) = m (arc QR) = *p*

m (arc PQ) + m (arc QR) = m (PQR) …(Arc addition)

$p + p = 200$

$2p = 200$

$p = \frac{200}{2}$

$p = 100^0$

(4) $\angle$ PQR = $\frac{1}{2}$ m (arc PQ) (By Inscribed angle)

$\angle$ PRQ) = $\frac{1}{2}$ x 100^0

$\angle$ PRQ) = 50^0

Q. 14 NAVNEET 68/3

Find measures of $\angle$ S & $\angle$O if ,□ NSQO is cyclic quadrilateral , given $\angle$ S = $(5p - 13)^0$, $\angle$ O = $(4p + 4)^o$

SOLUTION:

□ NSQO is cyclic

$\therefore$ By theorem of cyclic quadrilateral,

$\angle S + \angle O = 180^0$

$(5p - 13)^0 + (4p + 4)^0 = 180^0$

$(5p - 13) + (4p + 4) = 180$

$9p - 9 = 180$

$9p = 189$

$p = \frac{189}{9}$

$p = 21$

$\angle S = 5p - 13$

$\therefore \angle S = 5(21) - 13$

$\therefore \angle S = 105 - 13$

$\therefore \angle S = 92^0$

$\angle O = (4p + 4)$

$\angle O = 4(21) + 4$

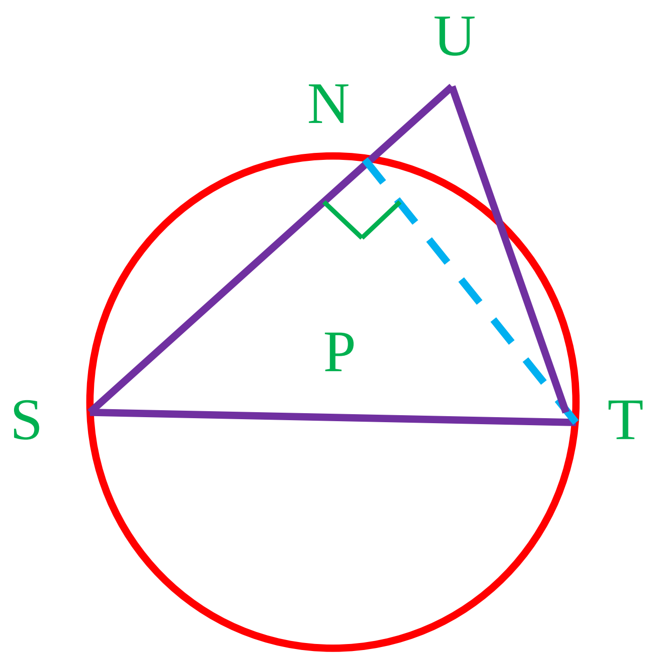
$\angle O = 84 + 4$

$\angle O = 88^0$

Ans.: $\angle S = 92^0$ & $\angle O = 88^0$

Q. 15 NAVNEET 68/4

In figure, seg ST is a diameter of the circle with center P. Point U lies in the exterior of the circle. Prove that $\angle$ SUT is an acute angle



SOLUTION:

At a point N, segment SU intersect circle. Draw seg NT … (Angle inscribed in the semicircle is a right angle)

$\angle$ SNT = 90^0

seg TN $\perp$ seg SU

Δ UNT is a right-angled triangle

$\angle$ UNT + $\angle$ NUT + $\angle$ UTN = 180^0 … (Sum of all triangles is 180^0)

$\therefore$ 90^0 + $\angle$ NUT + $\angle$ UTN = 180^0

$\angle$ NUT + $\angle$ UTN = $180^0 - 90^0$

$\angle$ NUT + $\angle$ UTN = 90^0

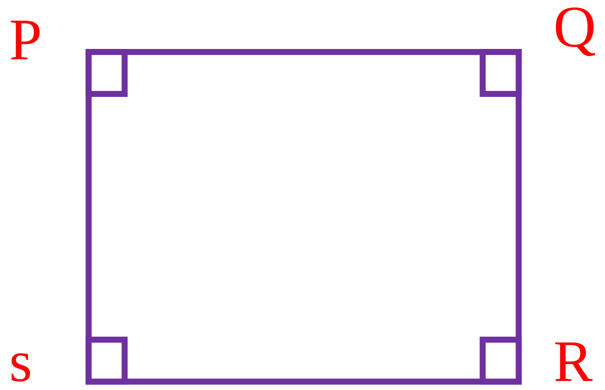
$\angle$ NUT < 90^0

i.e., $\angle$ SUT < 90^0 …. (S – N – U)

Q. 16 NAVNEET 68/5

Prove that any rectangle is a quadrilateral.

SOLUTION:



Given: □ PQRS is a rectangle

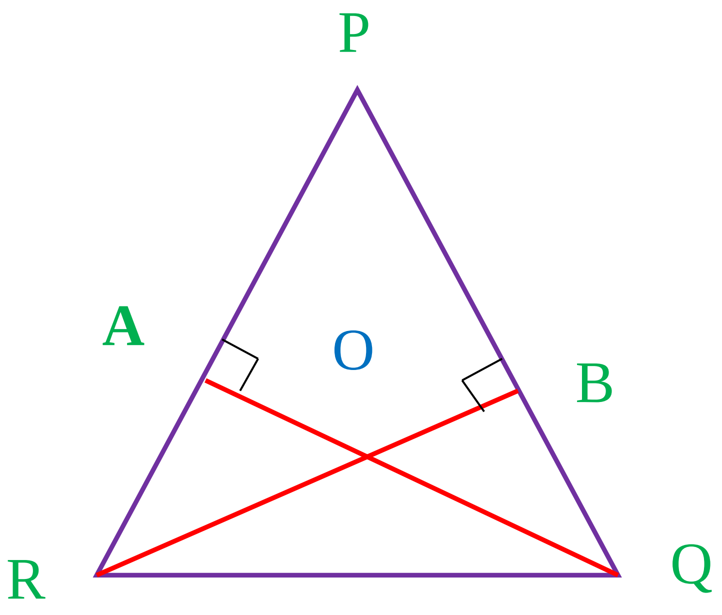
To prove: PQRS is a quadrilateral

$\therefore \angle P = \angle Q = \angle R = \angle S = 90^0$ … (measures of all the angles of a rectangle are right angle)

$\therefore$ □ PQRS is a cyclic … (By converse of cyclic quadrilateral theorem)

Q. 17 NAVNEET 68/4

Prove that (1) □ PAOB is cyclic (2) Points R, A, B, Q are concyclic, in the figure altitudes QA & RB of ΔWXY intersect at O.



SOLUTION:

1) $\angle PAO = \angle PBO = 90^0$

$\therefore \angle PAO + \angle PBO = 90^0 + 90^0 = 180^0$

∴ By converse cyclic theorem, □ PAOB is cyclic

2) $\angle RAQ = \angle RBQ = 90^0$ … (given)

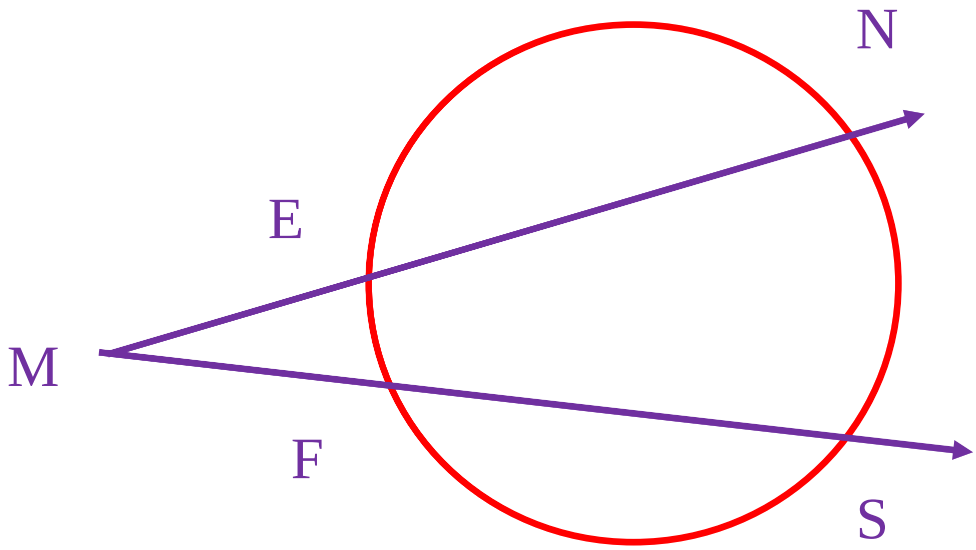
Points R and Q on the line RQ subtend equal angles at two distinct points A and B on the same of sideline RQ.

If two points given on the line subtend equal angles at two different points, then the four points are co-cyclic.

∴ Points P, A, O, B are co cyclic.

Q. 18 NAVNEET 69/7

For the given figure m (arc NS) = 60^0 and m (arc EF) = 16^0 Find measure of ∠NMS.



SOLUTION:

m (arc NS) = 60^0 and m (arc EF) = 16^0 … (Given)

∠ NMS has vertex in the exterior of the circle and intercepts arc EF and NS.

$\therefore \angle NMS = \frac{1}{2}\ [m(arcNS) - \ m(arc\ EF)]$

$\therefore \angle NMS = \frac{1}{2}\ [60 - \ 16]$

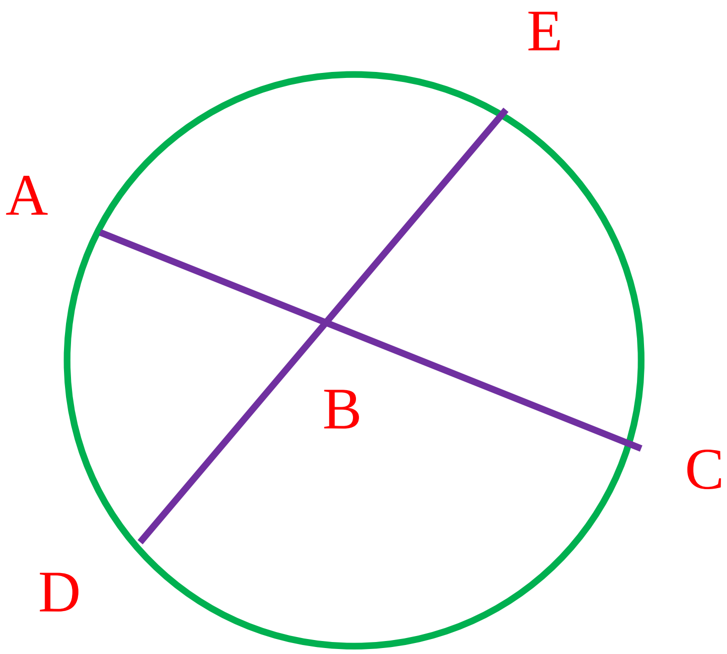
$\therefore \angle NMS = \frac{1}{2}\ [44]$

$\therefore \angle NMS = \ 22^0$

Ans.: ∠NMS = 22^0

Q. 19 navneet 69/9

In the figure chords AC & DE intersect each other at B. If $\angle ABE = 95^0$, m (arc AE) = 83^0 Find m (arc DC)



SOLUTION:

m (arc AE) = 83^0, $\angle ABE = 95^0$

$\angle ABE$ has vertex inside the circle and intercepts the arc AE and its vertically opposite $\angle$ DBC intercepts arc DC.

$$\angle ABE = \frac{1}{2}\,[m\,(\text{arc AE}) + m\,(\text{arc DC})]$$

$$\therefore\ 95^0 = \frac{1}{2}\,[m\,(\text{arc AE}) + m\,(\text{arc DC})]$$

$\therefore\ 95^0 \times 2 = 83^0 + m\,(\text{arc DC})$

$\therefore 190^0 = 83^0 + m\,(\text{arc DC})$

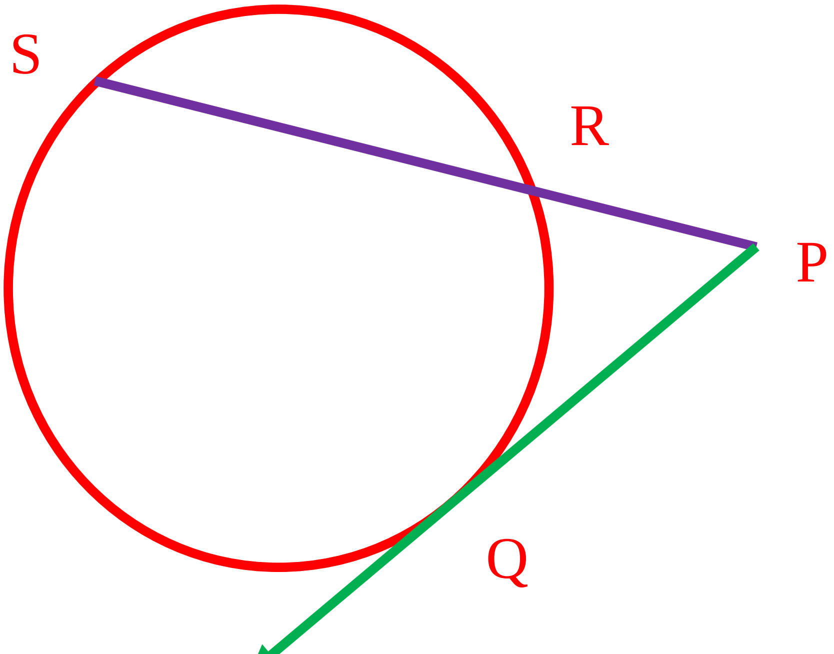
$\therefore\ m\,(\text{arc DC}) = 190^0 - 83^0$

$\therefore\ m\,(\text{arc DC}) = 107^0$

Ans.: $m(\text{arc DC}) = 107^0$

Q. 20 Navneet 71/1

Find PS and RS for the given figure in which ray PQ touches the circle at point Q. and PQ = 6, PR = 4.



SOLUTION:

Ray PQ is tangent touching the circle at point Q and the line PRS is secant intersecting the circle at points R & S.

$\therefore PQ^2$ = PR x PS … (By tangent secant theorem)

$\therefore 6^2$ = 4 x PS

$\therefore$ PS = $\frac{6 \text{ x } 6}{4}$

$\therefore$ PS = 9

PR + RS = PS … (P – R – S)

$\therefore$ 4 + RS = 9

$\therefore$RS = 9 – 4 = 5

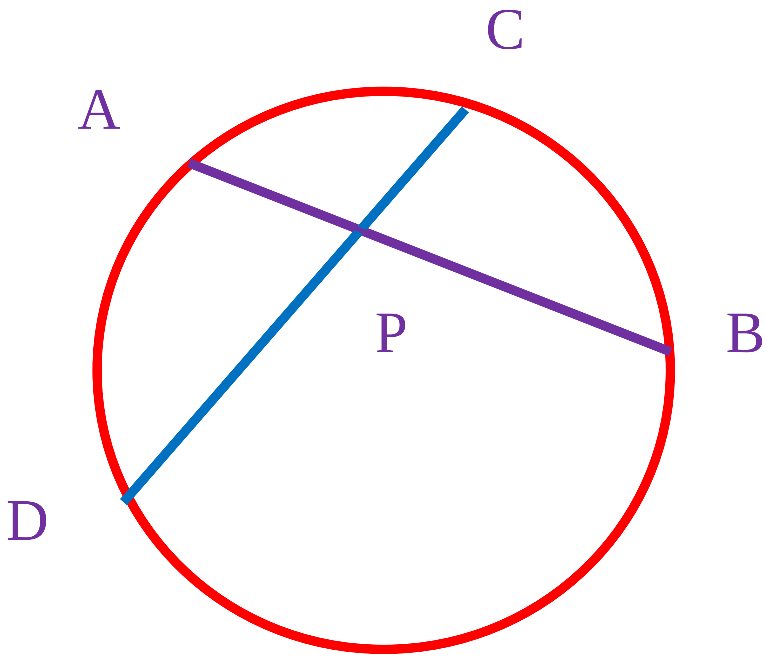
Ans.: PS = 9, RS = 5

Q. 21 NAVNEET

Chord AB & chord CD intersect at point P. Find the following:

1) If CP = 15, PD = 4, AP = 8, find PB

2) If CD = 18, AP = 9, PB = 8, find PD



SOLUTION:

1) Chords AB and CD intersect at each other at point P inside the circle.

∴ By theorem of internal division of chords,

PA x PB = PC x PD

∴ 8 x PB = 15 x 4

∴PB = $\frac{15 X 4}{8}$

∴PB = 7.5

2) Let PD = *p*

PC + PD = CD … (C – P – D)

∴ PC + *p* = 18

∴ PC = (18 - *p*)

Chords MN & RS intersect each other at point D inside the circle.

$\therefore$ 72 = 18 p - p^2

$\therefore p^2$- 18 p + 72 = 0

$\therefore p^2$- 18 p + 72 = 0

$\therefore p^2$- 12 p - 6 p + 72 = 0

$\therefore p(p-12)-6(p-12)$ = 0

$\therefore (p-12)(p-6)$ = 0

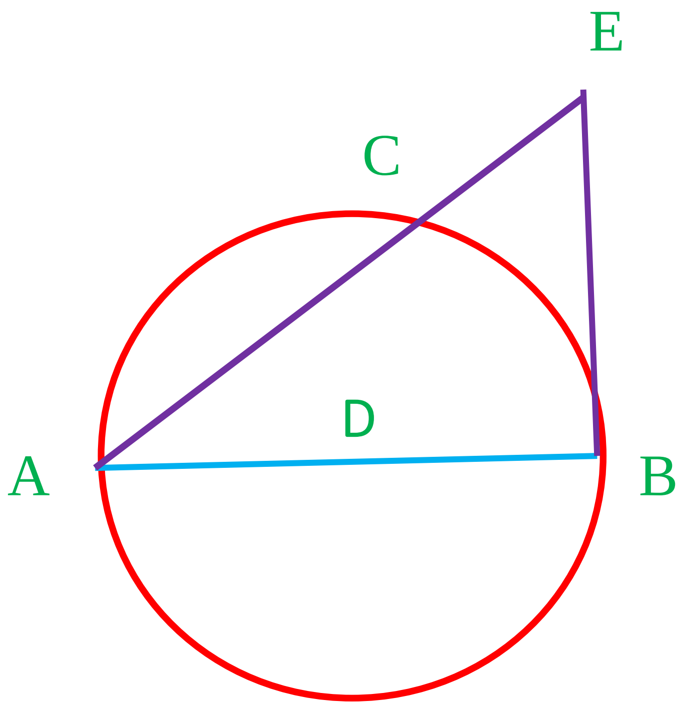
$\therefore (p-12) = 0$ or $(p-6)$ = 0

$p = 12$ or p = 6

SP $= 12$ or PD $= 6$

Q. 22 NAVNEET 73

Seg AB is a diameter & seg EB is a tangent segment. The radius of circle is r. Prove that EA x CA = 4r^2



SOLUTION:

Line EB is tangent to the circle touching the circle at point B and line ECA is secant intersecting the circle at points C and A.

∴ Use tangent secant theorem

$EB^2 = EC \times EA$ **… (1)**

In Δ EBA, ∠ EBA = 90^0

∴ by Pythagoras theorem

$EA^2 = EB^2 + AB^2$

$EA^2 = EB^2 + (2r)^2$ **… (Diameter = 2 X radius)**

$EA^2 = EB^2 + 4r^2$

$\therefore 4r^2 = EA^2 - EB^2$

$\therefore 4r^2 = EA^2 - EC \text{ x } EA$

$\therefore 4r^2 = EC(EA - EC)$ ....(E – C – A)

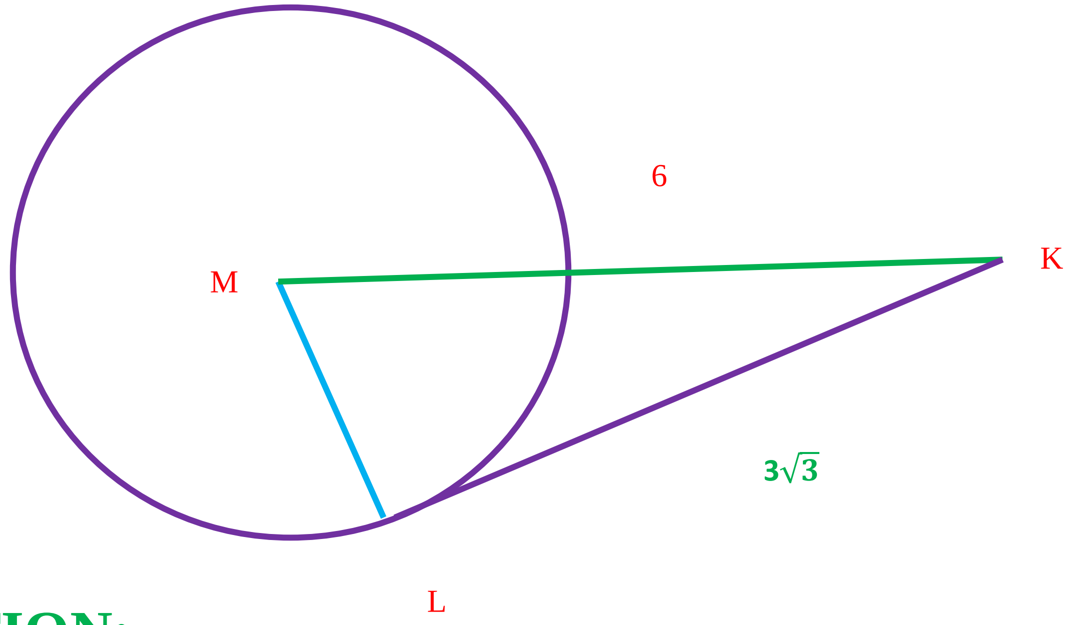
$EA \text{ x } CA = 4r^2$

Q. 23 navneet 74/3

Seg KL is a tangent segment of the circle with center M and is a. If MK= 6, KL = $3\sqrt{3}$), then find

1) Radius of circle

2) Measures of ∠ K and ∠ M



SOLUTION:

In Δ MLK, ∠ MLK = 90^0

By Pythagoras theorem,

$MK^2 = ML^2 + LK^2$

$6^2 = ML^2 + (3\sqrt{3})^2$

36 = $ML^2 + 9 \times 3$

36 = $ML^2 + 27$

$ML^2 = 36 - 27$

$ML^2 = 9$

ML = 3 … (Taking square roots of both sides)

Radius of circle = ML = 3

In Δ MLK,

ML = $\frac{1}{2}$ **MK**

$\therefore \angle$ **K =** 30^0 **… (Converse of** $30^0 - 60^0 - 90^0$**)**

In Δ MLK,

$\angle M + \angle L + \angle K = 180^0$

$\angle M + 30^0 + 90^0 = 180^0$

$\angle M + 30^0 + 90^0 = 180^0$

$\angle M + 120^0 = 180^0$

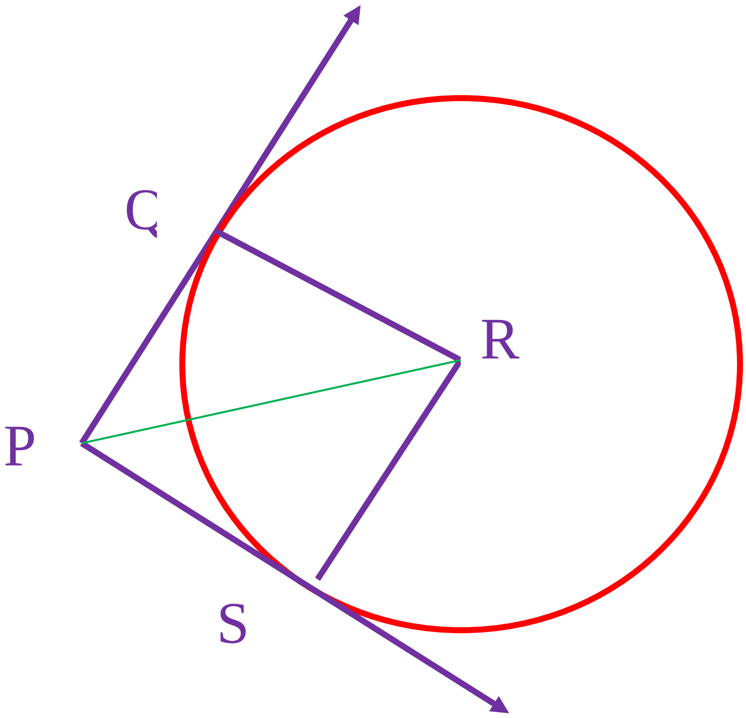
$\angle M = 180^0 - 120^0$

$\angle M = 60^0$

Ans.: Radius of circle is 3, $\angle K = 30^0$, $\angle M = 60^0$

Q. 24 NAVNEET 75/4

Seg PQ, seg PS are tangents segments in circle with center R. Radius of the circle is r and l (PQ) = r. Prove that □ PQRS is square



SOLUTION:

Draw seg RQ and seg RS

RQ = RS = r … (all radii of a circle equal)

PQ = r … (Given)

PQ = PS … (tangent secant theorem)

In □ PQRS

RQ = RS = PS = PQ = r (From (1) ,(2),(3))

In □ PQRS is a rhombus … (by definition)

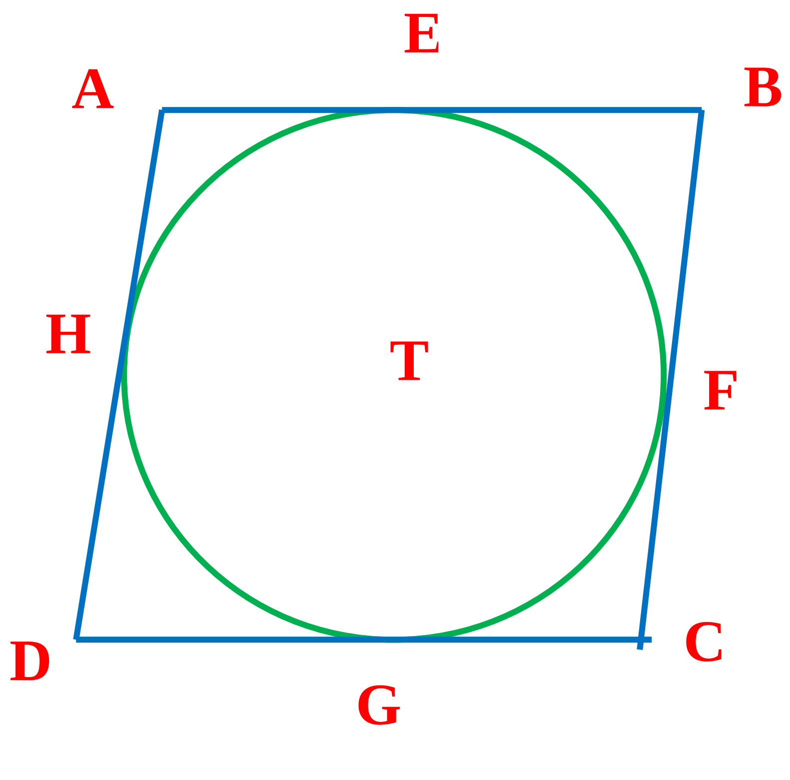
∠ RQP = 80^0

A rhombus is a square if its angle is a right angle

∴ □ PQRS is a square

Q. 25 NAVNEET 75

A parallelogram □ ABCD. circumscribes the circle T. Point E, F, G, H are touching points. If AE = 9, EB = 11, Find AD.



SOLUTION:

AE = 9, EB = 11, AH = AE = 9, BF = BE = 11

Let DH = DG = p, CG = CF = q

□ ABCD is a parallelogram.

∴ AB = CD … (opposite sides of parallelogram are equal)

AE + EB = CG +GD ... (A – E – B and C – G –D)

∴ 9 + 11 = $p + q$

$p + q = 20$ … (1)

AD = BC ... (opposite sides of parallelogram)

∴ AH + AD = BF + FC … (A – H – D & B – F – C)

∴ 9 + p = 11 + q

∴ $p - q = 2$ … (2)

Adding (1) & (2),

$p + q = 20$

$p - q = 1$

∴ 2 p = 21

∴ p = 10.5

DG = GH = 10.5

AD = AH + HD … (A –H –D)

∴ AD = 9 + 11

∴ AD = 20

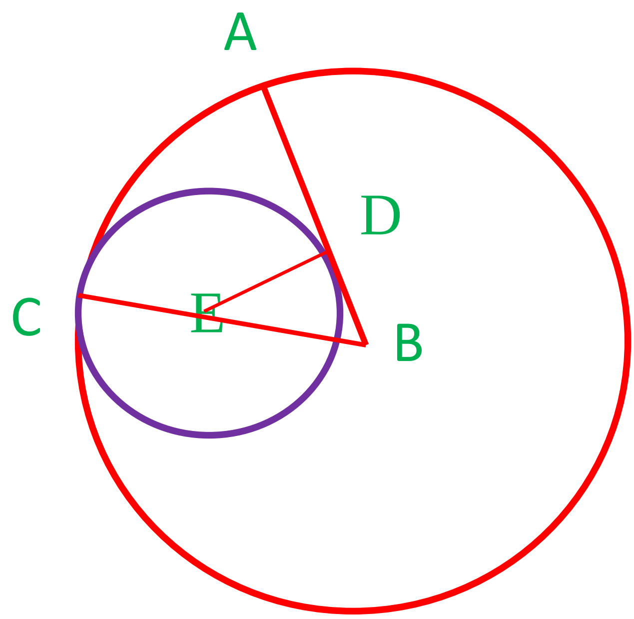
Q. 26 NAVNEET76/6

A circle with center B touches the circle with center E at point C. Radius AB touches the smaller circle at D, Radii of circles are 9 cm and 2.5 cm. Find the answers to the following questions and hence find the ratio BD : AD

1) Find the length of segment CB

2) Find the length of segment EB

3) Find the measure of ∠ EDB



SOLUTION:

Draw seg ED

Radius of larger circle is 9 cm

∴ AB = BC = 9 cm

Radius of smaller circle is 2.5 cm

ED = CE = 2.5 cm

By theorem of touching circle,

EB + CE = BC … (B – E – C)

∴ EB + 2.5 = 9

∴ EB = 9 - 2.5

∴ EB = 6.5 cm

In Δ BDE, ∠ BDE = 90^0 … (Tangent theorem)

$BE^2 = DB^2 + DE^2$

$\therefore 6.5^2 = BD^2 + 2.5^2$

$\therefore BD^2 = 6.5^2 - 2.5^2$

$\therefore BD^2 = 42.25 - 6.25$

$\therefore BD^2 = 36$

∴ BD = 6 cm … (Taking square roots on both the sides)

BD + AD = AB … (B – D – A)

∴ 6 + AD = 9

∴ AD = 9 – 6

∴ AD = 3 cm

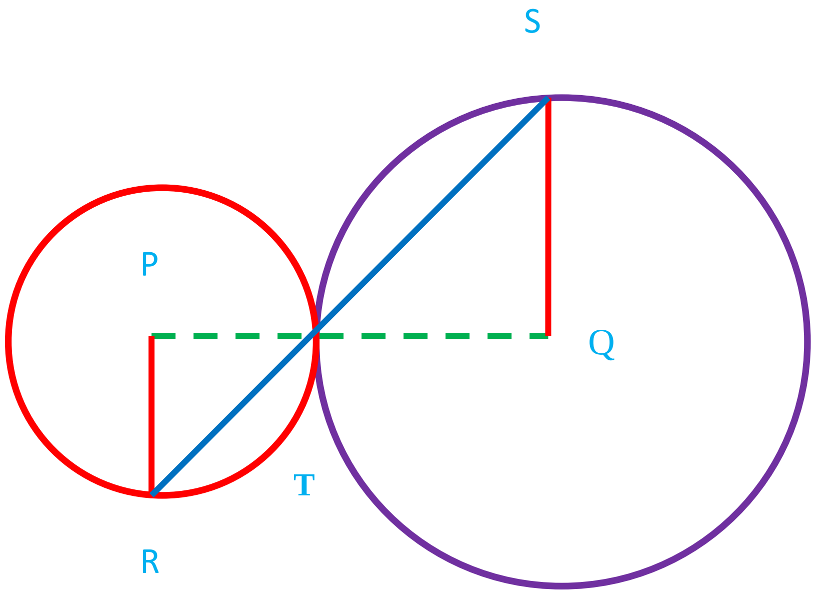
BD : AD = 6 : 3 = 2 : 1

Ans.:

Length of segment BC is 8 cm, length of segment BE is 6.5, ∠EDB = 90^0 and ED : AD = 2 : 1

Q. 27 NAWNET 76/7

Circles with centers P and Q touch each other at point T. A secant passing through T intersects the circles at points R and S respectively. Prove that radius PR ∥ radius QS. Fill in the blanks and complete the proof.



SOLUTION:

Draw segments PT and QT

By theorem of touching circles, points P, Q, T are collinear

∴∠ PTR ≅∠ STQ … (Opposite angles) … (1)

Let seg PR = seg PT … (Radii of the same circle)

∴ ∠ PRT ≅ ∠ PTR = α … (Isosceles triangle theorem) … (2)

Similarly,

seg QS ≅ seg QT … (Radii of the same circle)

∴ ∠ STQ ≅ ∠ QST = α … (Isosceles triangle theorem)

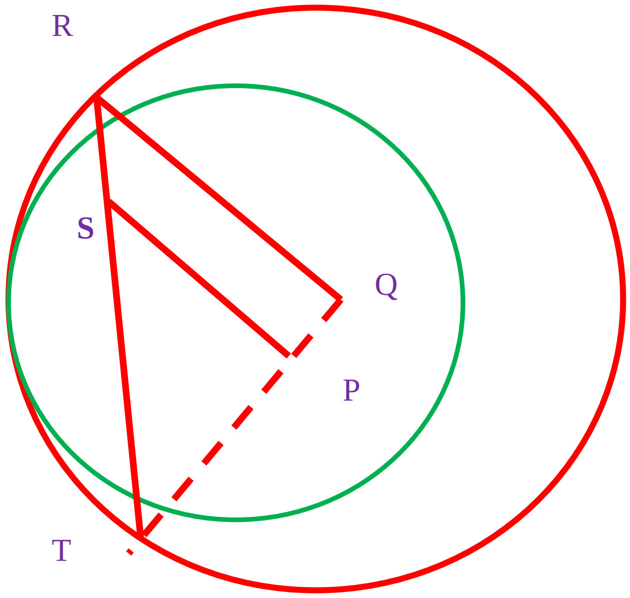
From (1), (2) and (3)

∴ ∠ PAT = ∠ QST

∴ Radius PR ∥ Radius SQ … (Alternate angle test for parallel line)

Q. 28 NAVNEET 76/8

Circles with centers P and Q touch internally at point T. Seg RQ is a chord of bigger circle, and it intersects smaller circle at point S. Prove that seg SP ∥ seg RQ.



SOLUTION:

Draw QT and PT … (Q – P – T)

In ΔQRT,

seg QR ≅ seg QT … (Radii of the same circle)

∴ ∠QRT ≅ ∠QTR … (Isosceles triangle theorem)

i.e. ∠ QRT ≅ ∠ PTS … (Q – P – T) and R – S– T) … (1)

Δ PST,

seg PS = seg QT … (Radii of same circle)

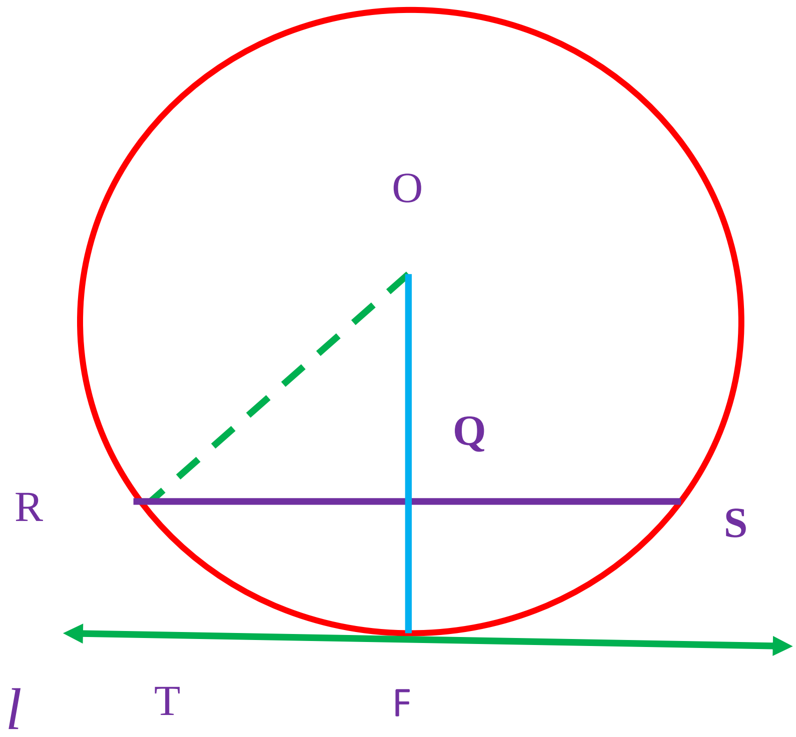
∴∠ PST ≅ ∠ QTA … (Isosceles triangle theorem)

∴∠QTR ≅ ∠ PST

seg SP || seg RQ

Q. 29

At point P, line *l* touches the circle with center O at point P. Q is the midpoint of radius OP. RS is a chord through Q such that chords RS ∥ line *l*. If RS = 6, find the radius of circle.



SOLUTION:

Draw segment seg OR. Consider point T as shown

In the figure ∠OPT = 90^0 … (Tangent theorem)

seg RS ∥ line *l* and OP is the transversal

∠ OQR ≅ ∠ OPT … (Corresponding angles theorem)

∴ ∠ OQR = 90^0

∴ seg OQ ⊥ chord RS

$\therefore$ QR = $\frac{1}{2}$ RS ... (perpendicular drawn from the center circle to the chord bisects the chord)

$\therefore$ QR = $\frac{1}{2}$ x 6

$\therefore$ QR = 3

Let the radius of the circle be 2*x*

$\therefore$ OR = OP = 2 *x* … (Radii of the same circle)

OQ = $\frac{1}{2}$ OP … (Q is the midpoint of seg OP)

$\therefore$ OQ = $\frac{1}{2}$ x 2*x*

$\therefore$ OQ = *x*

In Δ OQR, $\angle$ OQR = 90^0

by Pythagoras theorem,

$OR^2 = OQ^2 + QR^2$

$\therefore (2x)^2 = x^2 + 3^2$

$\therefore 4x^2 = x^2 + 9$

$\therefore 4x^2 - x^2 = 9$

$\therefore 3x^2 = 9$

$\therefore x^2 = 3$

$\therefore\ x = \sqrt{3}$

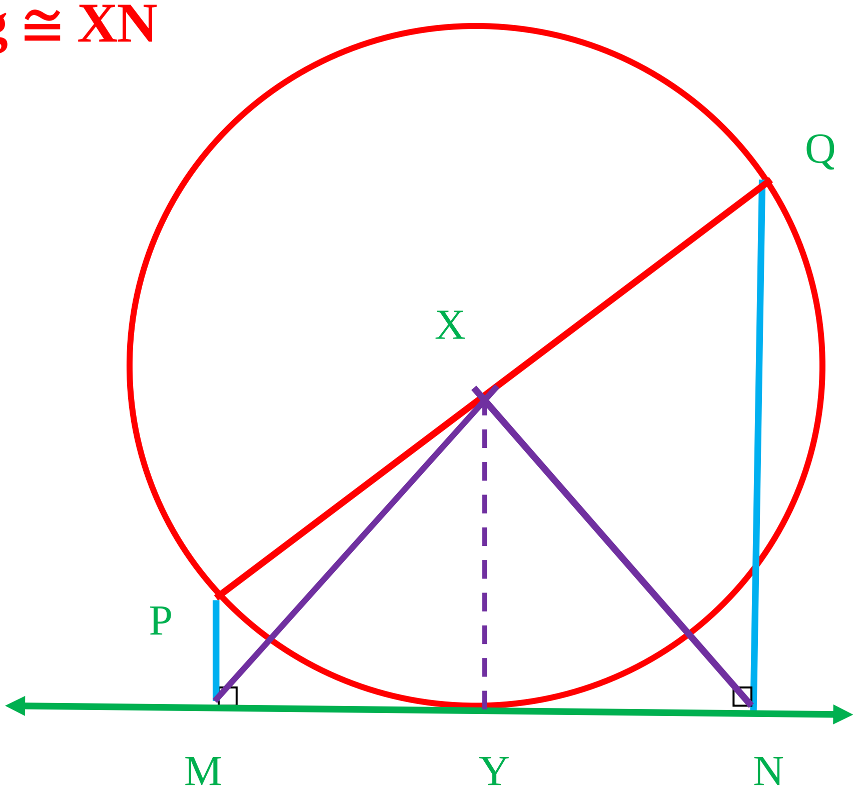
$\therefore\ x = \sqrt{3}$

Radius = 2x = 2 x $\sqrt{3}$ = 2$\sqrt{3}$

Ans.: Radius of circle is 2$\sqrt{3}$ cm

Q. 30 NAVNEET 77/19

Seg PQ is a diameter of circle with center X. Line MN is a tangent, which touches the circle at point Y. Seg XY ⊥ line MN and seg QN ⊥ line MN. Prove that seg XM seg ≅ XN



SOLUTION:

Draw seg XY, seg XM and seg XN

seg PM ⊥ line MN … (Given)

seg XY ⊥ line MN and

seg QN ⊥ line MN

∴ seg PM ∥ seg XY ∥ seg QN … (Perpendicular to the same line are parallel)

$\therefore \frac{MY}{YN} = \frac{PX}{XQ}$ **… (Property of three parallel lines and their transversals)**

But PX = XQ … (Radii of same circles)

$\therefore \frac{PX}{XQ} = 1$

∴ From (1) & (2) we get

$\frac{MY}{YN} = 1$

∴ MY = YN … (3)

In Δ XYM and Δ XYN,

seg YM ≅ seg YN … (common side)

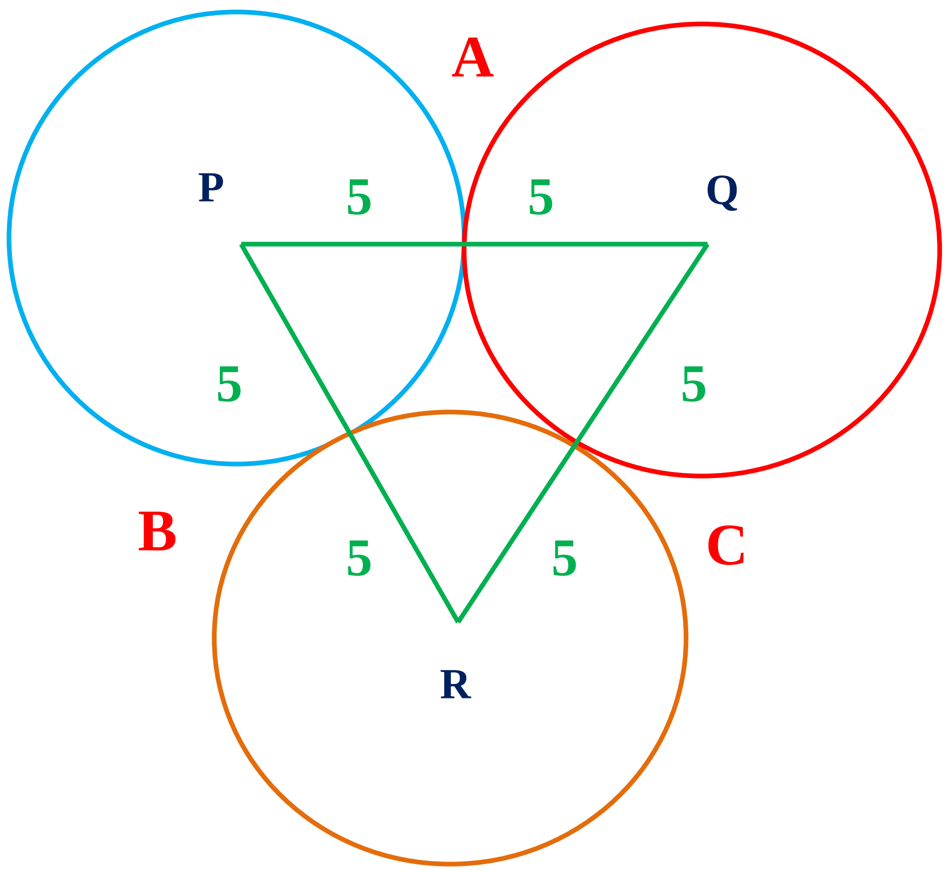
∠ XYM ≅ ∠ XYN … (Each measure 90^0)

seg XM ≅ seg CN … (SAS test of congruence)

∴ seg XM ≅ seg XN … (c. p. c. t)

Q. 31 NAVNEET 78/11

Draw circles with center P, Q & R each of radius 5 cm, such that each circle touches the other two circles



SOLUTION:

Let the circles with centers P, Q and R touch the points A, B and C as shown in figure.

By theorem of touching the circles, we get

P – A – Q, Q – C – R, and B – P – R

∴ PQ = PA + AQ = 5 + 5 = 10

Similarly, QR = 10 cm and PR = 10 cm

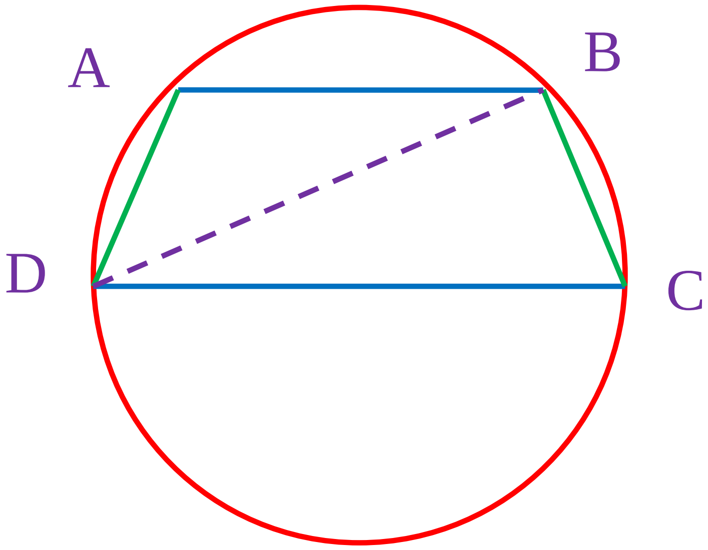
∴ We can construct Δ PQR with

PQ = QR = PR = 10 cm

With P, Q, R as centers, the required circles of radius 5 cm can be drawn.

Q. 32 NAVNEET 81/17

For a circle, chord AB ⊥ chord DC. Prove that Chord AD ≅ chord BC. Fill in the blanks and write the proof.



SOLUTION:

Draw seg GF

∠ ABD = ∠ BDC ... (Alternate angles) ... (1)

∠ ABD = $\frac{1}{2}$ m (arc AD)

... (Inscribed angle theorem) … (2)

∠ ADC = $\frac{1}{2}$ m (arc BC)

... (Inscribed angle theorem) … (3)

m (arc AD) = m (arc BC)

∴ From (1), (2), (3)

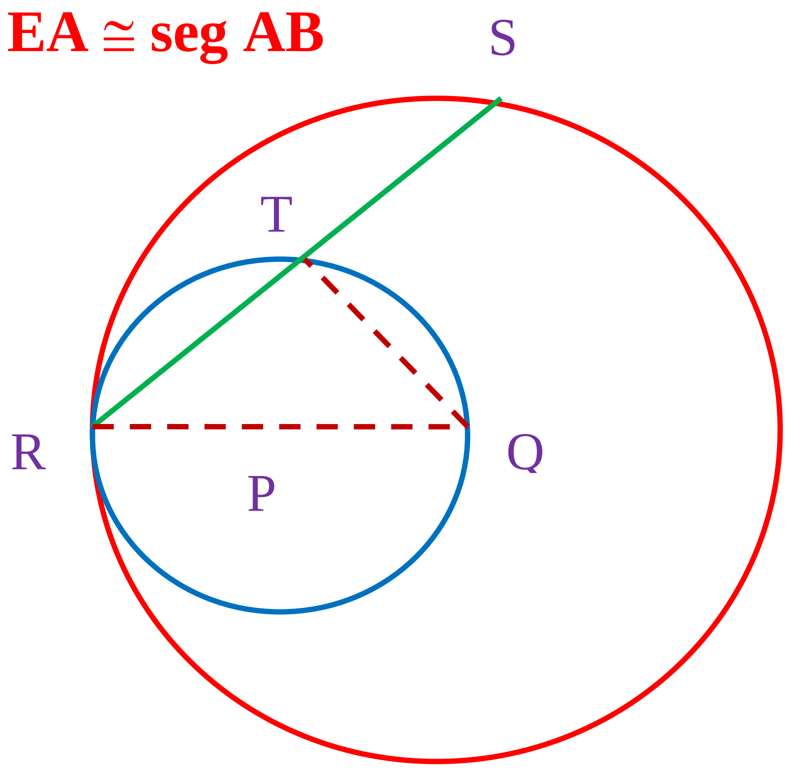
∴ chord AD ≅ chord BC

… (Corresponding chords of congruent arcs)

Ans.: FILLED IN THE BOXES

Q. 33 NAVNEET 82/19

Circles with centers C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at points. Prove that seg EA ≅ seg AB



SOLUTION:

Draw seg RQ and seg TQ

R – P – Q ... (By theorem of touching circles)

∴ seg RQ is the diameter of the circle.

∠ RTQ = 90^0 … (Angle inscribed in a semicircle is a right angle) ... (1)

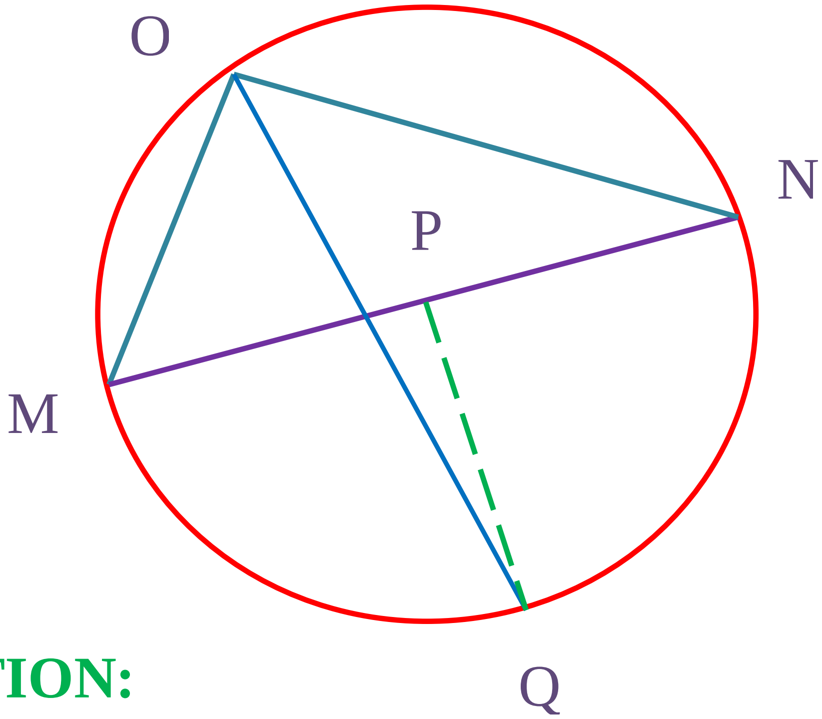
For larger circle, seg QT ⊥ chord RS {From (1)}

∴RT = TS …. (Perpendicular drawn from the center of the circle to the chord)

∴ seg RT ≅ seg RS

Q. 34 NAVNEET 82/20

Seg MN is diameter of circle with center P. The bisector of ∠ MON intersects the circle at point Q. Prove that seg MQ ≅ seg NQ. Complete the following proof by filling in the blanks.



SOLUTION:

Draw seg PQ

∴ ∠ MON^0 = 90^0

(Angle Inscribed in semicircle)

∠ OON = 45^0 OQ is the bisector of ∠ O

m (arc QN) = 90^0 **(Inscribed angle theorem)**

∠ QPN = 90^0

(Definition of Measure of an arc)

seg PM ≅ seg PN

….. (Radii of same circle) ……. (2)

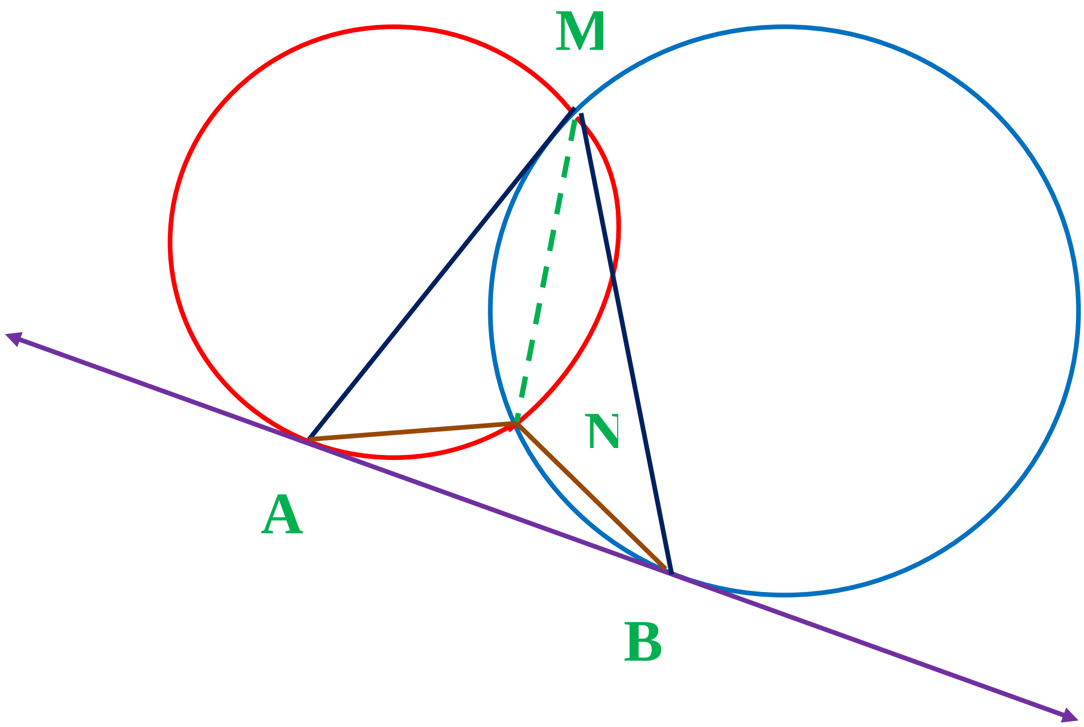
∴ Line PQ is perpendicular bisector of seg MN

……. {From (1) and (2)}

∴ seg MQ ≅ seg QN

Q. 35 NAVNEET 83/22

Two circles intersect each other at points M and N. Their common tangent AB touches the circle at A, B. Prove that ∠ANB + ∠AMB = 180^0



SOLUTION:

Draw seg MN

$\angle AMN = \angle NAB$ and .... (1)

$\angle NMO = \angle NAB$ .... (2)

(By tangent secant theorem)

In Δ ANB

$\angle ANB + \angle NAB + \angle NBA = 180^0$

(Sum of all angles of a triangle is 180^0)

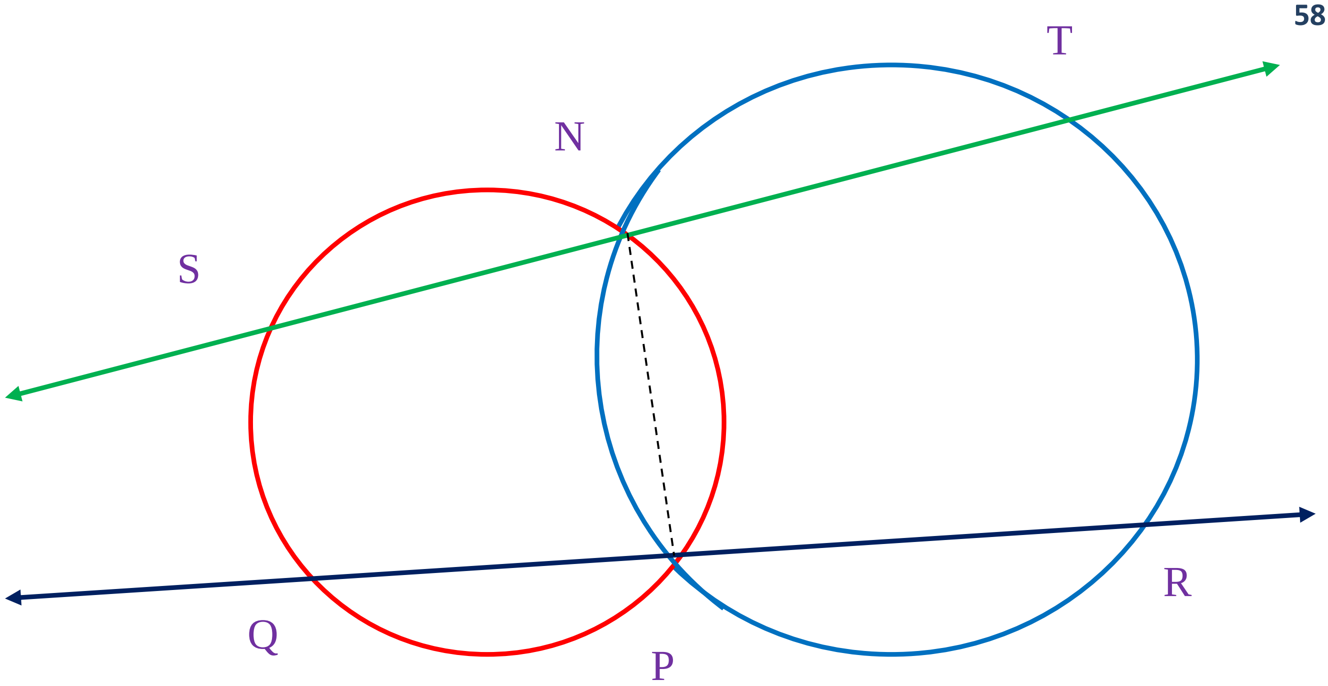
$\therefore \angle ANB + \angle AMN + \angle NMB = 180^0$ .. From (1)

$\therefore \angle ANB + (\angle AMN + \angle NMB) = 180^0$. From (2)

$\therefore \angle ANB + \angle AMB = 180^0$ … {Angle addition postulate}

Q. 36 NAVNEET 83/23

Two circles intersect each other at points N and P. Secants drawn through N and P intersect the circles at points S, T, Q and R respectively. Prove that seg TR ∥ seg SQ



SOLUTION:

Draw seg MN

□ NSQP is cyclic and ∠NPR is its exterior angle.

∠NPR = ∠NSQ … (1) … (corollary of cyclic quadrilateral theorem)

□ NSQP is cyclic

∴ ∠NPR + ∠MRP = 180^0 … (cyclic quadrilateral theorem)

∴ ∠NSQ + ∠NTR = 180^0 … (FROM (1)

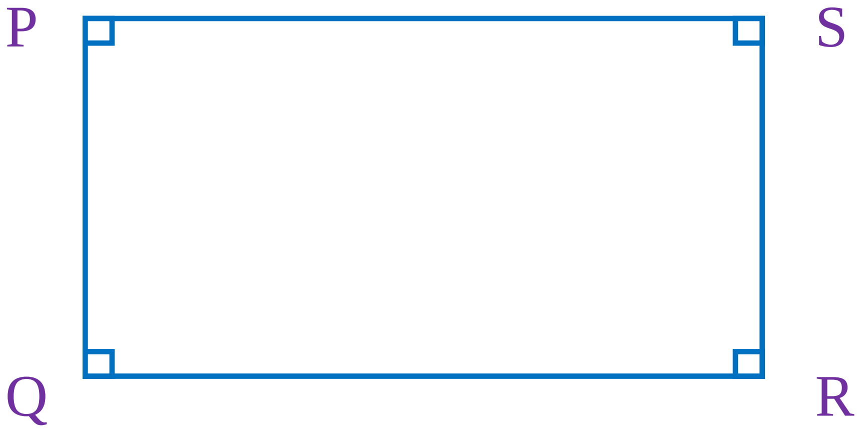
∴∠TSQ + ∠STQ = 180^0 …(R – M – S)

seg RP ∥ seg SQ … (Interior angle test for parallel lines)

Q. 37 NAVNEET 68/3

Prove that any rectangle is a cyclic quadrilateral

SOLUTION:



Given: □ PQRS is a rectangle

To prove: □ PQRS is cyclic

Proof: □ PQRS is a rectangle

$\therefore \angle P = \angle Q = \angle R = \angle S$ … (Angles of a rectangle)

$\therefore \angle P + \angle R = 90^0 + 90^0 = 180^0$

… (By converse of cyclic quadrilateral theorem)

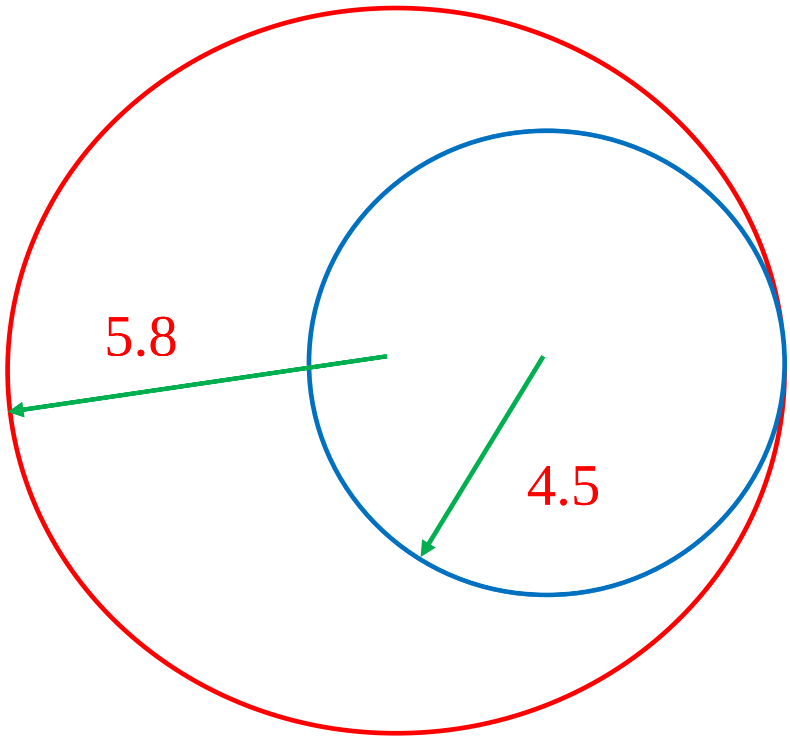
Q. 38

Two circles having radii 4.5 cm and 5.8 cm touch each other internally. Find the distance between their centers.

SOLUTION:

Let the radii of two circles be r_1 and r_2

r_1 = 4.5 and r_2 = 5.8 …. (given)



As in figure two circles are touching each other internally

∴ By theorem of touching circles, the centers and point of contact are collinear.

Distance in the centers of the circles is equal to the difference of their radii

∴ Distance in the centers of the circles

$= r_2 - r_1$

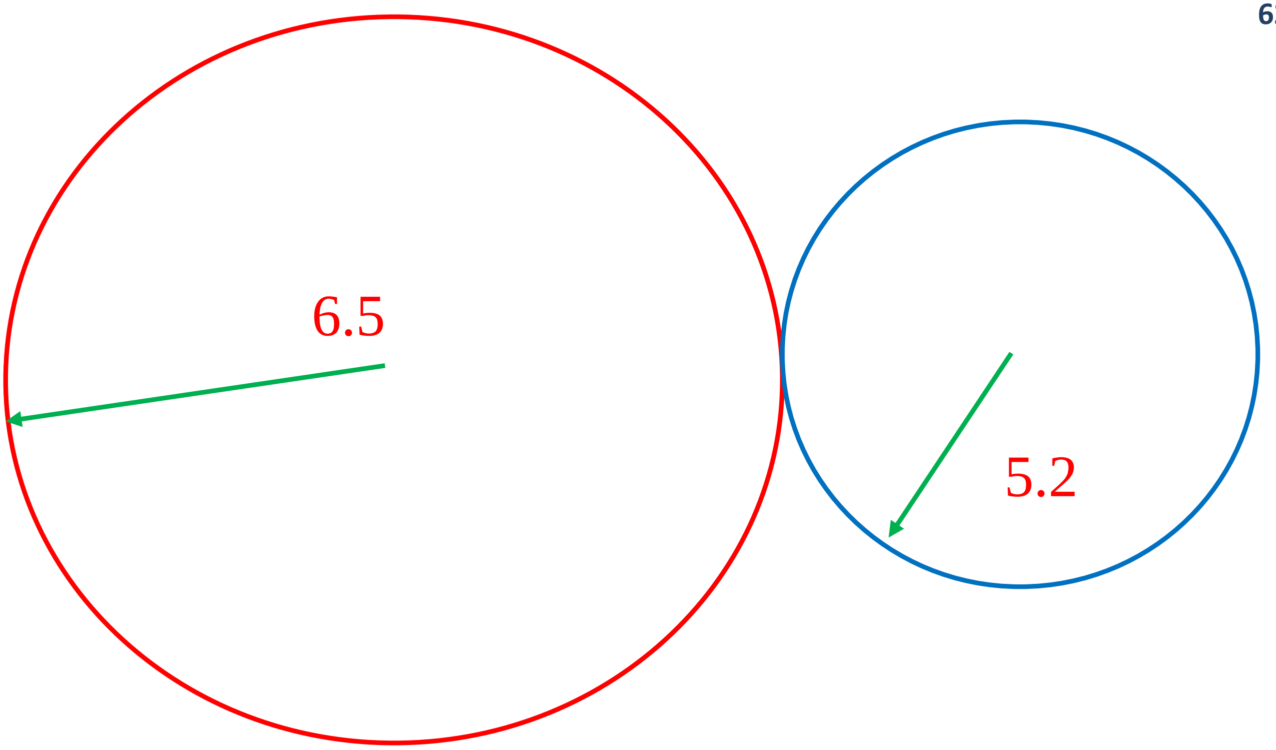
= 5.8 - 4.5

= 1.3 cm

Ans.: Distance in the centers of the circles is 1.3 cm

Q. 39

Two circles having radii 6.5 cm and 5.2 cm touch each other externally. Find the distance between their centers.



SOLUTION:

Let the radii of two circles be r_1 and r_2

$r_1 = 6.5$ and $r_2 = 5.2$ … (Given)

As in figure two circles are touching each other externally

$\therefore$ By theorem of touching circles, the centers and point of contact are collinear.

Distance in the centers of the circles is equal to the sum of their radii

$\therefore$ Distance in the centers of the circles

$= r_2 + r_1$

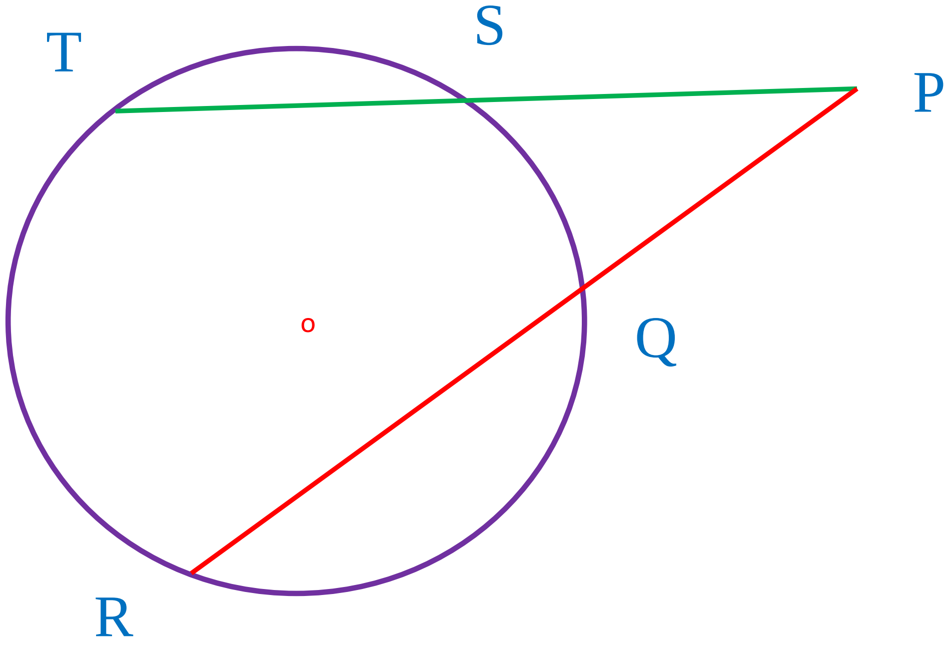
$= 5.2 + 6.5$

= 11.7 cm

Ans.: Distance in the canters of the circles is 9.7 cm

Q. 40 navneet (72/4)

If PQ = 3, QR = 5, PS = 4, find TS.



SOLUTION:

PR = PQ + QR ------ (P – Q – R)

∴ PR = 3 + 5

∴ PR = 8

Chord ST and chord QR intersect at point P outside the circle.

∴By theorem of external division of chords,

PS x PT = PQ x PR

$\therefore$ 4 x PT = 3 x 8

$\therefore$ PT = $\frac{3 \times 8}{4}$

$\therefore$ PT = 6

PS + TS = PT …….. (P – S – T)

$\therefore$ 4 + TS = 6

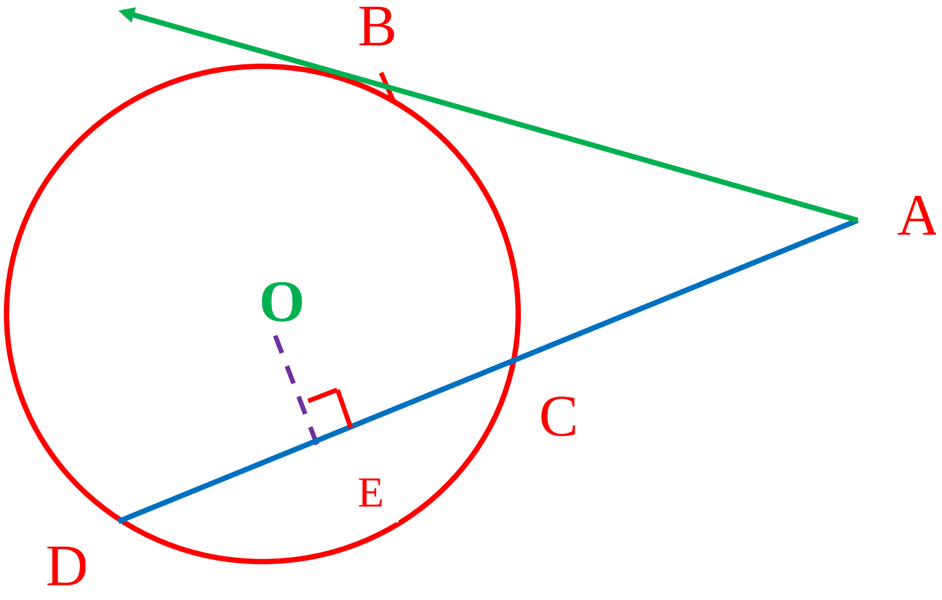
$\therefore$ TS = 6 - 4

$\therefore$ TS = 2

Ans.: TS = 2

Q. 41 navneet 72/4

O is the center of the circle and B is a point of contact. seg OE $\perp$ seg AD, AB = 6, AC = 4, find (1) AD (2) DC (3) DE



SOLUTION:

Ray AB is tangent to the circle at point B. Line ACD is secant intersecting the circle at points C and D.

By tangent secant theorem,

$AB^2 = \text{ AC x AD}$

$\therefore\ 6^2 = \text{ 4 x AD}$

$\therefore\ AD = \frac{6\ X\ 6}{4}$

$\therefore$ **AD = 9**

AC + DC = AD …. (A – C – D)

$\therefore$ **9 + CD = 4**

$\therefore$ **CD = 9 – 4**

$\therefore$ **CD = 5**

seg OE $\perp$ chord CD ……………. {given}

Perpendicular drawn from the center of circle to the chord bisects the chord.

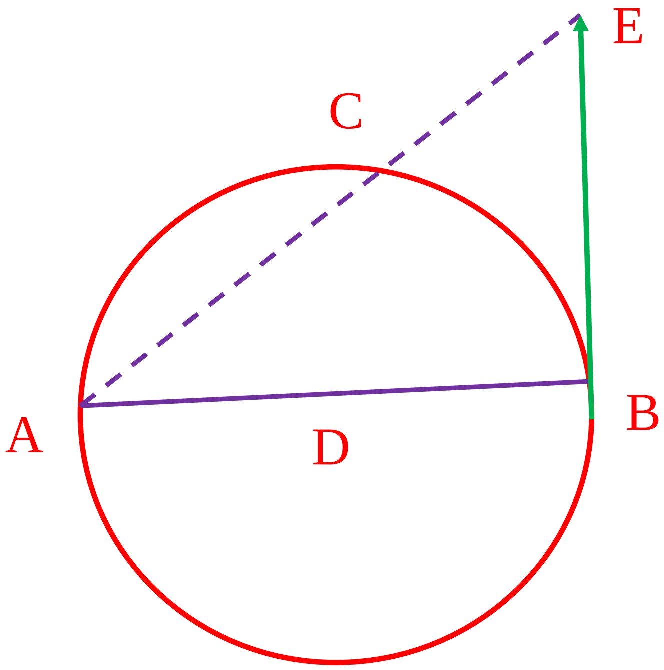
$\therefore$ DE = $\frac{1}{2}$ X 5

$\therefore$ DE = 2.5

Ans.: AD = 9, DC = 5, DE = 2.5

Q. 42 NAVNEET (73/5)

Seg AB is a diameter and seg EB is a tangent segment. The radius of circle is *p*. Prove that EA x CA = p^2



SOLUTION:

Line EB is a tangent to the circle touching the circle at point B line ECA is the secant intersecting the circle at points C and A.

∴ By tangent secant theorem

DB^2 **= EC x EA ………………… (1)**

In Δ EBA

$\angle$ EAB = 90^0 …. (By tangent theorem)

∴ By Pythagoras theorem

$EA^2 = EB^2 + (2p)^2$ **… (Diameter is twice the radius)**

$EA^2 = EB^2 + 4p^2$

$\therefore 4p^2 = EA^2 - EB^2$

$\therefore 4p^2 = EA^2 - EC \times EA$

$\therefore 4p^2 = EA\,(EA - EC)$

$\therefore 4p^2 = EA \times AC$ **…… (H – C – D)**

$EA \times AC = p^2$

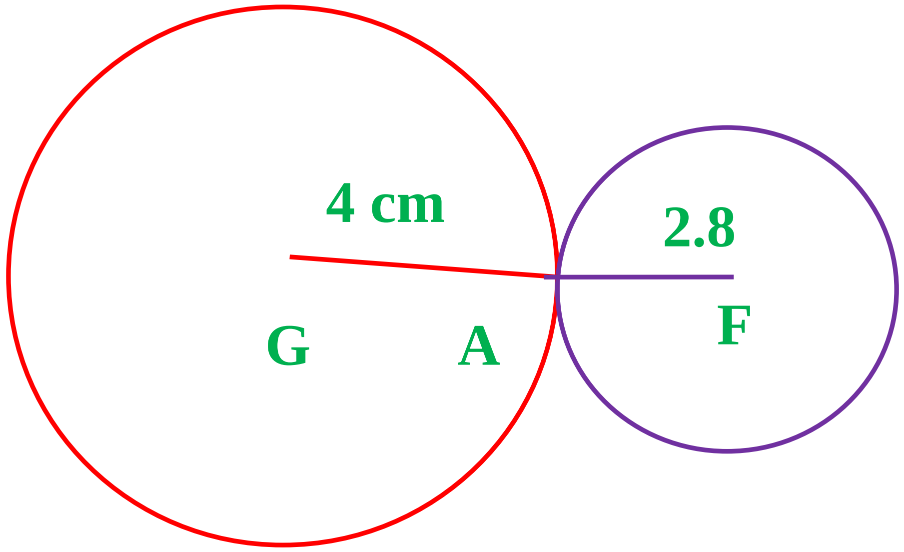
Proved $EA \times AC = p^2$

Q. 43

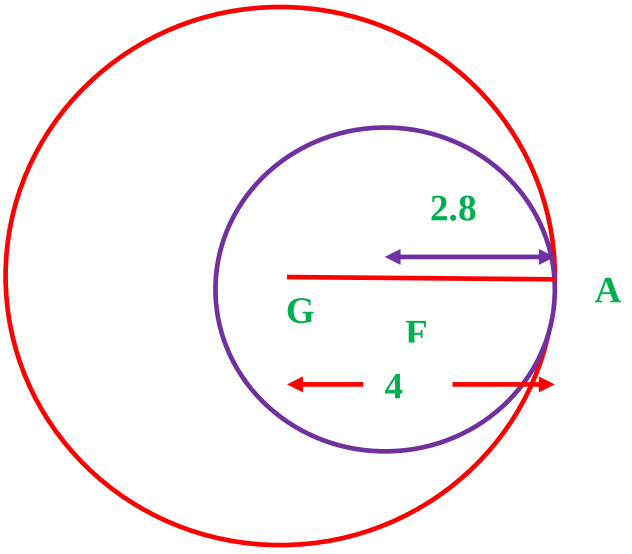
If radii of circles are 4 cm and 2.8 cm Draw figure of this circle (1) Externally (2) Internally

SOLUTION:

(1) Circles touch Externally



(2) Circles touch Internally

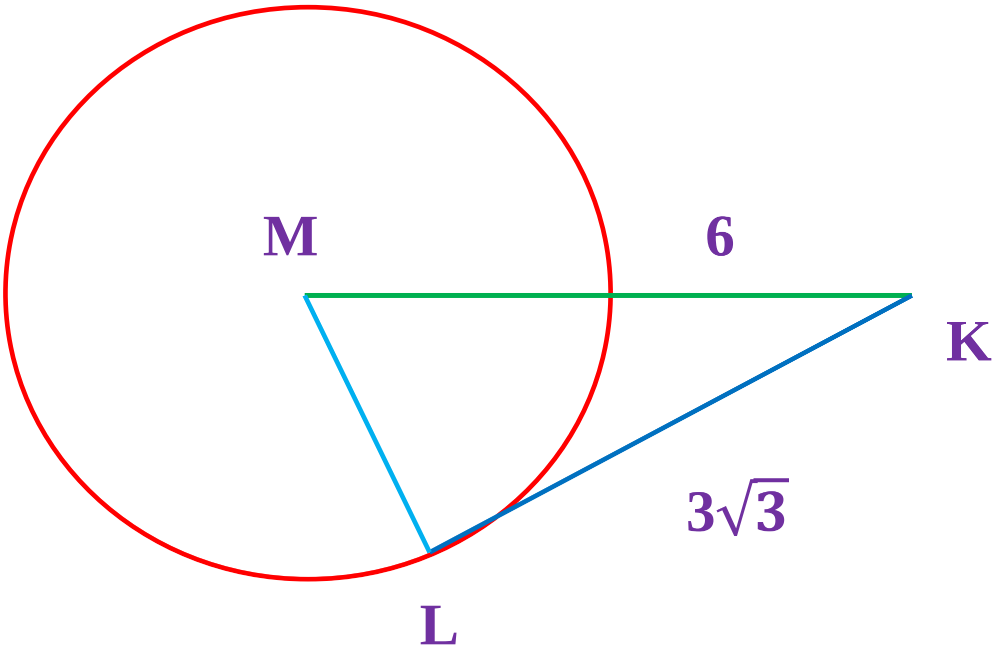


Q. 44 NAVNEET74/3

M is the circle of the center and seg KL is a tangent segment. If MK = 6, KL = $3\sqrt{3}$ then find

(1) Radius of the circle.

(2) Measures of ∠ K and ∠ M



SOLUTION:

In Δ MLK, ∠ MLK = 90^0

∴ By Pythagoras Theorem

$$MK^2 = ML^2 + LK^2$$

$$\therefore 6^2 = ML^2 + \left(3\sqrt{3}\right)^2$$

$$\therefore 36 = ML^2 + (9 X 3)$$

$\therefore$ 36 = $ML^2 + 27$

$\therefore$ $ML^2 = 36 - 27$

$\therefore$ $ML^2 = 9$

$\therefore$ $ML = 3$ (By taking square roots of both sides)

$\therefore$ Radius of circle = ML = 3

In Δ MLK

ML = $\frac{1}{2}$ MK

$\therefore \angle K = 30^0$… (By converse of $30^0 - 60^0 - 90^0$ triangle theorem)

In Δ MLK,

$\therefore \angle M + \angle K + \angle L = 180^0$

$\therefore \angle M + 30^0 + 90^0 = 180^0$

$\therefore \angle M + 120^0 = 180^0$

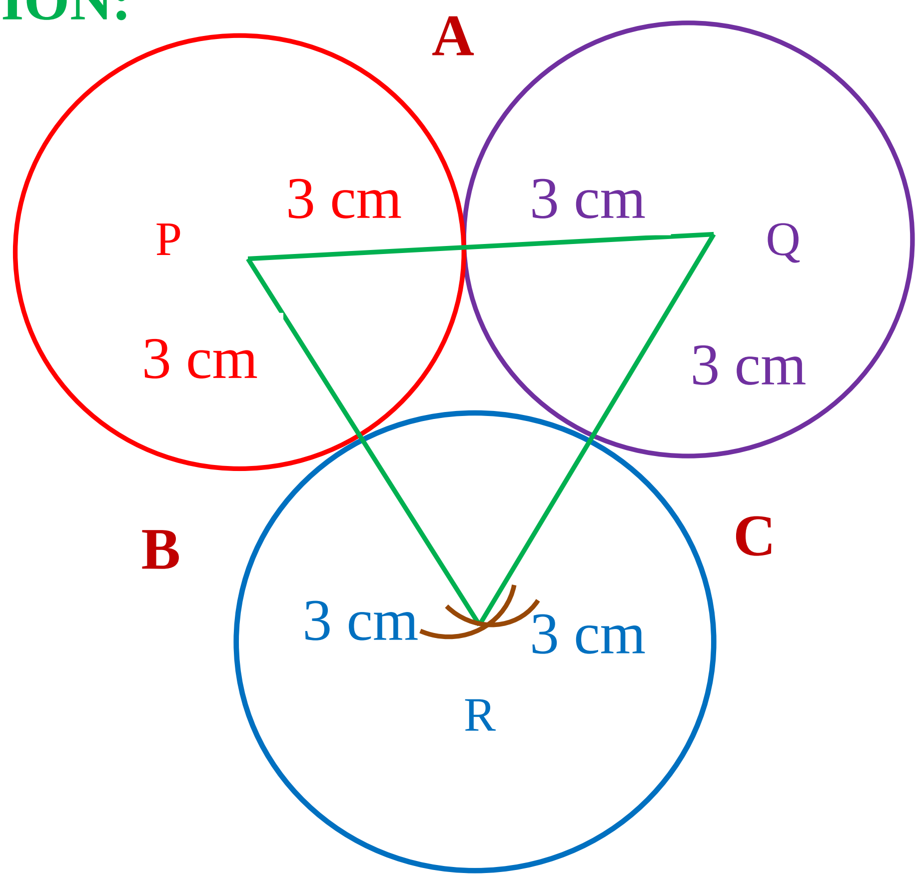
$\therefore \angle M = 60^0$

Ans. : Radius of circle is 3, $\angle K = 30^0$, $\angle M = 60^0$

Q. 45 navneet 78/11

Draw circles with centers P, Q and R each of radius 3 cm, such that each circle touches the other two circles.

SOLUTION:



Analysis

Let the circles with centers P, Q and R touch the points A, B, C as shown in figure.

By theorem of touching the circles, we get

A – P – B, B – R –C, and A – Q – C

∴ AB = AP + PB = 3 + 3 = 6

Similarly, BC = 6 cm and AC = 6 cm

∴ We can construct Δ ABC with

AB = BC = AC = 6 cm

With A, B, C as centers, the required circles of radius 3 cm can be drawn.

Q. 46 NAVNEET 84/24

Two circles intersect each other at points P and T. Their common tangent through T intersects at points Q and D. The segments of the circles at points B and S intersect each other at point R. Prove that □ PQRS is cyclic.

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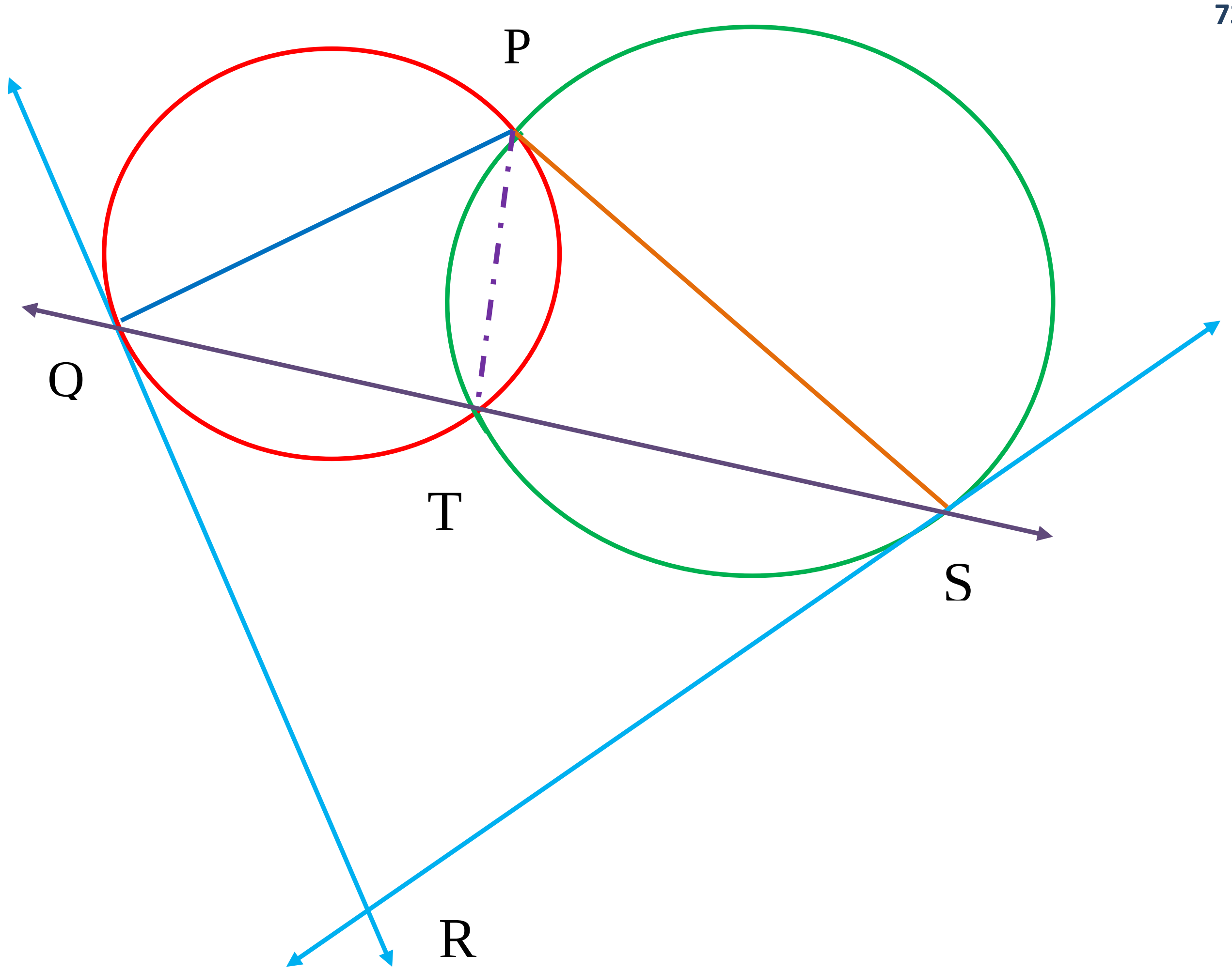


Proof:

Draw seg QP, seg PT, seg PS.

$\therefore \angle TQR = \angle QPT$ … (1)

$\angle TSR = \angle SPT$ … (2) … (By tangent secant theorem)

In ΔQRS,

$\angle SQR + \angle QSR + \angle QSR = 180^0$ … (Sum of all the angle of triangle is 180^0)

$\therefore \angle TQR + \angle TSR + \angle QRS = 180^0$ (Q – T – S)

∴ (∠QPT + ∠SPT) + ∠QRS = 180 ^{0} … {From (1) & (2) }

∴ ∠ QPS + ∠ QRS = 180 ^{0} ... (Angle addition theorem)

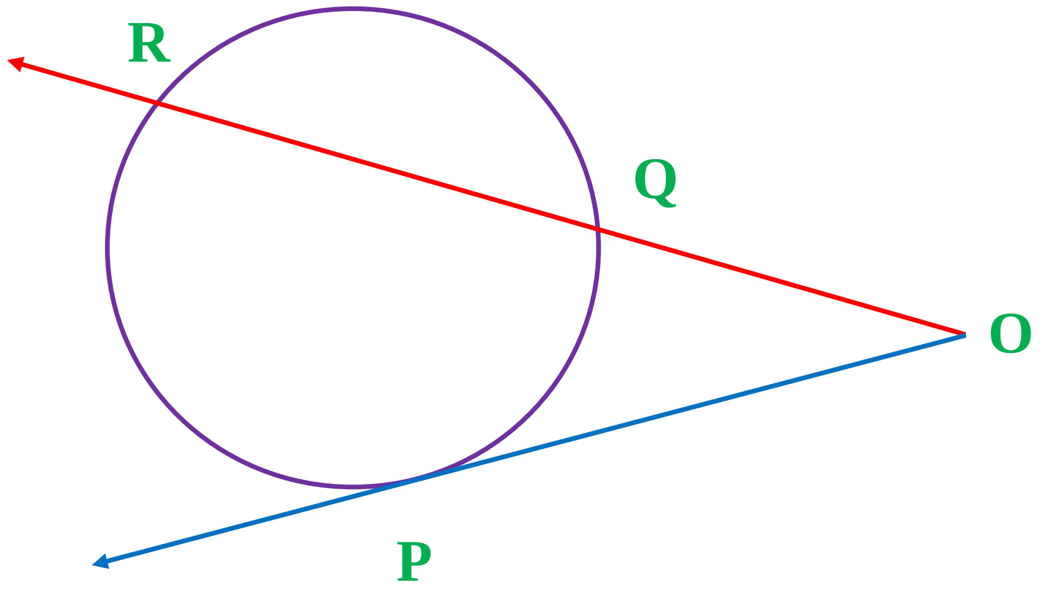
∴ □ PQRS is cyclic ... (Converse of cyclic quadrilateral theorem)

Q. 47 navneet 81/18

P is point of contact.

(1) If m (arc PR) = 120 ^{0}, ∠POR = 25 ^{0} find m (arc PQ)

(2) If OP = 14.4, OQ = 6.4, Find QR



SOLUTION:

(1) m (arc PR) = 120^0, ∠ POR = 25^0

∠ POR has its vertex outside the circle and intercepts arc PR and PQ

∴ $25^0 = \frac{1}{2}$ {m (arc PR) - m (arc PQ)}

∴ 25 x 2 = 120^0 − m (arc PQ)

∴ $50 = 120^0 - m$ (arc PQ)

∴ m (arc PQ) = $120 - 50$

∴ m (arc PQ) = 70^0

(2) Ray OP is tangent to the circle touching the circle at point P and line OQR is secant intersecting the circles at points Q and R.

∴ OP^2 = OQ x QR ... (By tangent secant segment theorem)

∴ 14.4^2 = 6.4 x QR

∴ QR = $\frac{14.4 \text{ X } 14.4}{6.4}$

∴ QR = 32.4

OQ + QR = OR ……….. (O – Q – R)

∴ 6.4 + QR = 32.4

∴ QR = 32.4- 6.4

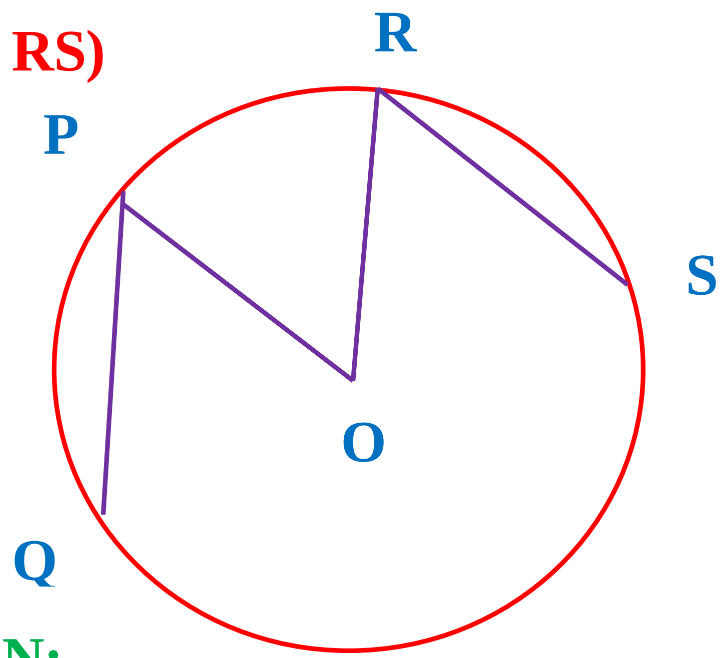
∴ QR = 26

Q. 48 navneet 79/14

O is the center of a circle, chord PQ ≅ chord PS, If $\angle$ POR = 60^0 and m (arc RS) = m (arc PQ) = 70^0. Find

(1) m (arc PR)

(2) m (arc RS)



SOLUTION:

1) $\angle$ POR = 60^0 and m (arc RS) = 70^0… (Given)

m (arc PR) = $\angle$ POR = 60^0

Chord PQ $\cong$ Chord RS … (Given)

$\therefore$ m (arc RS) = m (arc PQ) ... (Corresponding arc of congruent chords)

$\therefore$ m (arc RS) = m (arc PQ) $= 70^0$

2) m (arc PQ) + m (arc PR) + m (arc RS) + m (arc QS) $= 360^0$ … (Measure of a circle is 360^0)

$\therefore$ $70^0 + 60^0 + 70^0$ + m (arc QS) $= 360^0$

$\therefore$ 200^0 + m (arc QS) $= 360^0$

$\therefore$ m (arc QS) $= 360^0 - 200^0$

$\therefore$ m (arc QS) = 160^0

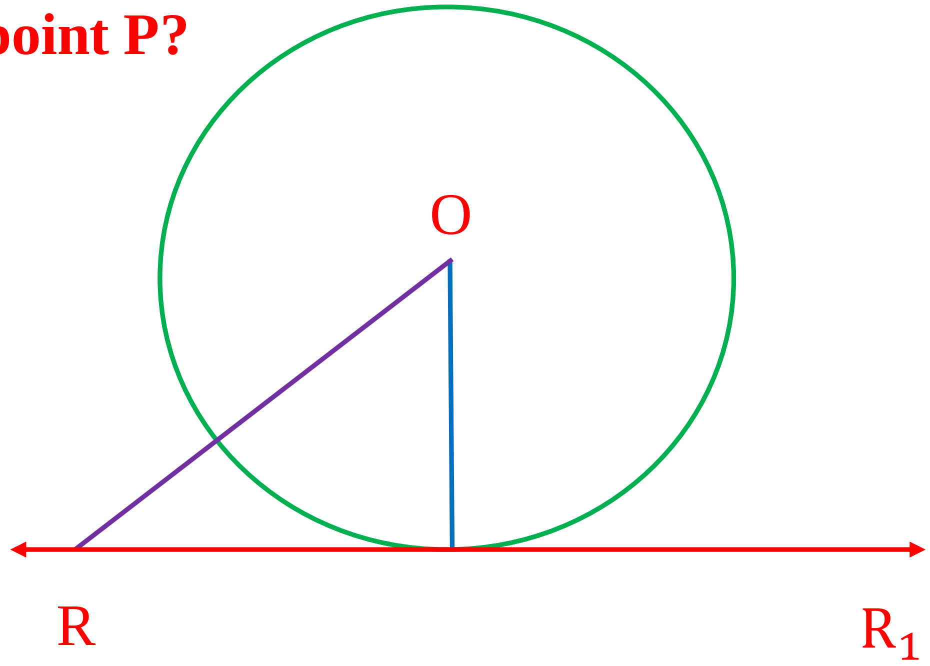
Q. 49 72/4

Line touches a circle with central O at point P. If radius of the circle is 9 cm, answer the following:

1) What is *d* (O, P) = Why?

2) If d (O, Q) = 8 cm

3) If d (O, R) = 15 cm, how many locations of point R are line l? At what distance will each of them be from point P?



SOLUTION:

(1) Radius of the circle is 9 cm ... (Given)

OP is the radius

$\therefore d$ (OP) = 9 cm

(2) d (O, Q) = 8 cm

If d (O, Q) < radius

$\therefore$Point P lies in the interior of the circle.

(3) If d (O, R) = 15 cm, then there are two possible locations of point R on line *l*, one towards the left

side of OP and another towards right side of OP . Let the locations be R and R_1 in Δ OPR.

∴∠ OPR = 90 ^{0} …. (Tangent theorem)

By Pythagoras theorem,

$OR^2 = OP^2 + PR^2$

$\therefore 15^2 = 9^2 + PR^2$

$\therefore 225 = 81 + PR^2$

$225 - 81 = PR^2$

144 = PR^2

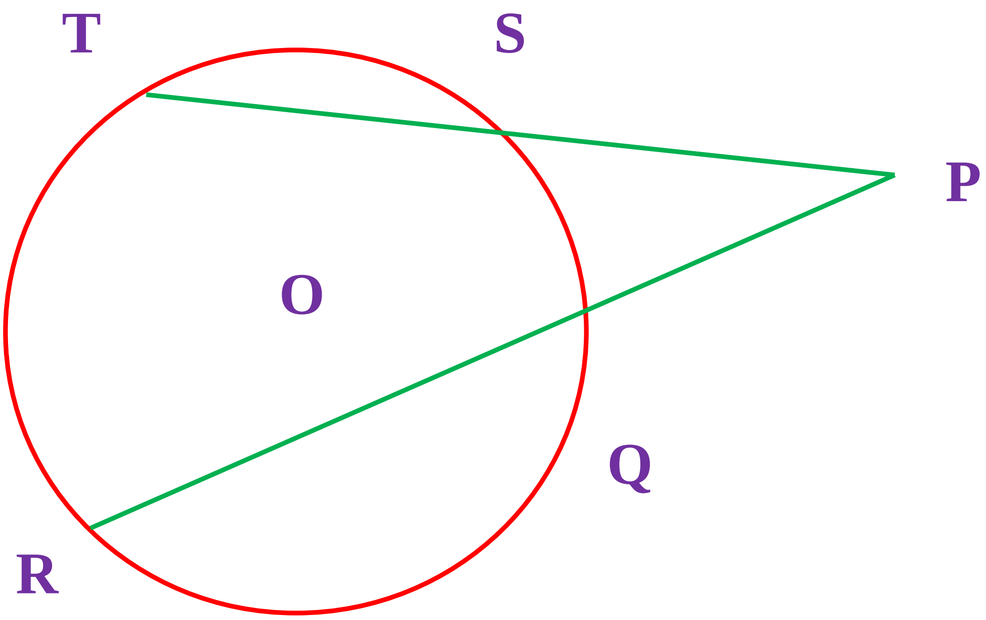
PR = 12 cm

Similarly, PR_1 = 12 cm

Ans.: Each location of point R will be at distance of 12 cm from point P

Q. 50

In figure, if PQ = 3, QR = 5, PS = 4, find TS.



SOLUTION:

PR = PQ + QR ….. (P – Q – R)

∴ PR = 3 + 5

∴ PR = 8

Chord ST and chord QR intersect at a point P outside the circle.

By theorem of external division of chords

PS x PT = PQ x PR

4 x PT = 3 x 8

$$PT = \frac{3 \times 8}{4}$$

PT = 6

PS + TS = PT … (P – S – T)

$\therefore 4 + TS = 6$

$TS = 6 - 4$

$TS = 2$