3. HCF and LCM

Practice Set 10

1. Which number is neither a prime number nor a composite number?

Ans. 1 is neither a prime number nor a composite number.

2. Which of the following are pairs of co-primes?

(i) 8, 14

Ans. : Factors of 8 : 1, 2, 4, 8

Factors of 14 : 1, 2, 7,14

Common divisor of 8 and 14 : 1, 2

∴ 8, 14 is not a pair of co-primes.

(ii) 4, 5

Ans. :

Factors of 4 : 1, 2, 4

Factors of 5 : 1, 5

Common divisor of 4 and 5 : 1

4 and 5 have only 1 as a common divisior.

∴ 4, 5 is a a pair of co-primes.

(iii) 17, 19

Ans. :

Factors of 17 : 1, 17

Factors of 19 : 1, 19

Common divisor of 17 and 19 : 1

17 and 19 have only 1 as a common divisior.

∴ 17, 19 is a a pair of co-primes.

(iv) 27, 15

Ans. :

Factors of 27 : 1, 3, 9, 27

Factors of 15 : 1, 3, 5,15

Common divisor of 27 and 15 : 1, 3

∴ 8, 14 is not a pair of co-primes.

3. List the prime numbers from 25 to 100 and say how many they are.

Ans. 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 are the prime numbers from 25 to 100 and there are 16 prime numbers.

4. Write all the twin prime numbers from 51 to 100.

Ans. (59,61) and (71,73) are the twin prime numbers from 51 to 100.

5. Write 5 pairs of twin prime numbers from 1 to 50.

Ans. (3, 5), (5, 7), (11, 13), (18, 19), (29, 30) are 5 pairs of twin prime numbers from 1 to 50.

6. Which are the even prime numbers?

Ans. Only 2 is the even prime number.

Practice Set 11

Factories the following numbers into primes.

(i) 32

Solution :

Vertical arrangement :

2	32
2	16
2	8
2	4
2	2
	1

$\therefore 32 = 2 \times 2 \times 2 \times 2 \times 2$

Horizontal arrangement :

$32 = 2 \times 16$

$= 2 \times 2 \times 8$

$= 2 \times 2 \times 2 \times 4$

$= 2 \times 2 \times 2 \times \times 2$

$\therefore 32 = 2 \times 2 \times 2 \times 2 \times 2$

(ii) 57

Solution :

Vertical arrangement :

3	57
19	19
	1

∴ 57 = 3 X 19

Horizontal arrangement :

57 = 3 × 19

∴ 57 = 3 × 19

(iii) 23

Solution :

Vertical arrangement :

1	23
23	23
	1

23 = 1 X 23

1 is not a prime number.

∴ 23 = 23

Horizontal arrangement :

23 = 1 × 23

1 is not prime number.

∴ 23 = 23

(iv) 150

Solution :

Vertical arrangement :

2	150
3	75
5	25
5	5
	1

∴ 150 = 2 X 3 X 5 X 5

Horizontal arrangement :

150 = 2 X 75

= 2 X 3 X 25

= 2 X 3 X 5 X 5

∴ 150 = 2 X 3 X 5 X 5

(v) 216

Solution :

Vertical arrangement :

2	216
2	108
2	54
3	27
3	9
3	3
	1

∴ 216 = 2 X 2 X 2 X 3 X 3 X 3

Horizontal arrangement :

216 = 2 X 108

= 2 X 2 X 54

= 2 X 2 X 2 X 27

= 2 X 2 X 2 X 3 X 9

= 2 X 2 X 2 X 3 X 3 X 3

∴ 216 = 2 X 2 X 2 X 3 X 3 X 3

(vi) 208

Solution :

Vertical arrangement :

2	208
2	104
2	52
2	26
13	13
	1

$\therefore 208 = 2 \times 2 \times 2 \times 2 \times 13$

Horizontal arrangement :

$208 = 2 \times 104$

$= 2 \times 2 \times 52$

$= 2 \times 2 \times 2 \times 26$

$= 2 \times 2 \times 2 \times 2 \times 13$

$\therefore 208 = 2 \times 2 \times 2 \times 2 \times 13$

(vii) 765

Solution :

Vertical arrangement :

3	765
3	255
5	85
17	17
	1

$\therefore 765 = 3 \times 3 \times 5 \times 17$

Horizontal arrangement :

$765 = 3 \times 255$

$= 3 \times 3 \times 85$

$= 3 \times 3 \times 5 \times 17$

$\therefore 765 = 3 \times 3 \times 5 \times 17$

(viii) 342

Solution :

Vertical arrangement :

2	342
3	171
3	57
19	19
	1

∴ 342 = 2 X 3 X 3 X 19

Horizontal arrangement :

342 = 2 X 171

= 2 X 3 X 57

= 2 X 3 X 3 X 19

∴ 342 = 2 X 3 X 3 X 19

(ix) 377

Solution :

Vertical arrangement :

13	377
29	29
	1

∴ 377 = 13 X 29

Horizontal arrangement :

377 = 13 X 29

13 and 29 are the prime numbers.

∴ 377 = 13 X 29

(x) 559

Solution :

Vertical arrangement :

13	559
43	43
	1

∴ 559 = 13 X 43

Horizontal arrangement :

559 = 13 X 43

13 and 43 are the prime numbers.

∴ 559 = 13 X 43

1. Find the HCF.

(i) 25, 40

Solution :

Vertical arrangement :

5	25
5	5
	1

2	40
2	20
2	10
5	5
	1

5 is a common factor in 25, 40.

∴ HCF of 25, 40 is 5.

Horizontal arrangement :

$25 = \underline{5} \times 5$

$40 = 2 \times 2 \times 2 \times \underline{5}$

5 is a common factor in 25 and 40

∴ HCF of 25, 40 is 5

(ii) 56, 32

Solution :

Vertical arrangement :

2	56
2	28
2	14
7	7
	1

2	32
2	16
2	8
2	4
2	2
	1

$HCF = 2 \times 2 \times 2 = 8$

∴ HCF of 56, 32 is 8.

Horizontal arrangement :

$56 = \underline{2} \times \underline{2} \times \underline{2} \times 7$

$32 = \underline{2} \times \underline{2} \times \underline{2} \times 2 \times 2$

$= 2 \times 2 \times 2$

$= 8$

∴ HCF of 56, 32 is 8.

(iii) 40, 60, 75

Solution :

Vertical arrangement :

5	40	60	75
	8	12	15

5 is a common factor in 40, 60 and 75.

∴ HCF of 40, 60, 75 is 5.

Horizontal arrangement :

$40 = 2 \times 20$

$= 2 \times 2 \times 10$

$= 2 \times 2 \times 2 \times \underline{5}$

$60 = 2 \times 30$

$= 2 \times 2 \times 15$

$= 2 \times 2 \times 3 \times \underline{5}$

$75 = 3 \times 25$

$= 3 \times 5 \times \underline{5}$

5 is a common factor in 40, 60 and 75.

∴ HCF of 40, 60, 75 is 5.

(iv)16, 27

Solution :

Vertical arrangement :

2	16
2	8
2	4
2	2
	1

3	27
3	9
3	3
	1

There is no common prime factors in 16 and 27.

∴ HCF of 16, 27 is 1.

Horizontal arrangement :

$16 = 2 \times 2 \times 2 \times 2$

$27 = 3 \times 3 \times 3$

There is common prime factors in 16 and 27.

∴ HCF of 16, 27 is 1.

(v) 18, 32, 48

Solution :

Vertical arrangement :

2	18	32	48
	9	16	24

2 is a common factor in 18, 32 and 48.

∴ HCF of 18, 32, 48 is 2 .

Horizontal arrangement :

18 = 2 X 9

= 2 X 3 X 3

32 = 2 X 16

= 2 X 2 X 8

= 2 X 2 X 2 X 4

= 2 X 2 X 2 X 2 X 2

48 = 2 X 24

= 2 X 2 X 12

= 2 X 2 X 2 X 6

∴ HCF of 18, 32, 48 is 2

(vi) 105, 154

Solution :

3	105
5	35
7	7
	1

105 = 3 × 5 × 7

2	154
7	77
11	11
	1

154 = 2 × 7 × 11

7 is a common factor in 105 and 154.

∴ HCF of 105, 154 is 7.

(vii) 42, 45, 48

Solution :

2	42
3	21
7	7
	1

$42 = 2 \times \underline{3} \times 7$

3	45
3	15
5	5
	1

$45 = 3 \times \underline{3} \times 5$

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2 \times 2 \times 2 \times 2 \times \underline{3}$

3 is a common factor in 42, 45 and 48.

∴ HCF of 42, 45, 48 is 3.

(viii) 57, 75, 102

Solution :

3	57
19	19
	1

$57 = \underline{3} \times 19$

3	75
5	25
5	5
	1

$75 = \underline{3} \times 5 \times 5$

2	102
3	51
17	17
	1

$102 = 2 \times \underline{3} \times 17$

3 is a common factor in 57, 75 and 102.

∴ HCF of 57, 75, 102 is 3.

(ix) 56, 57

Solution :

2	56
2	28
2	14
7	7
	1

56 = 2 X 2 X 2 X 7

3	57
19	19
	1

57 = 3 X 19

There are no common prime factors in 56 and 57.

∴ HCF of 56, 57 is 1.

(x) 777, 315, 588

Solution :

3	777
7	259
37	37
	1

777= 3 X 7 X 37

3	315
3	105
5	35
7	7
	1

315 = 3 X 3 X 5 X 7

2	588
2	294
3	147
7	49
7	7
	1

588 = 2 X 2 X 3 X 7 X 7

∴ HCF of 777, 315, 588 = 3 X 7 = 21.

(2) Find the HCF by the division method and reduce to the simplest form.

(i) $\frac{275}{525}$

Solution:

$$
\begin{array}{r|l|l}
275 & 525 & 1 \\
 & -\ 275 & \\
\hline
250 & 275 & 1 \\
 & -\ 250 & \\
\hline
025 & 250 & 10 \\
 & -250 & \\
\hline
 & 000 &
\end{array}
$$

Simplest form : $\frac{275}{525} = \frac{275 \div 25}{525 \div 25} = \frac{11}{21}$

$\therefore$ The HCF is 25 and the simplest form of the fraction is $\frac{11}{21}$

(ii) $\frac{76}{133}$

Solution :

$$
\begin{array}{r|l|l}
76 & 133 & 1 \\
 & -\ 76 & \\
\hline
57 & 76 & 1 \\
 & -\ 57 & \\
\hline
19 & 57 & 3 \\
 & -\ 57 & \\
\hline
 & 00 &
\end{array}
$$

Simplest form : $\frac{76}{133} = \frac{76 \div 19}{133 \div 19} = \frac{4}{7}$

$\therefore$ The HCF is 19 and the simplest form of the fraction is $\frac{4}{7}$

(iii) $\frac{161}{69}$

Solution:

$$
\begin{array}{r|l|l}
69 & 161 & 2 \\
 & -138 & \\
\hline
 & 23 \;|\; 69 & 3 \\
 & -\,69 & \\
\hline
 & 00 &
\end{array}
$$

Simplest form : $\frac{161}{69} = \frac{161 \div 23}{69 \div 23} = \frac{7}{3}$

$\therefore$ The HCF is 23 and the simplest form of the fraction is $\frac{7}{3}$

Practice Set 13

1. Find the LCM :

(i) 12, 15

Solution : $12 = 2 \times 2 \times \underline{3}$; $15 = \underline{3} \times 5$

LCM = $2 \times 2 \times 3 \times 5 = 60$

$\therefore$ The LCM of 12 and 15 is 60.

(ii) 6, 8,10

Solution :

6 = 2 X 3

8 = 2 X 2 X 2

10 = 2 X 5

LCM = 2 X 2 X 3 X 5 X 2 = 120

∴ The LCM of 6, 8 and 10 is 120.

(iii) 18, 32

Solution :

18 = 2 X 3 X 3

32 = 2 X 2 X 2 X 2 X 2

LCM = 2 X 2 X 2 X 2 X 2 X 3 X 3 = 288

∴ The LCM of 18 and 32 is 288.

(iv) 10, 15, 20

Solution :

10 = 2 X 5

15 = 3 X 5

20 = 2 X 2 X 5

LCM = 2 X 3 X 2 X 5 = 60

∴ The LCM of 6, 8 and 10 is 60.

(v) 45, 86

Solution :

45 = 3 X 3 X 5

86 = 2 X 43

LCM = 3 X 3 X 5 X 2 X 43 = 3870

∴ The LCM of 45 and 86 is 3870.

(vi) 15, 30, 90

Solution :

15 = 3 X 5

30 = 3 X 5 X 2

90 = 3 X 5 X 3 X 2

LCM = 3 X 5 X 2 X 3 = 90

∴ The LCM of 15, 30 and 90 is 90.

(vii) 105, 195

Solution :

105 = 3 X 5 X 7

195 = 3 X 5 X 13

LCM = 3 X 5 X 7 X 13 = 1365

∴ The LCM of 105, 195 is 1365.

(viii) 12, 15, 45

Solution :

12 = 2 X 2 X 3

15 = 3 X 5

45 = 3 X 5 X 3

LCM = 2 X 2 X 3 X 5 X 3 = 180

∴ The LCM of 12, 15 and 45 is 180.

(ix) 63, 81

Solution :

63 = 3 X 7 X 3

81 = 3 X 9 X 3

LCM = 3 X 7 X 9 X 3 = 567

∴ The LCM of 63 and 81 is 567.

(x) 18, 36, 27

Solution :

18 = 2 X 3 X 3

36 = 3 X 3 X 2 X 2

27 = 3 X 3 X 3

LCM = 2 X 2 X 3 X 3 X 3 = 108

∴ The LCM of 18, 36 and 27 is 108.

2. Find the HCF and LCM of the numbers given below. Verify that their product is equal to the product of the given numbers.

(i) 32, 37

Solution :

32 and 37 are co-prime numbers.

∴ Their HCF is 1 ……..(i)

LCM = 32 X 37 = 1184 …….(ii)

The product of two numbers = HCF × LCM

32 X 37 = 1 × 1184 …from (i) and (ii)

1184 = 1184

The product of 32 and 37 is equal to the product of the HCF and LCM.

(ii) 46, 51

Solution :

46 and 51are co-prime numbers.

∴ Their HCF is 1 ……..(i)

LCM = 46 X 51 = 2346 …….(ii)

The product of two numbers = HCF × LCM

46 X 51 = 1 × 2346 …from (i) and (ii)

2346 = 2346

The product of 46 and 51 is equal to the product of the HCF and LCM.

(iii) 15, 60

Solution :

$15 = 3 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$

∴ Their HCF is $3 \times 5 = 15$ ……..(i)

LCM = $2 \times 2 \times 3 \times 5 = 60$ …….(ii)

The product of two numbers = HCF × LCM

$15 \times 60 = 15 \times 60$ …from (i) and (ii)

$900 = 900$

The product of 15 and 60 is equal to the product of the HCF and LCM.

(iv) 18, 63

Solution :

$18 = 2 \times 3 \times 3$ and $63 = 3 \times 3 \times 7$

$\therefore$ Their HCF is $3 \times 3 = 9$ ……..(i)

LCM = $3 \times 3 \times 2 \times 7 = 126$ …….(ii)

The product of two numbers = HCF × LCM

$18 \times 63 = 9 \times 126$ …from (i) and (ii)

$1134 = 1134$

The product of 18 and 63 is equal to the product of the HCF and LCM.

(v) 78, 104

Solution :

$78 = 2 \times 3 \times 13$ and $104 = 2 \times 2 \times 2 \times 13$

$\therefore$ Their HCF is $2 \times 13 = 26$ ……..(i)

LCM = $2 \times 2 \times 2 \times 3 \times 13 = 312$ …….(ii)

The product of two numbers = HCF × LCM

$78 \times 104 = 26 \times 312$ …from (i) and (ii)

$8112 = 8112$

The product of 78 and 104 is equal to the product of the HCF and LCM.

Practice Set 14

1. Choose the right option.

(i) The HCF of 120 and 150 is .....

(1) 30 (2) 45 (3) 20 (4) 120

Ans : (1) 30

Explanation : $120 = 2 \times 60$

$= 2 \times 2 \times 30$

$= 2 \times 2 \times 2 \times 15$

$= \underline{2} \times 2 \times 2 \times \underline{3} \times \underline{5}$

$150 = 2 \times 75$

$= 2 \times 3 \times 25$

$= \underline{2} \times \underline{3} \times \underline{5} \times 5$

$\therefore$ HCF $= 2 \times 3 \times 5 = 30$

$\therefore$ Option (1) 30 is the answer.

(ii) The HCF of this pair of numbers is not 1.

(1) 13,17 (2) 29, 20 (3) 40, 20 (4) 14,15

Ans : (3) 40, 20

Explanation : 13,17 ; 29, 20 ; 14,15 are the pair of coprime numbers.

$\therefore$ their HCF is 1.

Except 40 and 20 the others are coprime numbers.

$\therefore$ Option (3) 40, 20 is the answer.

2. Find the HCF and LCM.

(i) 14 , 28

Solution :

2	14	28
2	7	14
7	7	7
	1	1

HCF = $2 \times 7 = 14$

LCM = $2 \times 2 \times 7 = 28$

(ii) 32 , 16

Solution :

2	32	16
2	16	8
2	8	4
2	4	2
2	2	1
	1	1

HCF = 2 X 2 X 2 X 2 = 16

LCM = 2 X 2 X 2 X 2 X 2 = 32

(iii) 17, 102 , 170

Solution :

17	17	102	170
2	1	6	10
3	1	3	5
5	1	1	5
	1	1	1

HCF = 17

LCM = 2 X 3 X 5 X 17 = 510

(iv) 23 , 69

Solution :

23	23	69
3	1	3
	1	1

HCF = 23

LCM = 3 X 23 = 69

(v) 21, 49 , 84

Solution :

3	21	49	84
7	7	49	28
7	1	7	4
2	1	1	4
2	1	1	2
	1	1	1

HCF = 7

LCM = 2 X 2 X 3 X 7 X 7 = 588

3. Find the LCM.

(i) 36 , 42

Solution :

2	36	42
2	18	21
3	9	21
3	3	7
7	1	7
	1	1

LCM = 2 X 2 X 3 X 3 X 7 = 252

(ii) 15, 25 , 30

Solution :

5	15	25	30
3	3	5	6
5	1	5	2
2	1	1	2
	1	1	1

LCM = 2 X 3 X 5 X 5 = 150

(iii) 18, 42 , 48

Solution :

2	18	42	48
3	9	21	24
2	3	7	8
2	3	7	4
2	3	7	2
3	3	7	1
7	1	7	1
	1	1	1

LCM= 2 X 2 X 2 X 2 X 3 X 3 X 7 = 1008

(iv) 4, 12 , 20

Solution :

2	4	12	20
2	2	6	10
3	1	3	5
5	1	1	5
	1	1	1

LCM = 2 X 2 X 3 X 5 = 60

(v) 24, 40, 80 ,120

Solution :

2	24	40	80	120
2	12	20	40	60
2	6	10	20	30
2	3	5	10	15
3	3	5	5	15
5	1	5	5	5
	1	1	1	1

LCM = 2 X 2 X 2 X 2 X 3 X 5 = 240

4. Find the smallest number which when divided by 8, 9, 10, 15, 20 gives a remainder of 5 every time.

Solution :

Let us find the LCM of the 8, 9, 10, 15, 20

2	8	9	10	15	20
2	4	9	5	15	10
2	2	9	5	15	5
3	1	9	5	15	5
3	1	3	5	5	5
5	1	1	5	5	5
	1	1	1	1	1

LCM = 2 X 2 X 2 X 3 X 3 X 5 = 360

To the LCM we add the remainder obtained every time.

Hence, that number = LCM + remainder

= 360 + 5

= 365

∴ The smallest number is 365.

5. Reduce the fractions $\frac{348}{319}, \frac{221}{247}, \frac{437}{551}$ to the lowest terms.

Solution :

(i) $\frac{348}{319}$

To reduce the number to its simplest form, we will find the common factors of 348 and 319. Let us find their HCF by the division method.

$$
\begin{array}{l}
319\overline{)\,348\,(1} \\
\quad -\,319 \\
\hline
\quad 029\overline{)\,319\,(11} \\
\qquad -\,319 \\
\hline
\qquad\;\; 000
\end{array}
$$

Here, 29 is the HCF. That is the numerator and denominator are both divisible by 29.

$\therefore$ Simplest form : $\frac{348}{319} = \frac{348 \div 29}{319 \div 29} = \frac{12}{11}$

(ii) $\frac{221}{247}$

To reduce the number to its simplest form, we will find the common factors of 221 and 247. Let us find their HCF by the division method.

$$
\begin{array}{l}
221\overline{)247}\,(1 \\
\quad -221 \\
\quad \overline{026)221}\,(8 \\
\quad\quad -208 \\
\quad\quad \overline{013)26}\,(2 \\
\quad\quad\quad -26 \\
\quad\quad\quad \overline{00}
\end{array}
$$

Here, 13 is the HCF. That is the numerator and denominator are both divisible by 13.

$\therefore$ Simplest form : $\frac{221}{247} = \frac{221 \div 13}{247 \div 13} = \frac{17}{19}$

(iii) $\frac{437}{551}$

To reduce the number to its simplest form, we will find the common factors of 437 and 551. Let us find their HCF by the division method.

$$
\begin{array}{l}
437\overline{)\,551\,(1} \\
\quad -\,437 \\
\quad\overline{} \\
\quad 114\overline{)\,437\,(3} \\
\quad\quad -\,342 \\
\quad\quad\overline{} \\
\quad\quad 095\overline{)\,114\,(1} \\
\quad\quad\quad -\,095 \\
\quad\quad\quad\overline{} \\
\quad\quad\quad 019\overline{)\,95\,(5} \\
\quad\quad\quad\quad -\,95 \\
\quad\quad\quad\quad\overline{} \\
\quad\quad\quad\quad\quad 00
\end{array}
$$

Here, 19 is the HCF. That is the numerator and denominator are both divisible by 19.

$\therefore$ Simplest form : $\frac{437}{551} = \frac{437 \div 19}{551 \div 19} = \frac{23}{29}$

6. The LCM and HCF of two numbers are 432 and 72 respectively. If one of the numbers is 216, what is the other ?

Solution :

Given : LCM = 432, HCF = 72, One of the number = 216

Suppose the other number be x.

The product of two number = HCF X LCM

$$\therefore 216 \times x = 72 \times 432$$

$$\therefore 216 \times x = 31104$$

$$\therefore x = \frac{31104}{216}$$

$$\therefore x = 144$$

$\therefore$ The other number is 144

7. The product of two two-digit numbers is 765 and their HCF is 3. What is their LCM ?

Solution :

Given : The product of two two-digit numbers is 765,

HCF = 3

The product of two number = HCF X LCM

$$765 = 3 \times \text{LCM}$$

$$\therefore \text{LCM} = \frac{765}{3}$$

$$\therefore \text{LCM} = 255$$

$\therefore$ Their LCM is 255.

8. A trader has three bundles of string 392 m, 308 m and 490 m long. What is the greatest length of string that the bundles can be cut up into without any left-over tring ?

Solution:

We have to find the HCF of the given lengths of the string.

Let us find the greatest possible length of each piece of string.

2	392
2	196
2	98
7	49
7	7
1	

2	308
2	154
7	77
11	11
	1

2	490
5	245
7	49
7	7
	1

The HCF $= 2 \times 7 = 14$

$\therefore$ The greatest length of each piece of string is 14 m.

9 . Which two consecutives even numbers have an LCM of 180 ?

Solution: The LCM of two consecutive even numbers is 180

The HCF of two consecutive even numbers is 2.

The product of two numbers = HCF X LCM

The product of two numbers = 2 X 180

∴ The product of two numbers = 360

The difference between two consecutive numbers is 2 so find the factors of 360 whose difference is 2.

360 = 18 X 20

∴ The required consecutive even numbers are 18 and 20.

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