CHAPTER - 4

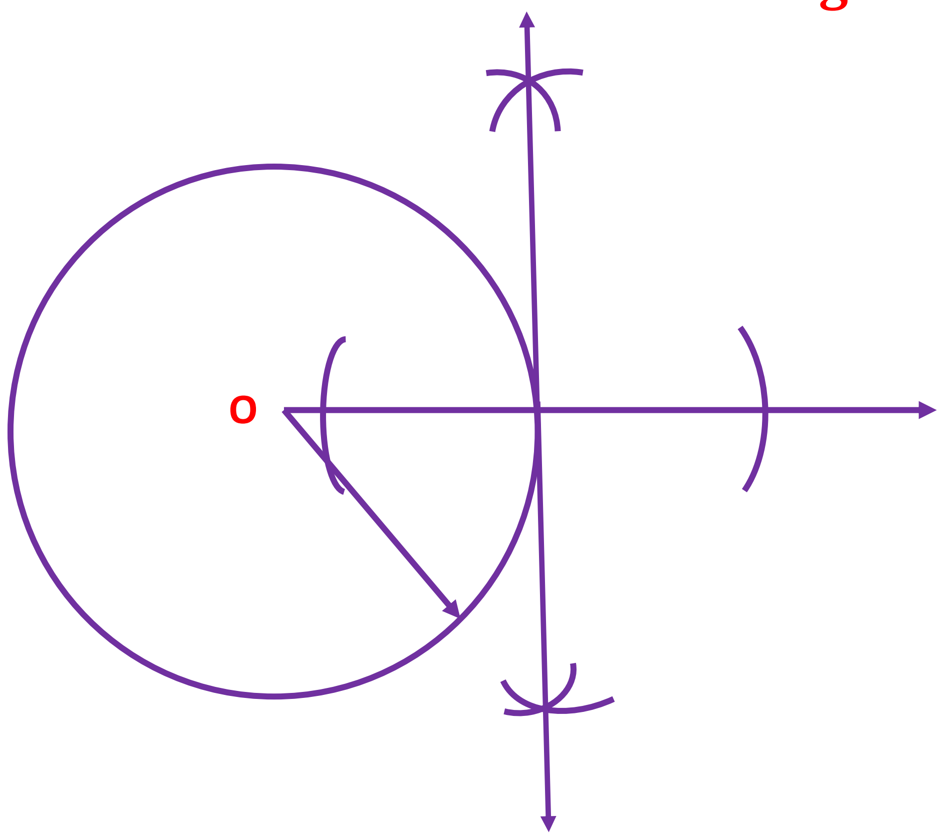
GEOMETRIC CONSTRUCTIONS

LONG QUESTIONS AND

Q. 1 (JIVANDEEP 186)

Draw a circle with O as center and with radius 6.2 mm. Take any point M on the circle and draw tangent to the circle.

SOLUTION:



1) Draw a circle with O as center and 6.2 cm radius

2) Take any point M on the circle

3) Draw OM as radius of circle and extend beyond M

4) Draw perpendicular PQ to OM

5) The straight-line PQ is the tangent to the circle passing through point M

Q. 2

Δ ABC ~ Δ LMN. In Δ ABC, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm. Construct Δ ABC and Δ LMN such that $\frac{BC}{MN} = \frac{5}{4}$.

SOLUTION:

ANALYSIS:

For Δ ABC, the lengths of three sides are known.

$\therefore$ Δ ABC can be constructed.

Δ ABC ~ Δ LMN

$\therefore$ $\frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN}$ … (Corresponding sides of similar triangles are in proportion)

$\therefore$ $\frac{5.5}{LM} = \frac{6}{MN} = \frac{4.5}{LN} = \frac{5}{4}$

$\therefore \frac{5.5}{LM} = \frac{5}{4}$

$\therefore LM = \frac{5.5 \times 4}{5}$

$\therefore$ LM = 1.1 x 4

$\therefore$ LM = 4.4 cm

$\therefore \frac{6}{MN} = \frac{5}{4}$

$\therefore MN = \frac{6 \times 4}{5}$

$\therefore LM = \frac{24}{5}$

$\therefore$ LM = 4.8 cm

$\therefore \frac{4.5}{LN} = \frac{5}{4}$

$\therefore LN = \frac{4.5 \times 4}{5}$

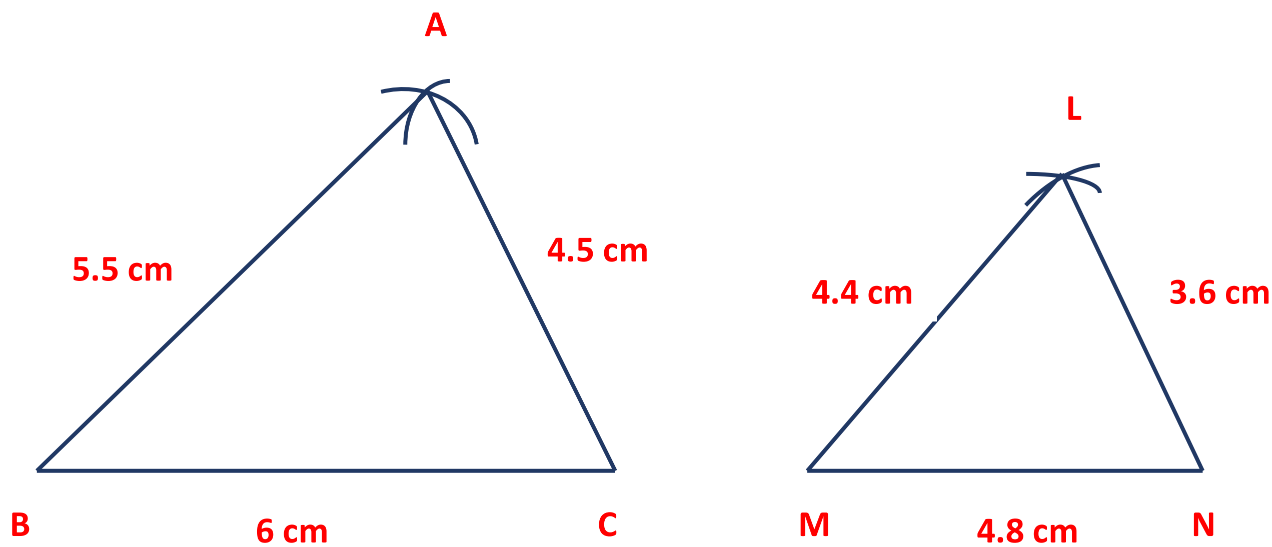
$\therefore LN = \frac{18}{5}$

$\therefore$ LN = 3.6 cm

For Δ LMN, the lengths of three sides are known.

∴ Δ LMN can be constructed.

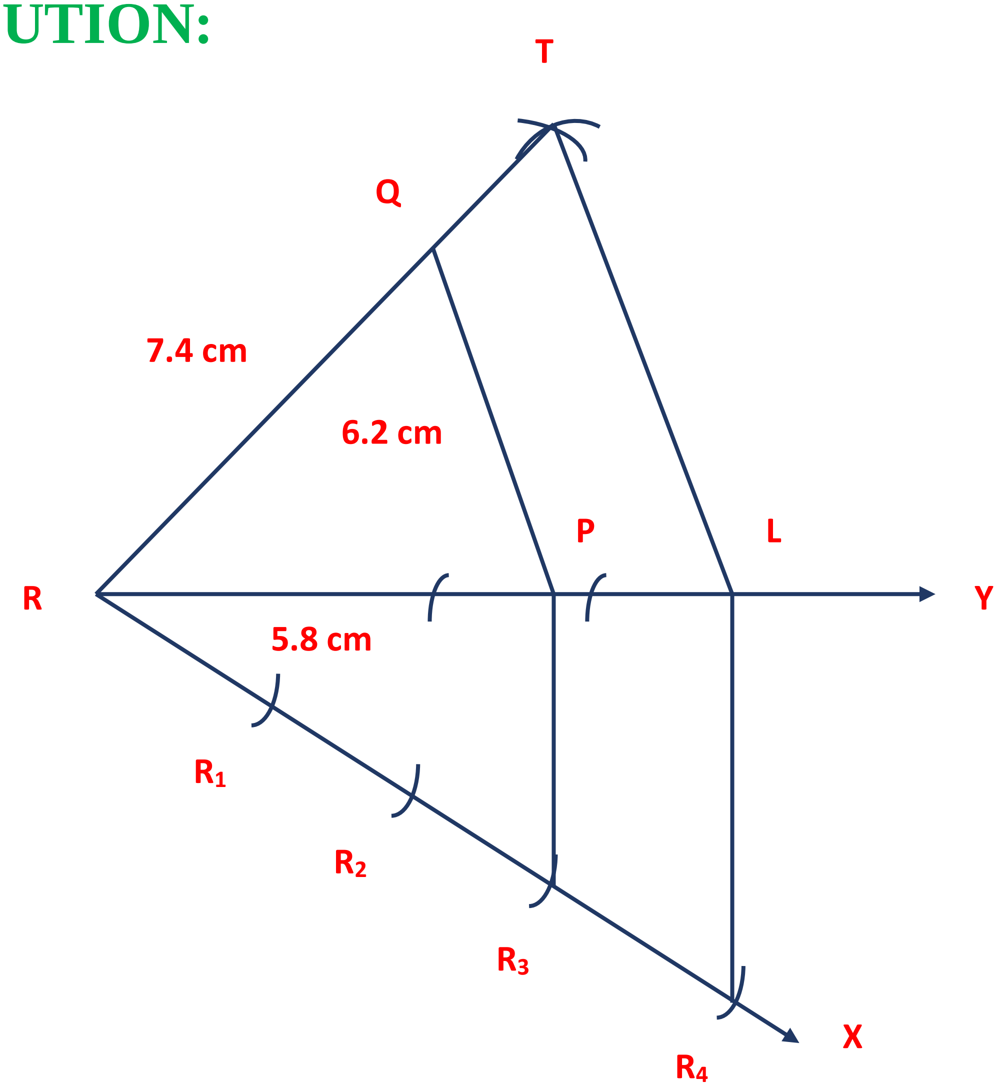
CONSTRUCTION:



Q. 3

Δ PQR ~ Δ LTR. In Δ PQR, PQ = 6.2 cm, QR = 7.4 cm, PR = 5.8 cm. Construct Δ PQR and Δ LTR such that $\frac{PQ}{LT} = \frac{3}{4}$

SOLUTION:



Q. 4

Δ RST ~ Δ XYZ. In Δ RST, RS = 4.5 cm, ∠ RST = 50°, ST = 5.7 cm. Construct Δ RST and Δ XYZ such that $\frac{RS}{XY} = \frac{3}{5}$

SOLUTION:

ANALYSIS:

For Δ RST, the lengths of two sides and included angle are known.

∴ Δ RST can be constructed.

Δ RST ~ Δ XYZ

$\therefore \frac{RS}{XY} = \frac{ST}{YZ} = \frac{3}{5}$ **… Corresponding sides of similar triangles are in proportion)**

$\therefore \frac{4.5}{XY} = \frac{5.7}{YZ} = \frac{3}{5}$

$\therefore \frac{4.5}{XY} = \frac{3}{5}$ $\frac{5.7}{YZ} = \frac{3}{5}$

$\therefore XY = \frac{4.5 \times 5}{3}$ $\therefore YZ = \frac{5.7 \times 5}{3}$

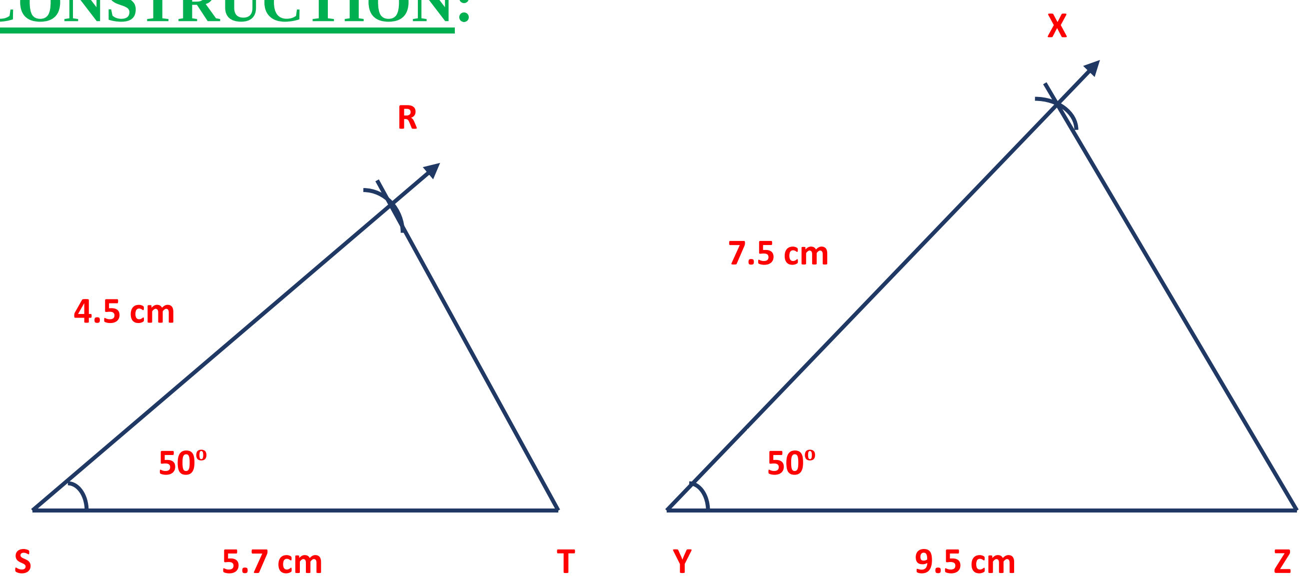
$\therefore$ XY = 7.5 cm **$\therefore$ YZ = 9.5 cm**

$\angle$ RST = $\angle$ XYZ = 50º … (Corresponding angles of similar triangles are congruent)

For Δ XYZ, the lengths of two sides and included angle are known.

$\therefore$ Δ XYZ can be constructed.

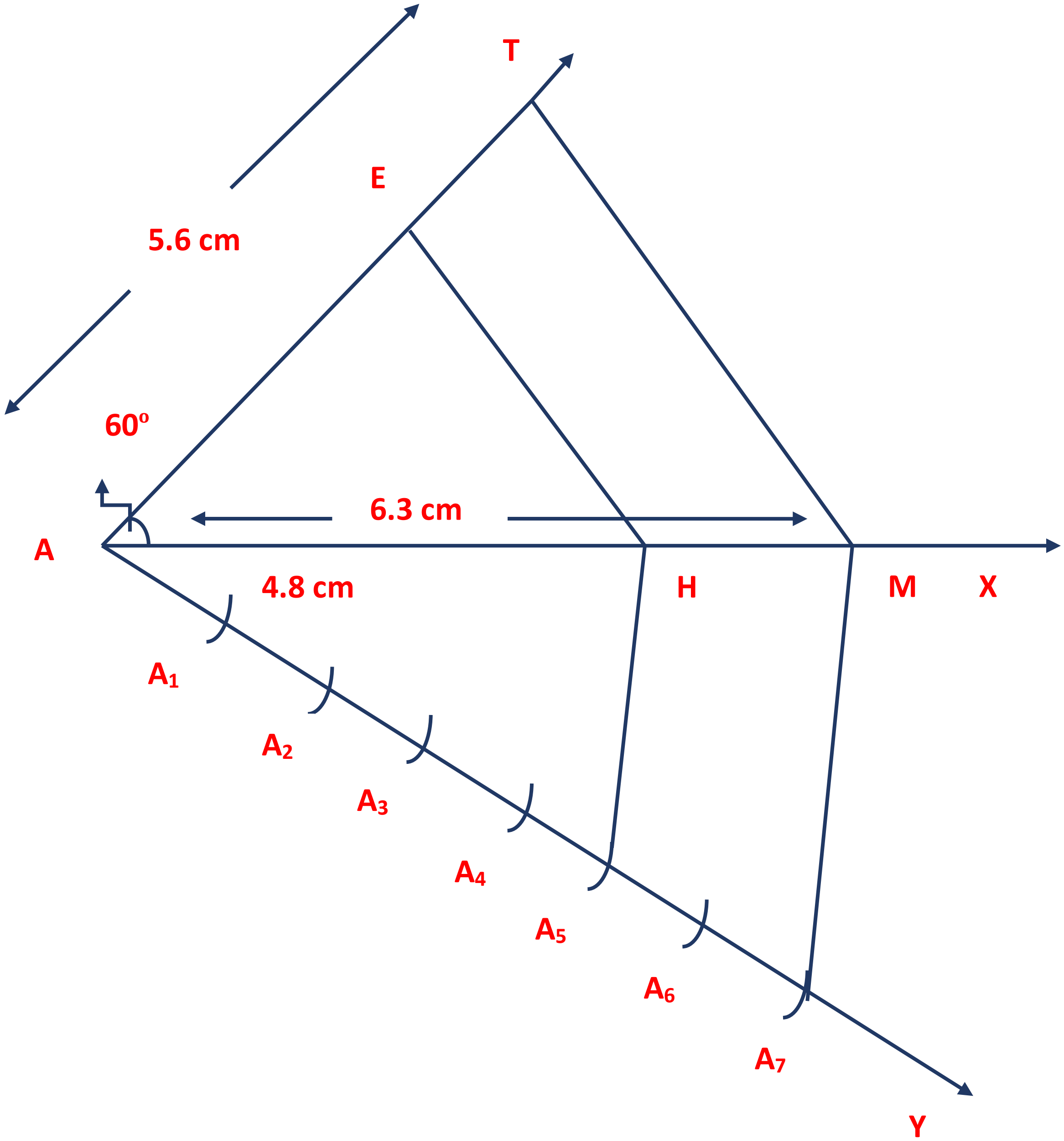
CONSTRUCTION:



Q. 5

Δ AMT ~ Δ AHE. In Δ AMT, AM = 6.3 cm, $\angle$ TAM = 60º, AT = 5.6 cm. $\frac{AM}{AH} = \frac{7}{5}$ Construct Δ AHE

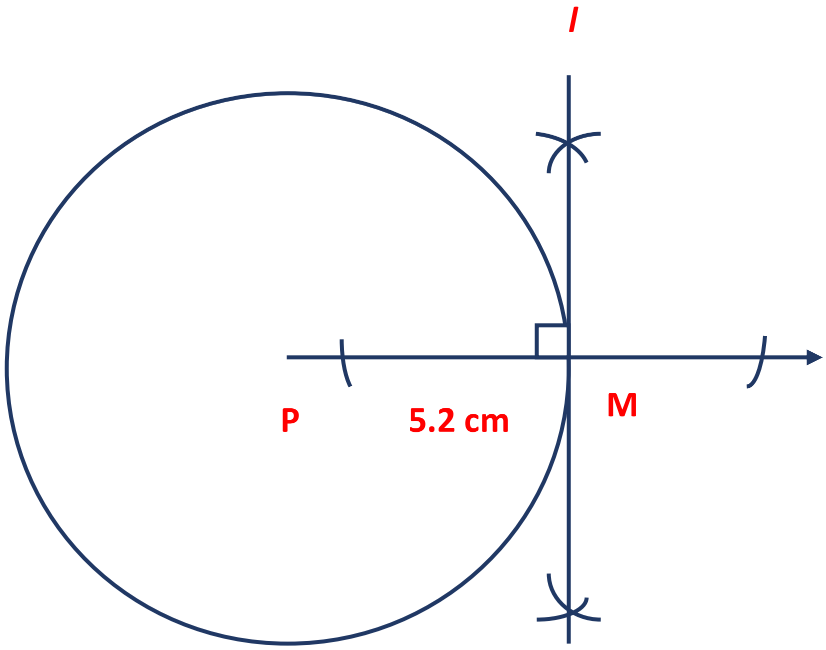
SOLUTION:



Q. 6

Construct a tangent to a circle with center P and radius 5.2 cm at any point M on it.

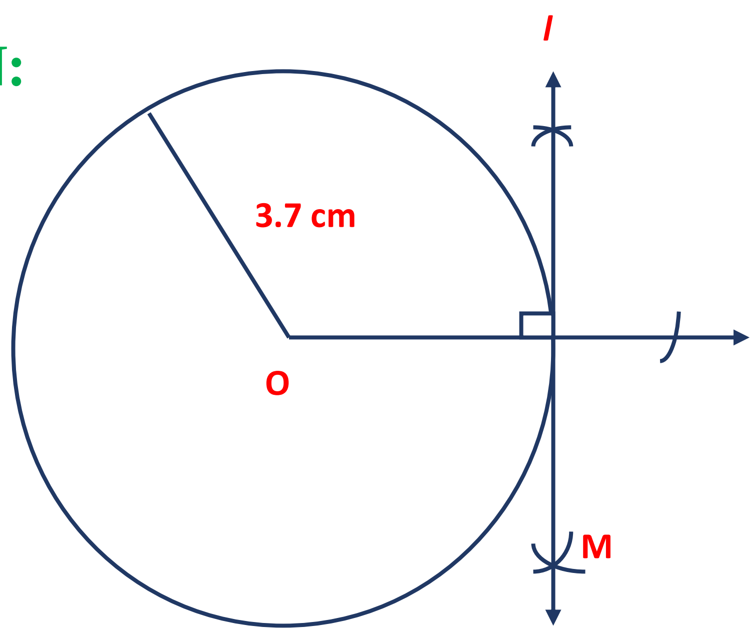
SOLUTION:



Q. 7

Draw a circle of radius 3.7 cm. Draw a tangent to the circle at any point on it.

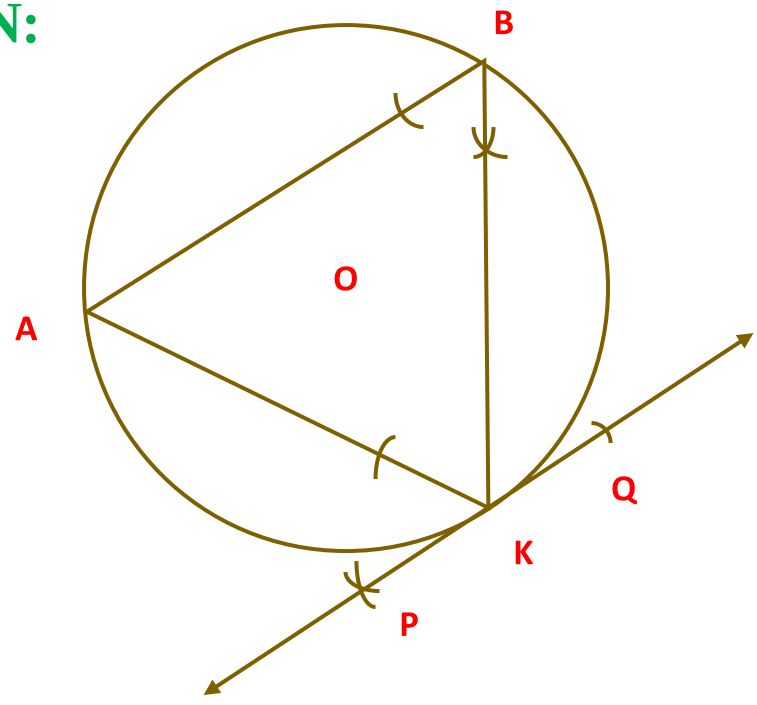
SOLUTION:



Q. 8

Draw a circle of radius 5.6 cm. Draw a tangent to the circle at any point on it without using the center.

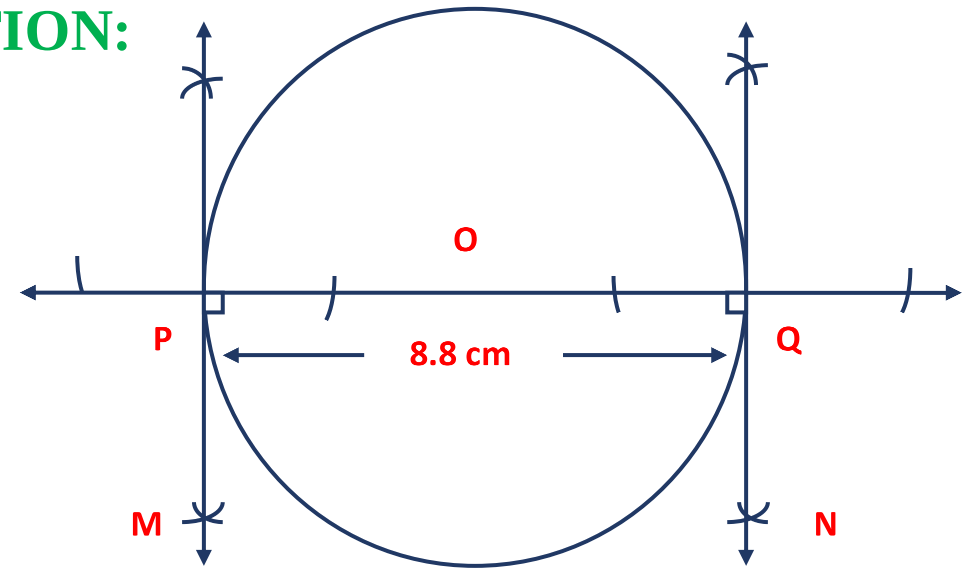
SOLUTION:



Q. 9

Draw a circle of radius 4.4 cm. Draw a chord PQ of length 8.8 cm. Draw tangents to the circle at points P and Q. Write your observations about the tangents.

SOLUTION:

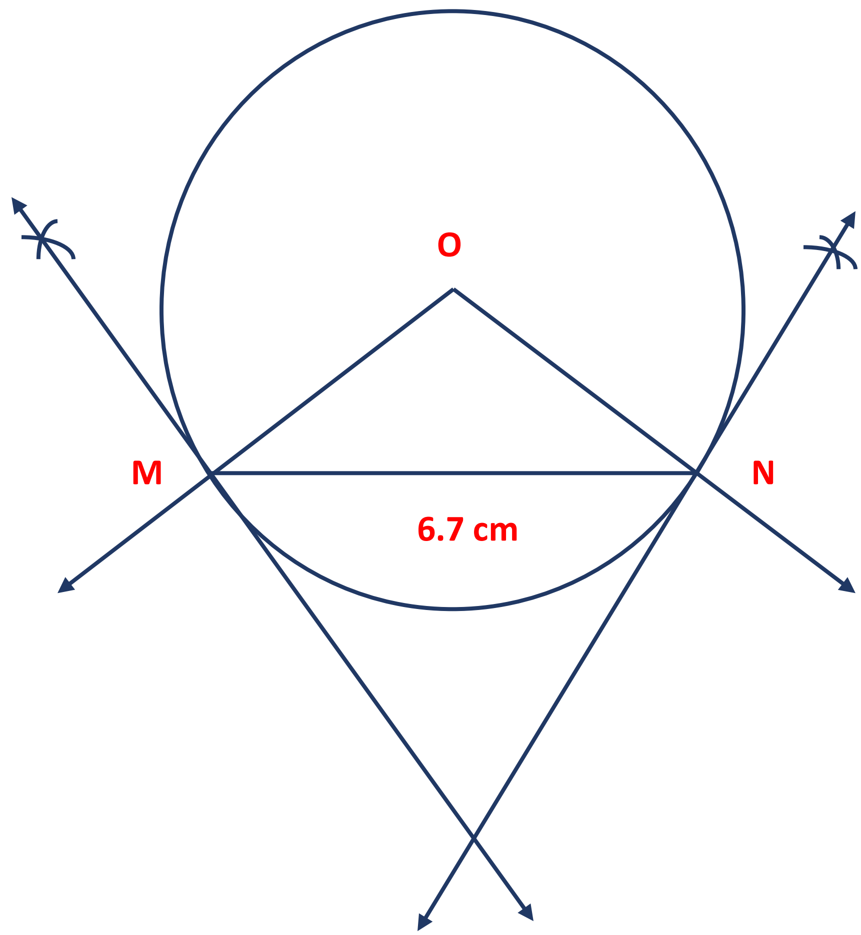


The tangents drawn at points P and Q are parallel.

Q. 10

Draw a circle with radius 4.4 cm. Draw a chord MN of length 6.7 cm in it. Construct tangents at point M and N to the circle.

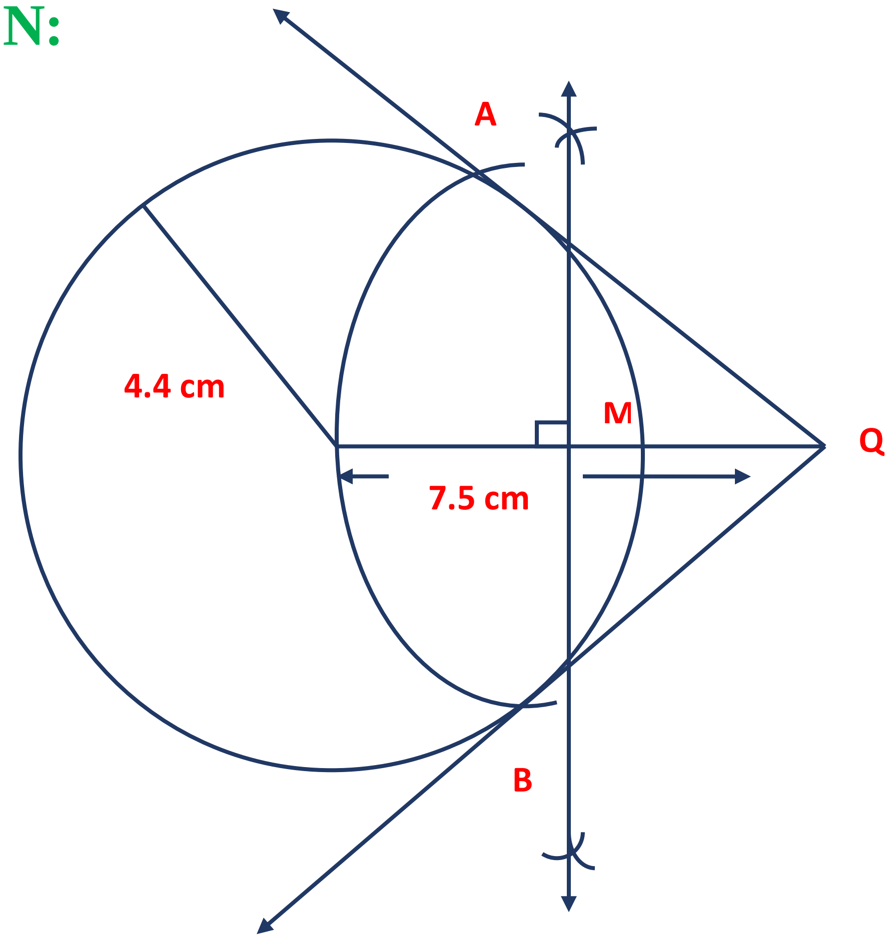
SOLUTION:



Q. 11

Draw a circle with center P and radius 4.4 cm. Take a point Q at a distance 7.5 cm from the center. Construct tangents to the circle from point Q.

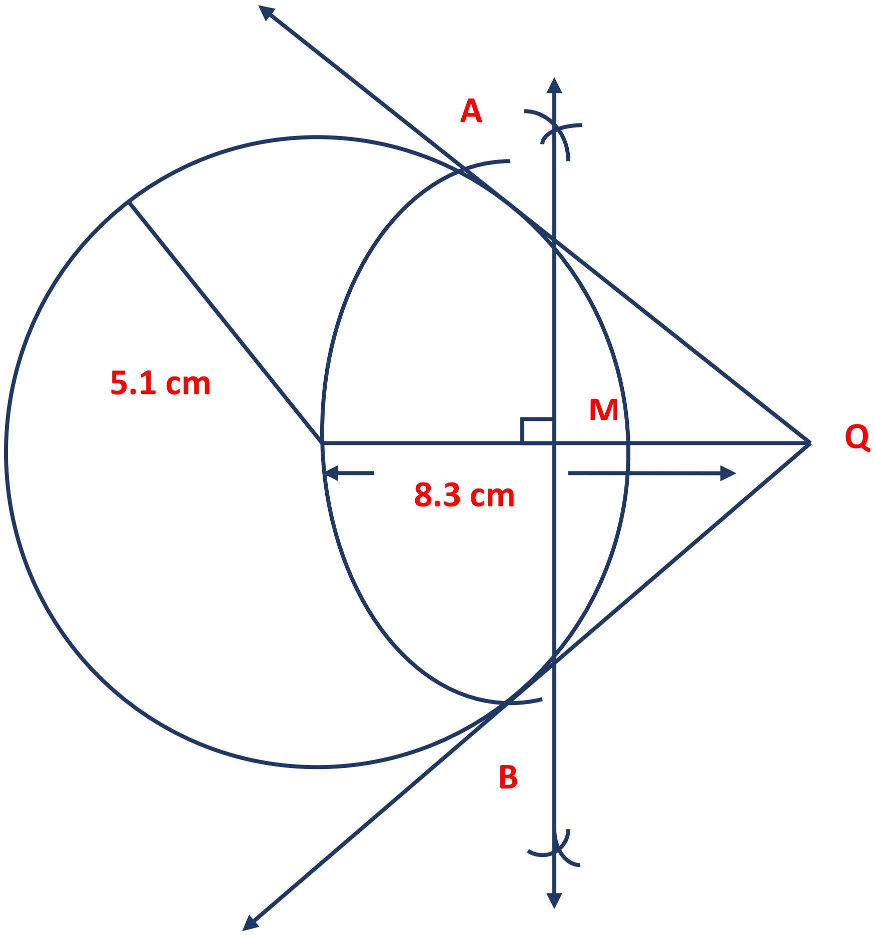
SOLUTION:



Q. 12

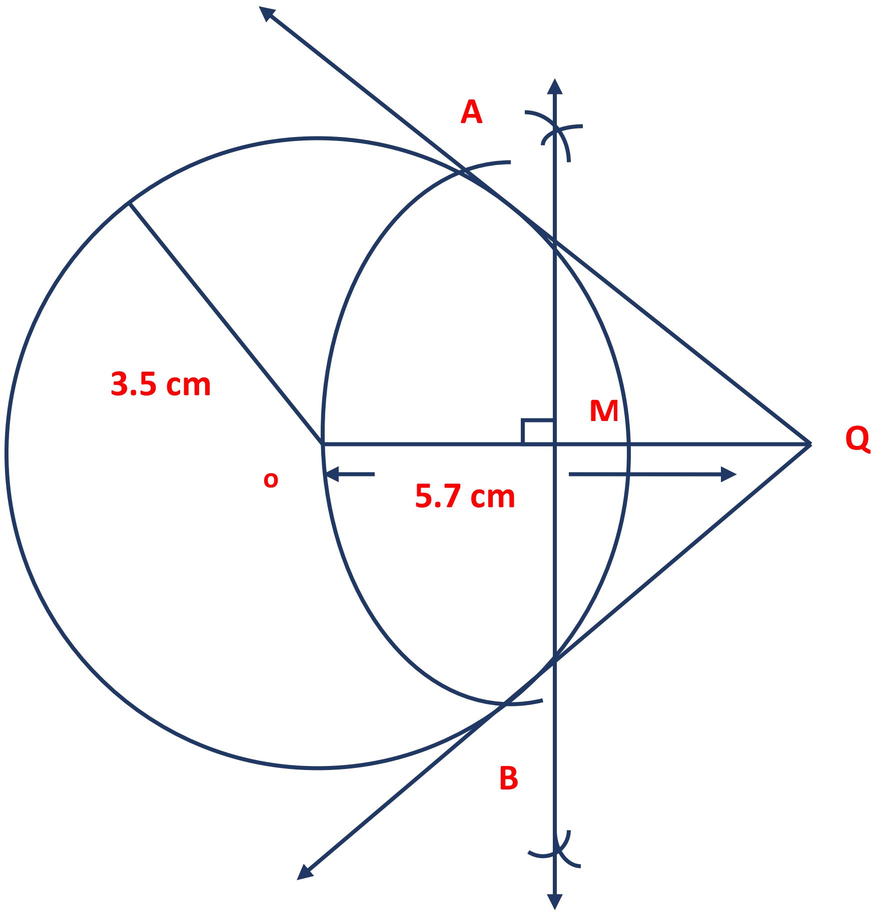
Draw a circle with radius 5.1 cm. Construct tangents to the circle from a point at a distance 8.3 cm from the center.

SOLUTION:



Q. 13

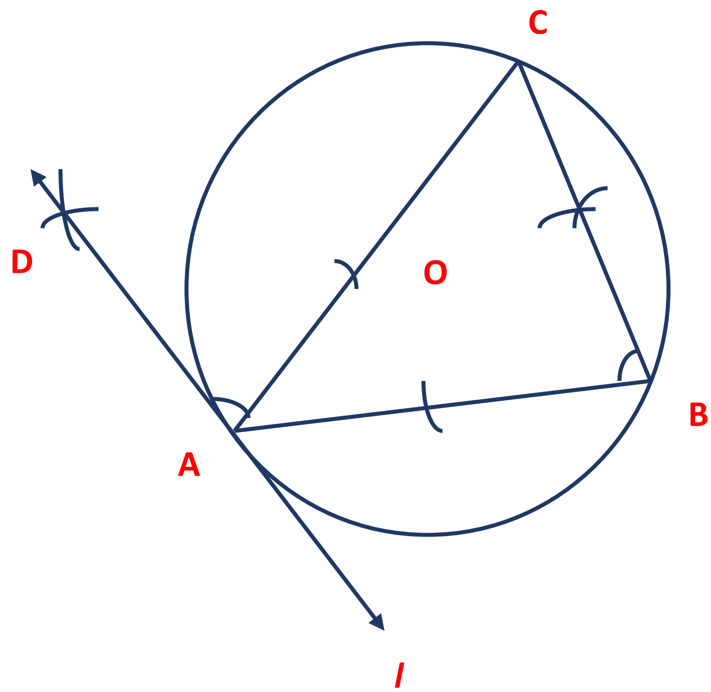
Draw a circle with center O and radius 3.5 cm. Take a point P at a distance 5.7 cm from the center. Draw tangents to the circle from point P.



Q. 14

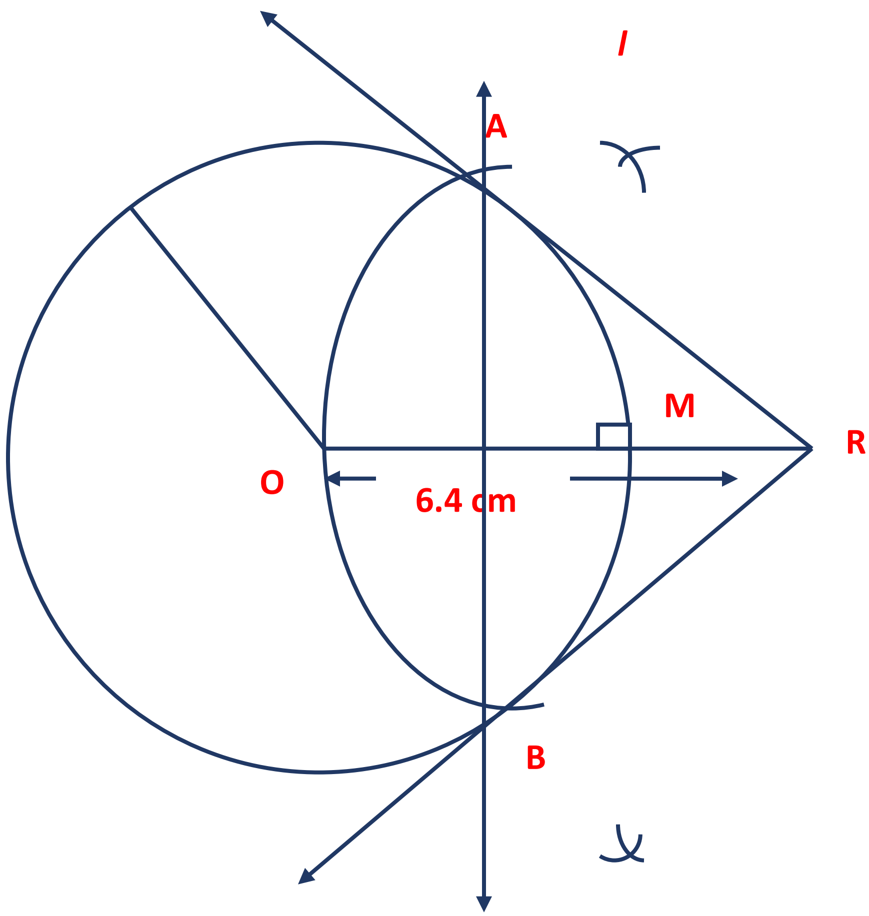
Draw any circle. Take any point A on it and construct tangent at A without using the center of the circle.

SOLUTION:



Q. 15

Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R.



Q. 16

Draw a circle with center P. Draw an arc AB of 100º measure. Draw tangents to the circle at point A and B.

SOLUTION:

Analysis:

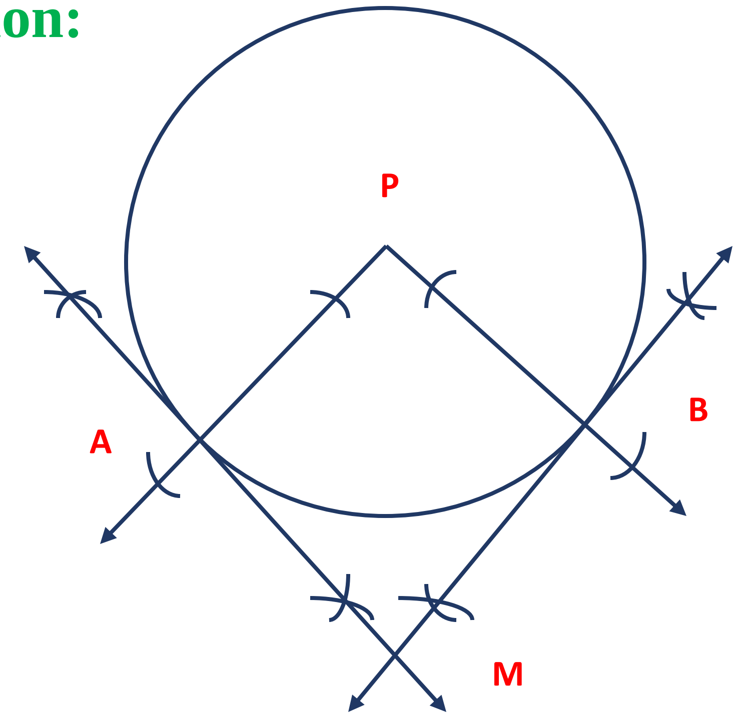
m (arc AB) = 100º … (Given)

∠ APB = m (arc AB) … Definition of measure of minor arc)

∴ ∠ APB = 100°

Central ∠ APB can be drawn in the circle and thus points A and B can be located on the circle. Tangents at A and B can thus be constructed.

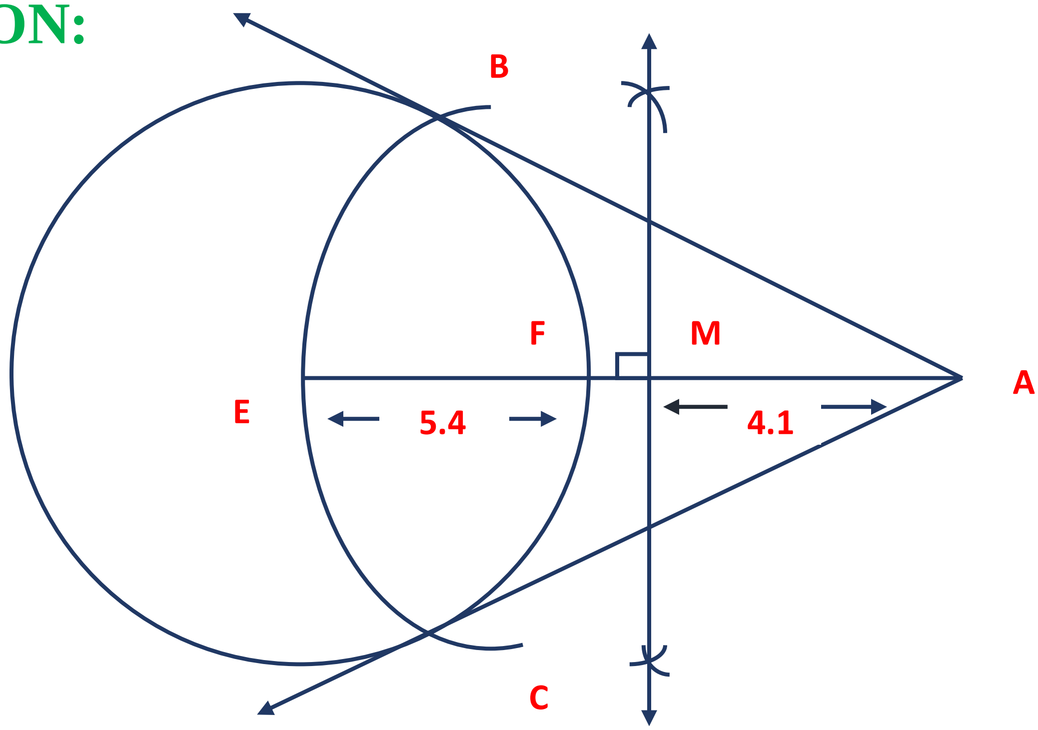
Construction:



Q. 17

Draw a circle of radius 5.4 cm and center E. Take a point F on the circle. Take another point A such that E – F – A and FA = 7.1 cm. Draw tangents to the circle from point A.

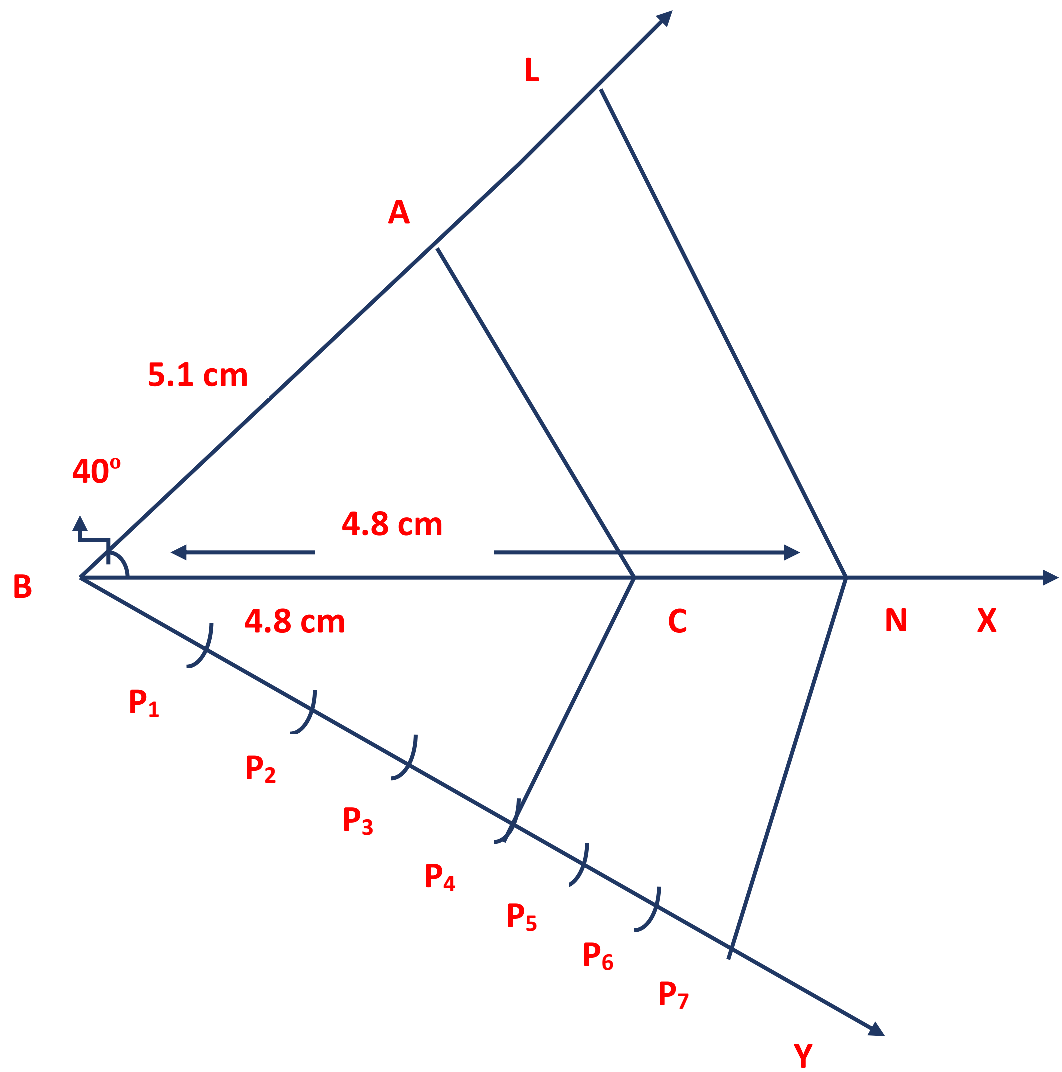
SOLUTION:



Q. 18

Δ ABC ~ Δ LBN. In Δ ABC, AB = 5.1 cm, ∠ B = 40º, BC = 4.8 cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct Δ ABC and Δ LBN.

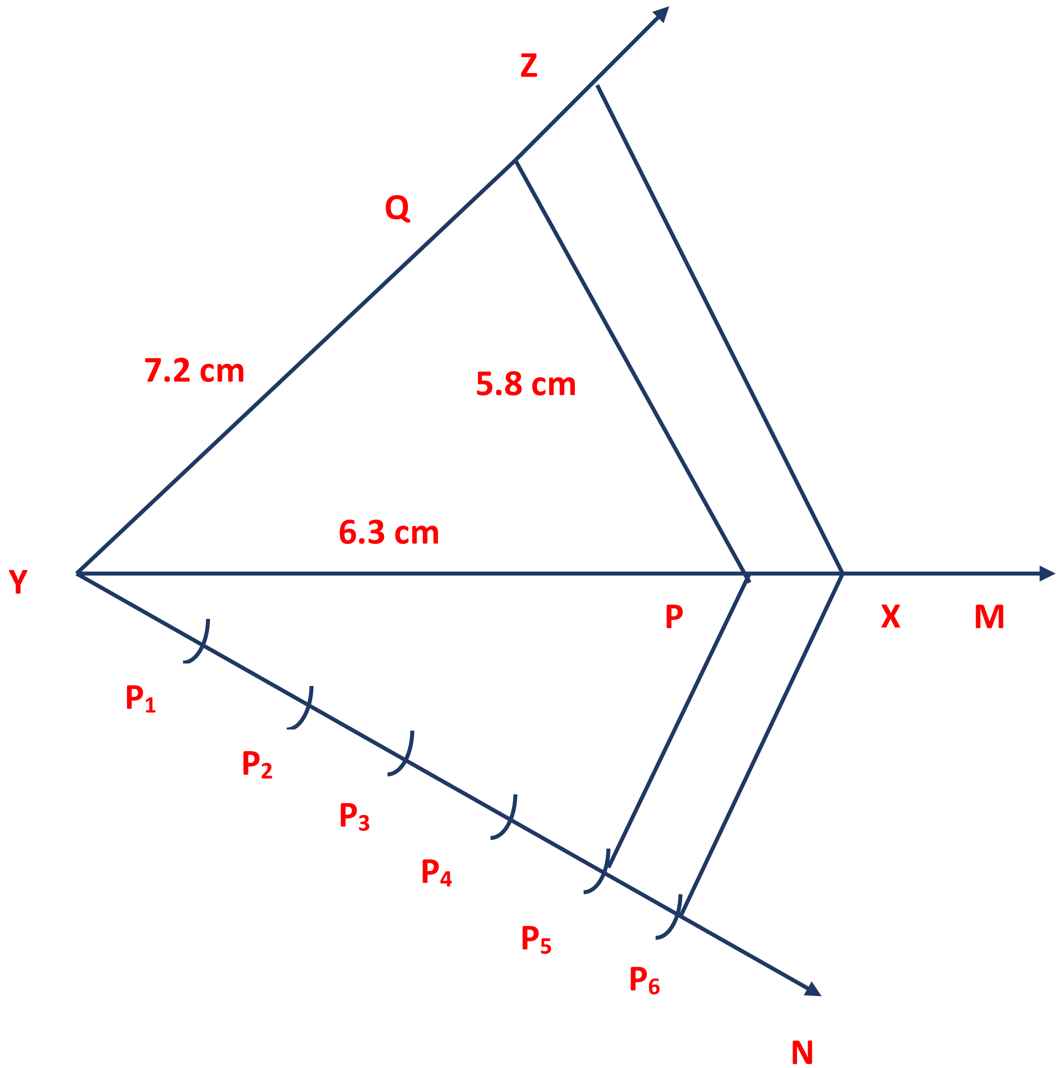
SOLUTION:



Q. 19

Construct Δ PYQ such that PY = 6.3 cm, YQ = 7.2 cm, PQ = 5.8 cm. If $\frac{YZ}{YQ} = \frac{6}{5}$, then construct Δ XYZ similar to Δ PYQ.

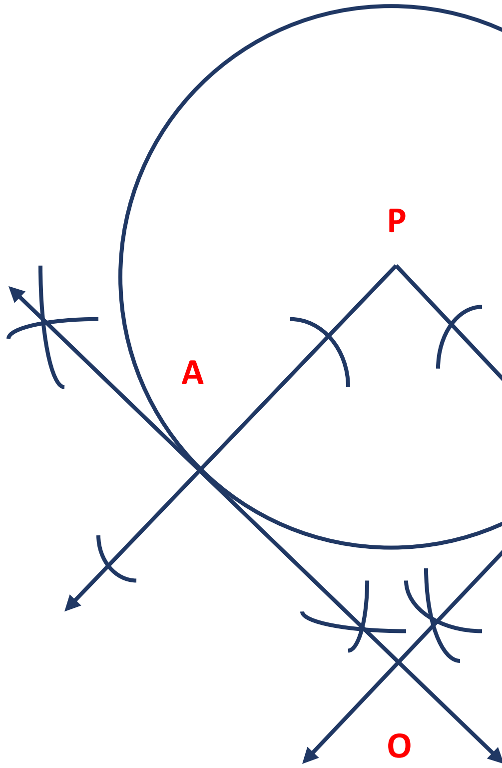
SOLUTION:



Q. 20

Draw a circle with center P. Draw an arc AB of 100^{o} measure. Draw tangents to the circle of point A and point B.

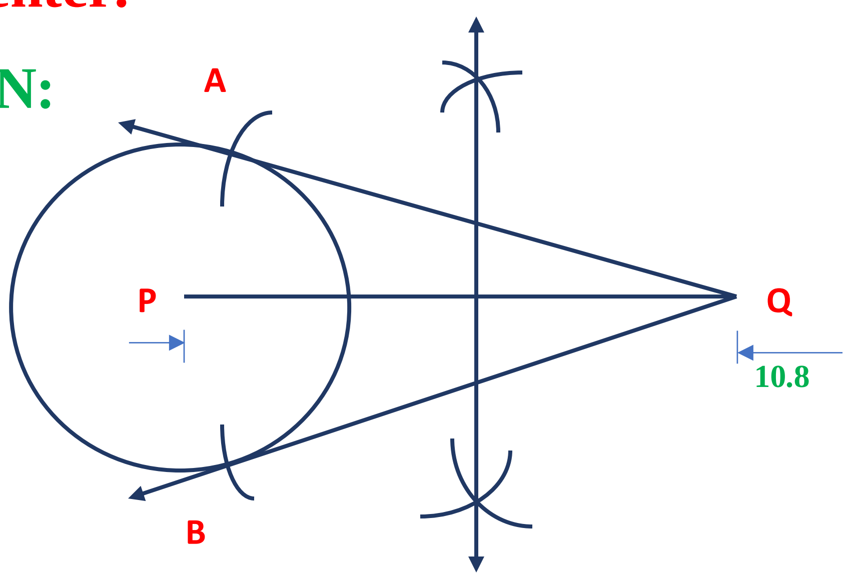
SOLUTION:



Q. 21

Draw tangents to the circle with center P and radius 4.9 cm from a point Q which is at distance 10.8 cm from the center.

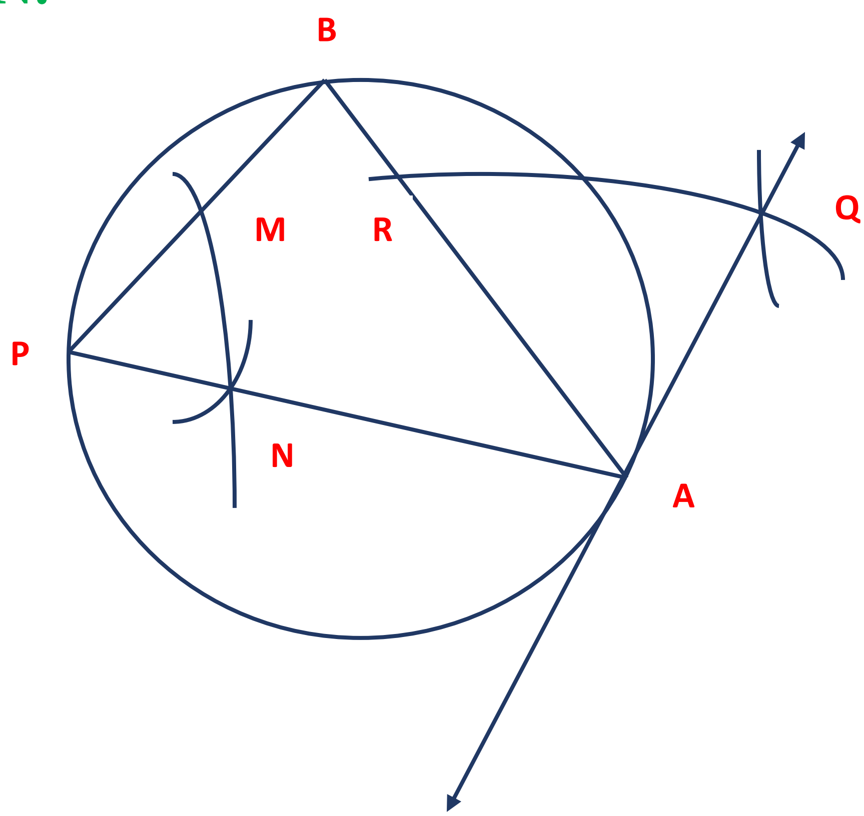
SOLUTION:



Q. 22

Draw any circle. Take any point A on it and construct tangent at A without using the center of the circle.

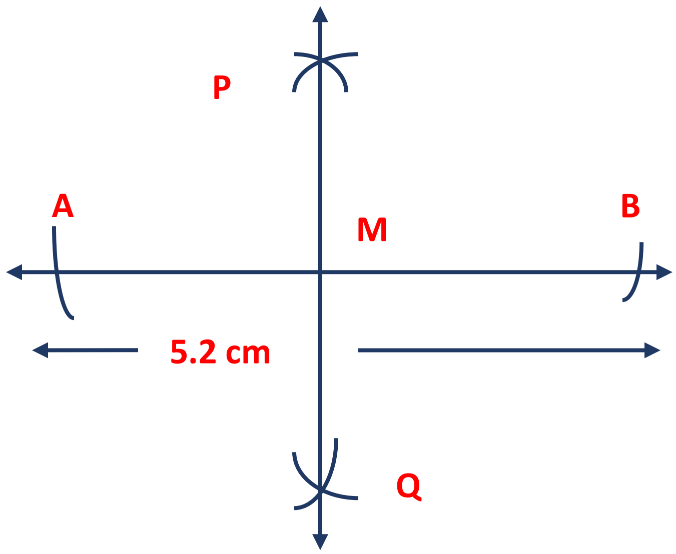
SOLUTION:



Q. 23

Draw seg AB of length 5.2 cm. Construct its perpendicular bisector.

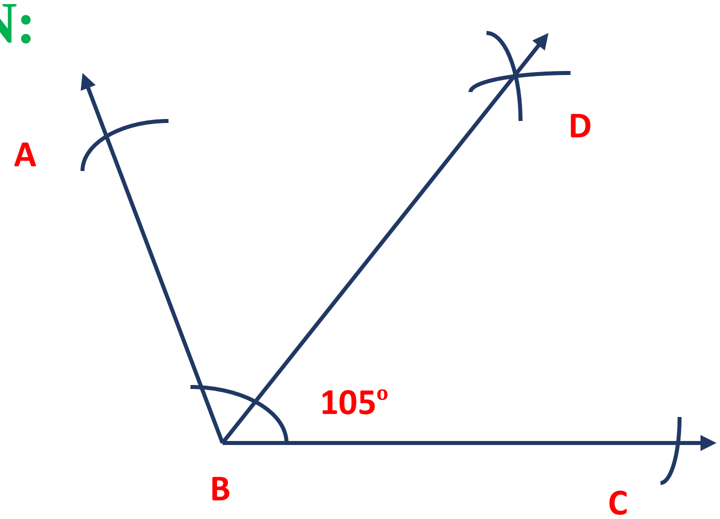
SOLUTION:



Q. 24

Draw ∠ ABC = 105°, construct its bisector.

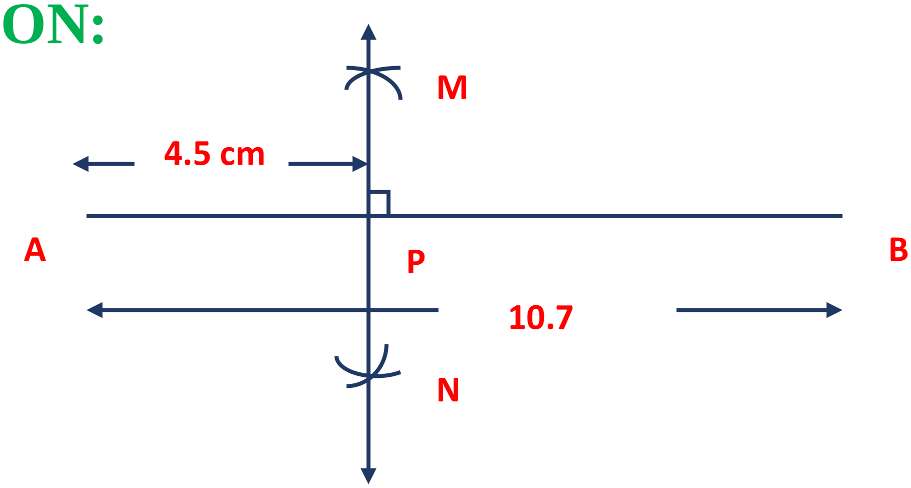
SOLUTION:



Q. 25

Draw AB = 10.7 cm. Take a point P on it such that A – P – B and AP = 4.5 cm. Through P draw a line MN perpendicular to seg AB.

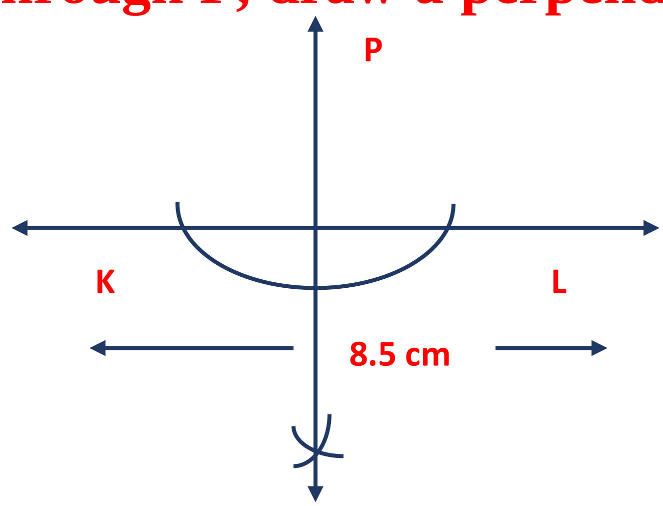
SOLUTION:



Q. 26

Draw line KL such that KL = 8.5 cm. Consider point P outside it. Through P, draw a perpendicular to line KL.

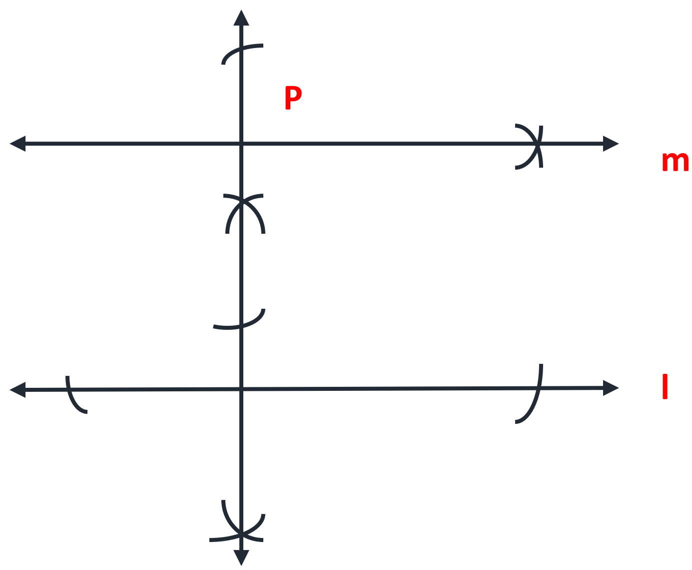
SOLUTION:



Q. 27

Draw a line *l* take a point P outside it. Draw a line m ∥ line *l* passing through point P.

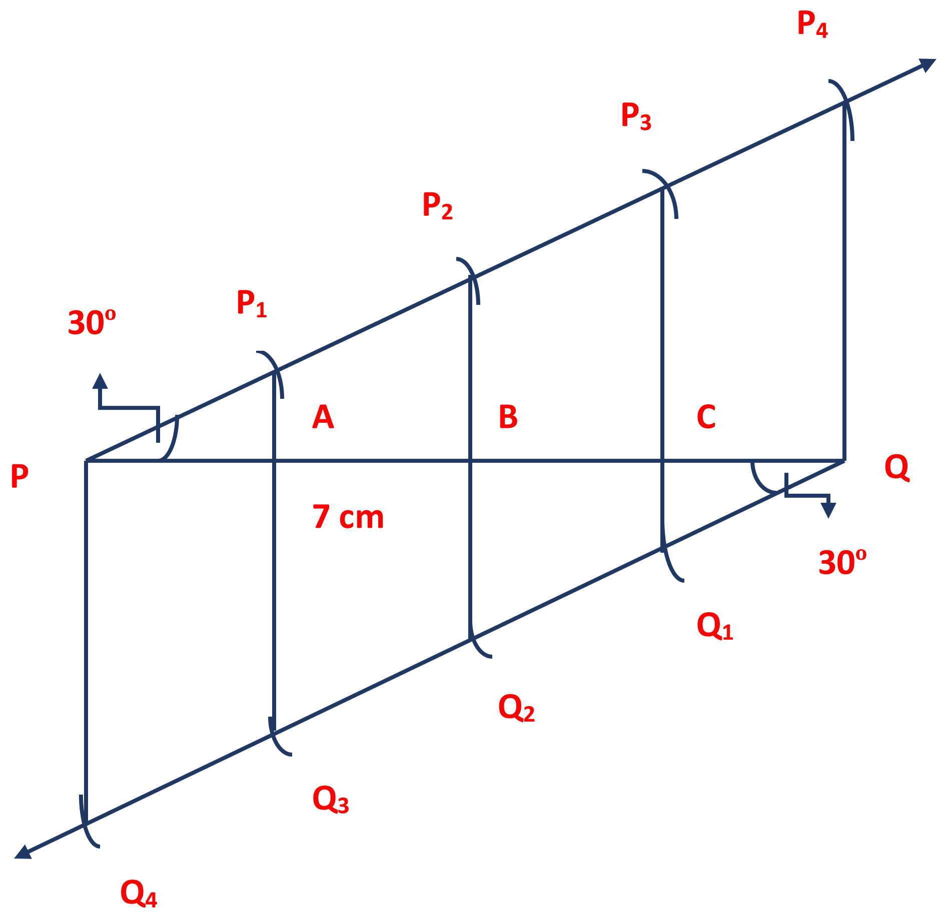
SOLUTION:



Q. 28

Draw segment PQ of length 7 cm. Divide it into 4 equal parts.

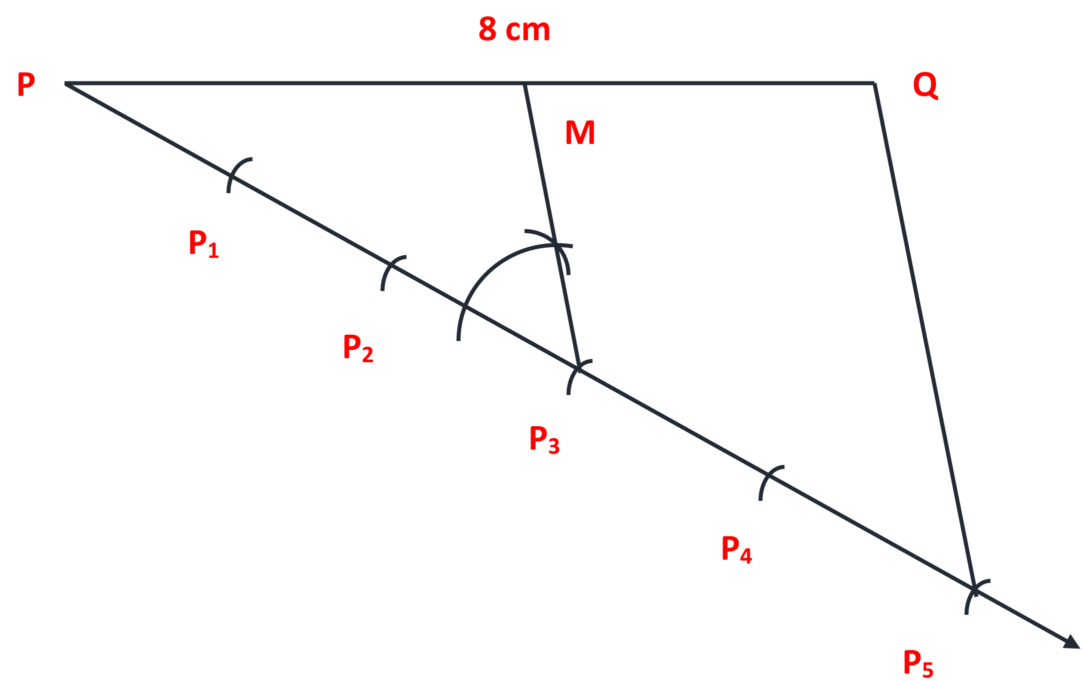
SOLUTION:



Q. 29

Draw segment PQ of length 8 cm. Divide it in the ratio 3 : 2

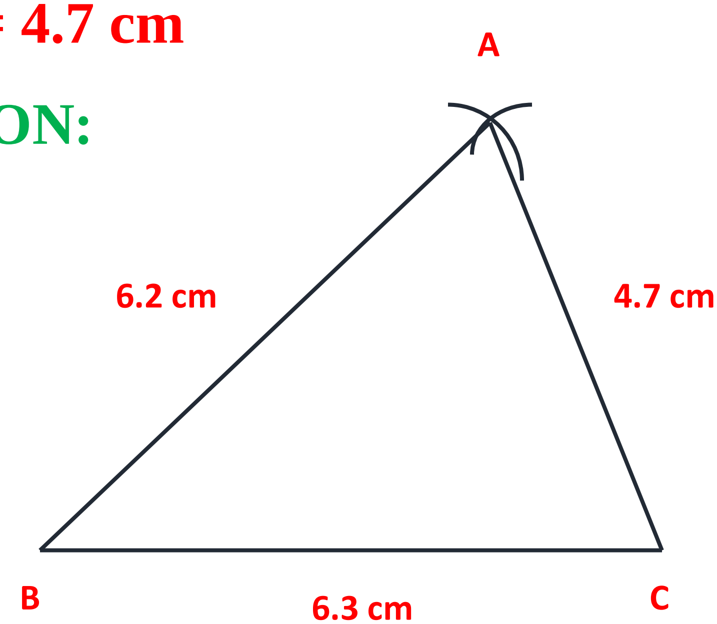
SOLUTION:



Q. 30 NAWNEET 92

Construct Δ ABC such that AB = 6.2 cm, BC = 6.3 cm, and AC = 4.7 cm

SOLUTION:



Q. 31

Δ XYZ ~ Δ DEF. In Δ XYZ, XY = 5.1 cm, YZ = 3.9 cm, XZ = 6 cm. XY: DE = 3 : 2. Construct Δ XYZ and Δ DEF.

SOLUTION:

For Δ XYZ, the length of the three sides are given.

∴ Δ XYZ can be constructed.

Δ XYZ and Δ DEF are similar.

∴ Their corresponding sides are in proportion.

$$\therefore \frac{XY}{DE} = \frac{YZ}{EF} = \frac{XZ}{DF} = \frac{3}{2}$$

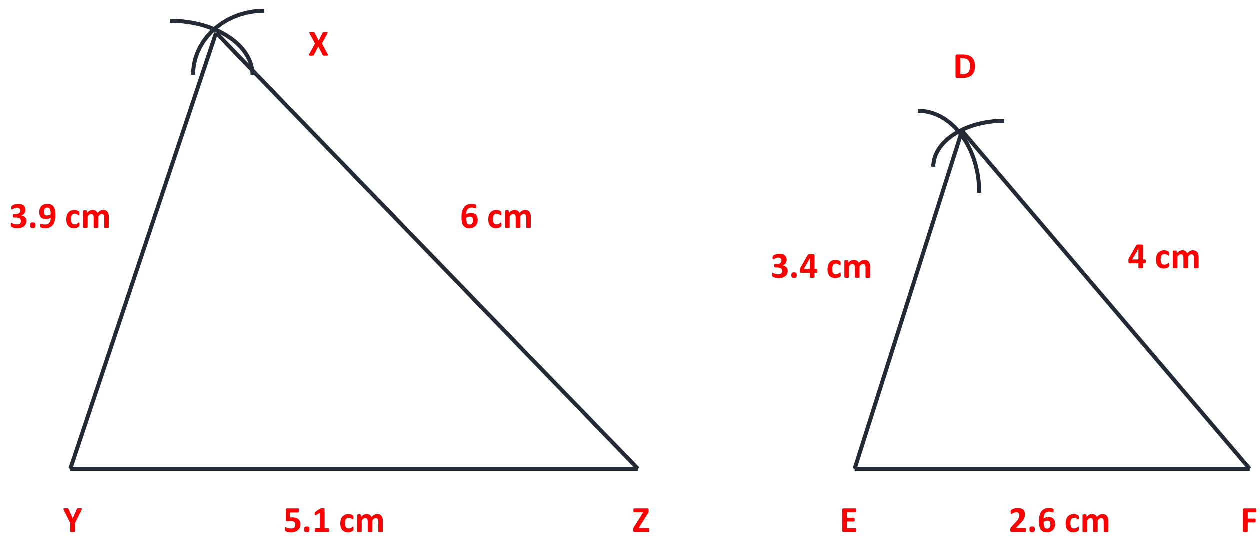
$$\therefore \frac{5.1}{DE} = \frac{3.9}{EF} = \frac{6}{DF} = \frac{3}{2}$$

$\therefore \frac{5.1}{DE} = \frac{3}{2}$	$\frac{3.9}{EF} = \frac{3}{2}$	$\frac{6}{DF} = \frac{3}{2}$
$\therefore DE = \frac{5.1 \times 2}{3}$	$\therefore EF = \frac{3.9 \times 2}{3}$	$\therefore DF = \frac{6 \times 2}{3}$
∴ DE = 3.4 cm	∴ EF = 2.6 cm	∴ DF = 4 cm

For Δ DEF, the length of three sides are not known.

∴ Δ DEF can be constructed.

Construction:



Q. 32NAVNEET 94/3

Δ PQR ~ Δ PMN. In Δ PQR, PQ = 5 cm, QR = 6 cm, and PR = 7 cm. Construct Δ PQR and Δ PMN such that $\frac{PR}{PN} = \frac{3}{5}$

SOLUTION:

The length of three sides of Δ PQR are known.

∴ Δ PQR can be constructed.

Δ PQR ~ Δ PMN such that $\frac{PR}{PN} = \frac{5}{7}$

$\therefore$ Sides of Δ PMN are smaller than corresponding sides of Δ PQR.

and $\angle$ QPR $\cong$ $\angle$ MPN … (Corresponding angles of similar triangles)

$\therefore$ Δ PQR and Δ PMN can have common angle P.

If we divide PR into 5 equal parts, then PN would be equal to three equal parts. Thus point N can be located on seg PR.

As, $\angle$ PRQ $\cong$ $\angle$ PNM … (Corresponding angles of similar triangles)

$\therefore$ At point N, we draw line NM ∥ side QR intersecting side PQ at M. Thus, we obtain Δ PMN.

<u>Steps of Construction:</u>

1. Construct Δ PQR such that PQ = 5 cm, PR = 7 cm and QR = 6 cm.

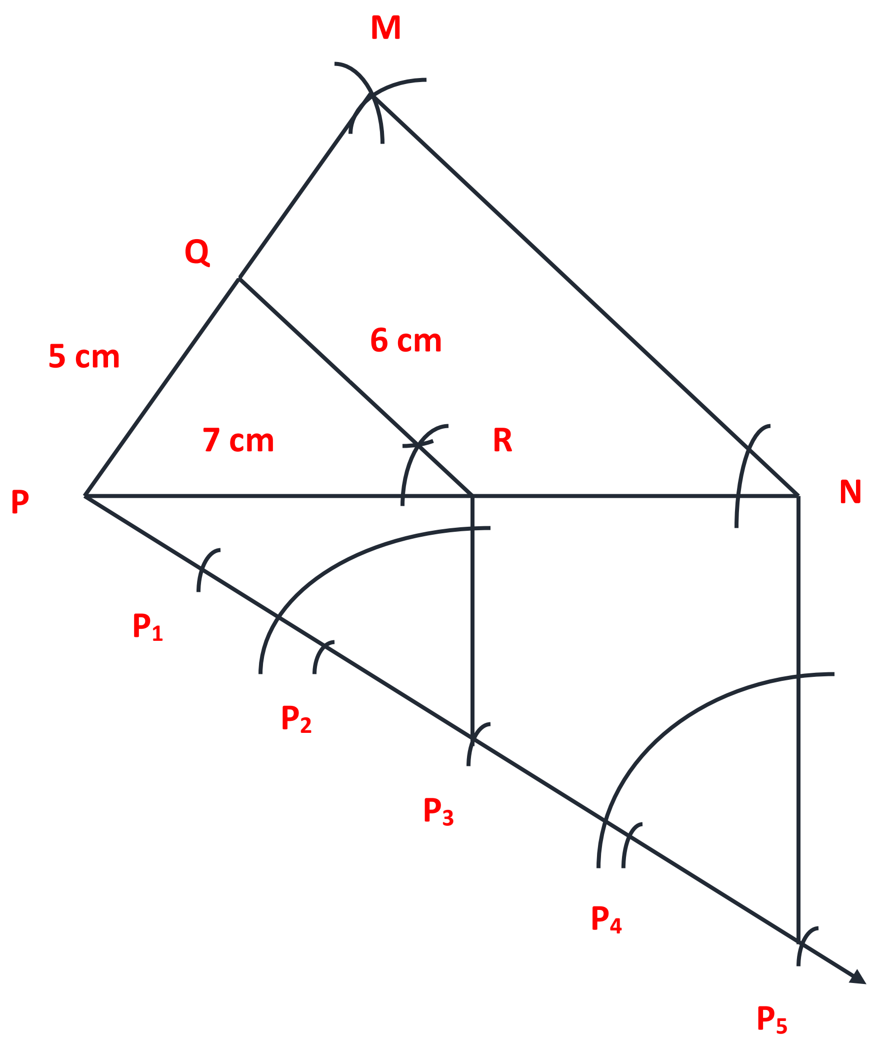
2. Divide seg PR in 5 equal parts

3. Name the endpoint of the third part as N

4. Now, draw a line parallel to QR through N. Mark the point of intersection of the parallel line with PQ as M.

5. Δ PMN is the required triangle similar to Δ PQR.

CONSTRUCTION:



Q. 33

Draw a circle of radius 3 cm. Mark a point P on the circle. Draw a tangent to the circle through point P using the center of the circle.

SOLUTION:

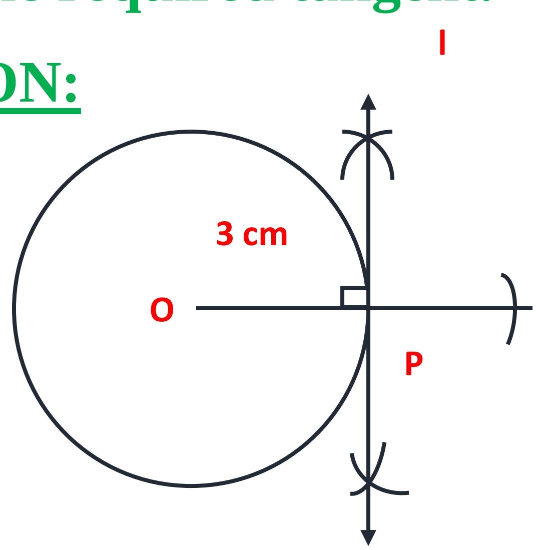
A circle of radius 3 cm can be drawn.

Let the center of the given circle be O and the line *l* be the required tangent.

We know, converse of tangent theorem states that, 'A line perpendicular to radius at its outer end is tangent'.

$\therefore$ We construct perpendicular to radius OP at point P, then line *l* is the required tangent.

CONSTRUCTION:

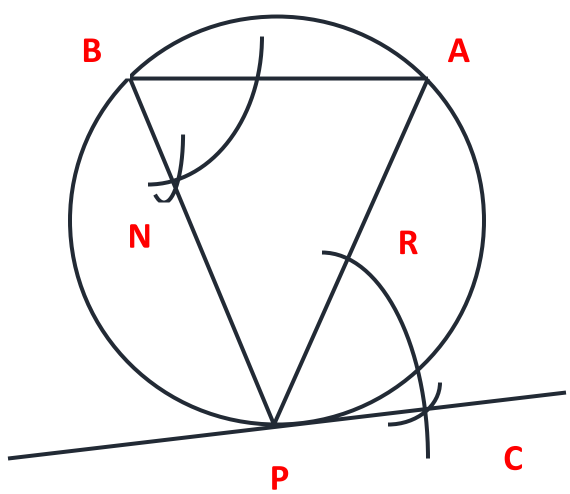


Q. 34 navneet 96

Draw a circle of radius 6 cm. Take any point P on it. Draw tangent to the circle through point P without using the center of the circle.

SOLUTION:

Through P, a chord can be drawn. Let it be PA. Draw any ∠ PBA in the alternate segment. Now an ∠ APC can be constructed congruent to ∠ ABP, then by converse of tangent secant angle theorem, line PC is the required tangent.



Q. 35 navneet 97

Draw a circle of radius 5.5 cm and centre O. Mark a point P at a distance of 8 cm from the centre. Draw tangents to the circle from point P.

SOLUTION:

A circle of radius 5.5 cm can be drawn and point P at a distance of 8 cm can be located. Suppose tangents through P at point A and B, then ∠ OAP = ∠ OBP = 90º … (Tangent Theorem)

we know, 'angle inscribed in a semicircle is right angle'

∴ A and B lie on the semicircular arcs whose diameter is OP.

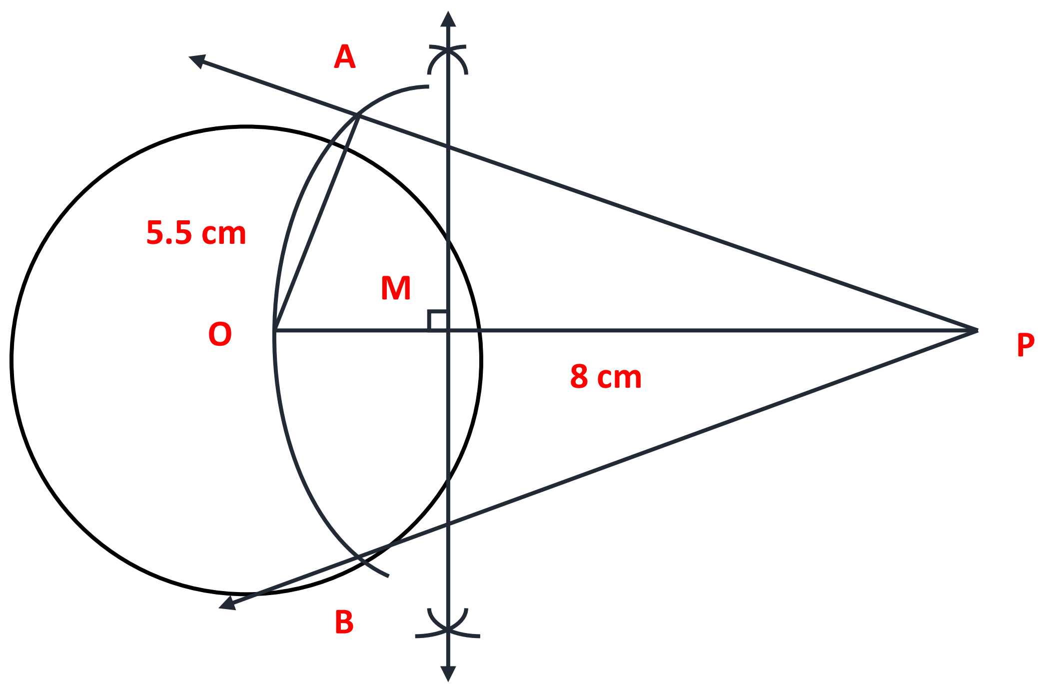
A and B therefore would be the points of intersection of those semicircular arcs with the circle.

∴ On drawing the perpendicular bisector on seg OP we can obtain the center and the radius of the semicircular arcs.

Points of intersection of semicircular arcs and the circle are points A and B.

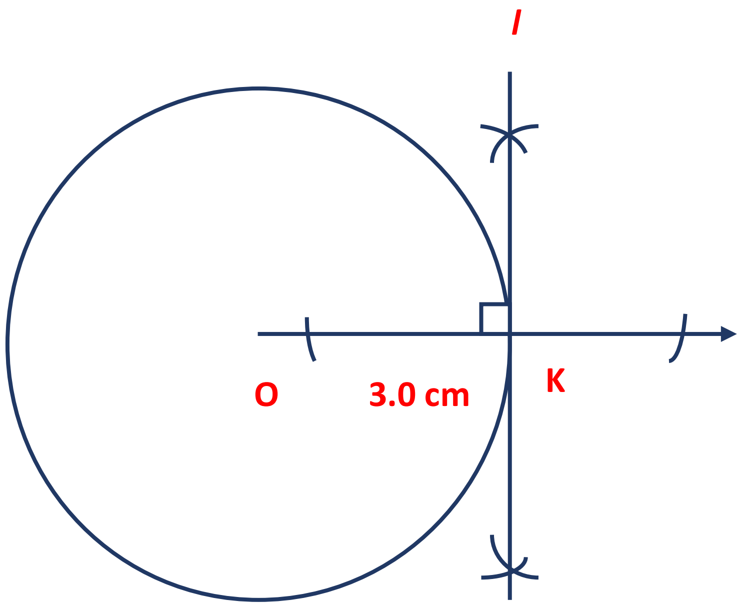
∴ Tangents PA and PB can be drawn.

<u>Construction:</u>



Q. 36

Construct a tangent to a circle with centre 0 and radius 3.0 cm at any point K on it.



Construction:

1. Draw a circle with Centre radius 3 cm and Centre as O.
2. Name any point on the circle as k
3. Join OK and draw a tangent to the circle through the point k

Q. 37

In ΔMNP, MP = 6.6 CM, $\angle$MNP = 70^0, seg ND is height of triangle and ND = 5 cm, then draw ΔMNP

Solution:

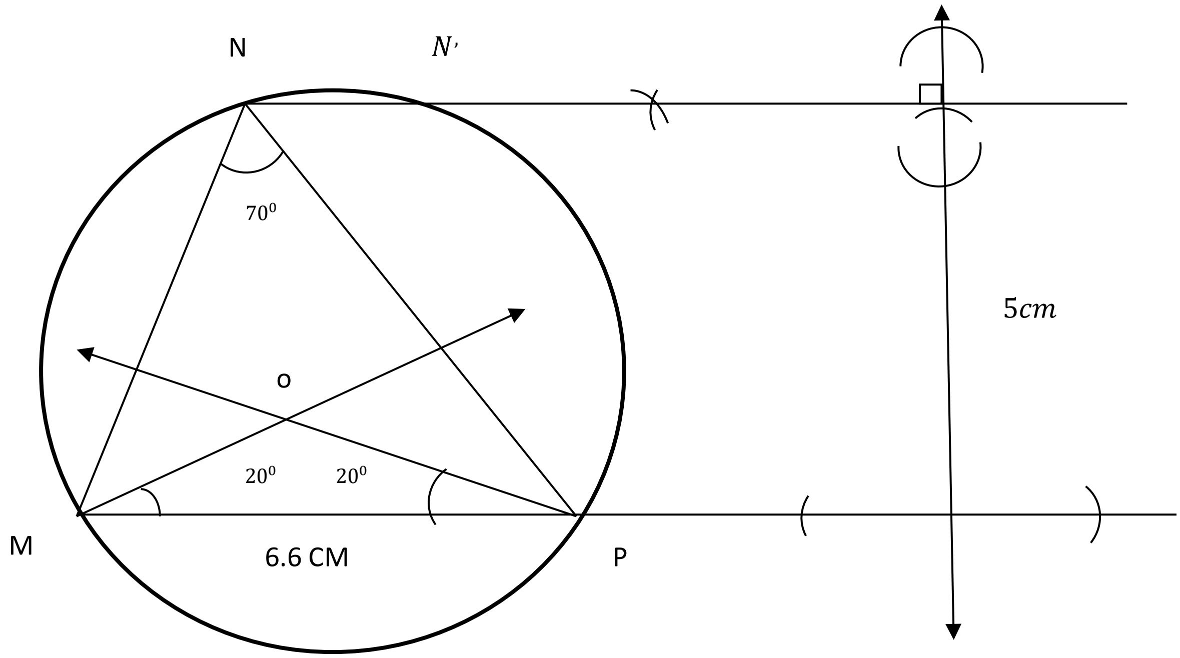
$\angle$MNP = 70^0 ,MP = 6.6 CM,

Point N lies on arc MNP

Seg MP makes an angle of $70^0$with point N

Construction:

1. **Draw seg MP = 6.6 cm**
2. **Draw angles $\angle$OMP and $\angle$OPM**
3. **Draw a line parallel to seg MP at a distance of 5 cm. This line will intersect at M and N to the arc**
4. **Draw seg MN and MP. ΔMNP is the triangle of the given measurement**



Q. 38

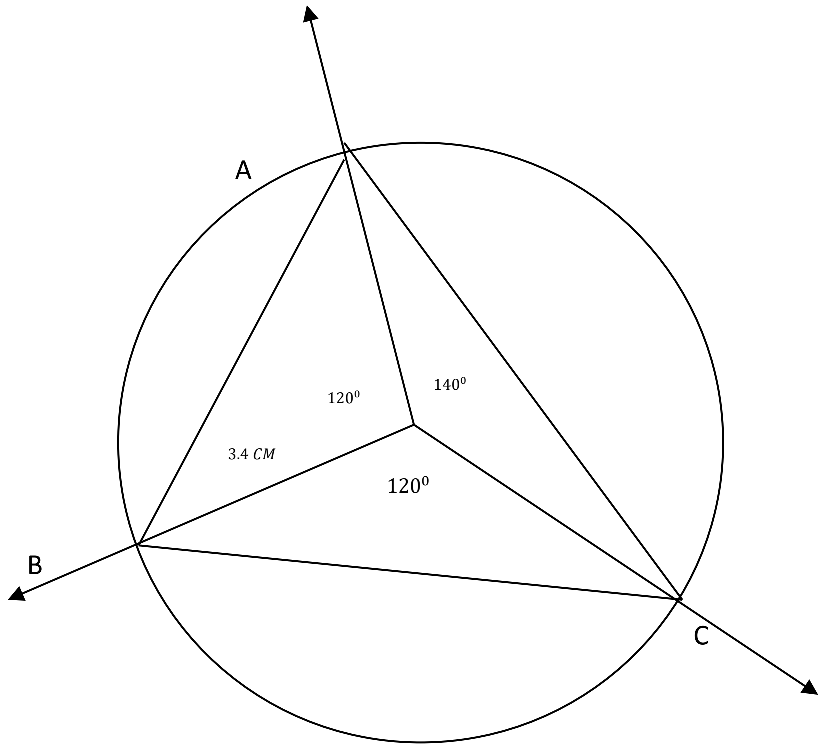
Construct ΔABC with circle circumscribing radius of 3.4cm $\angle A = 60^0 \angle B = 70^0 \angle C = 50^0$

Construction:

1. Draw a circle with Centre O and radius as 3.4 cm

2. Draw $\angle AOB = 100^0$, $\angle BOC = 120^0$

3. Join AB, AC, BC

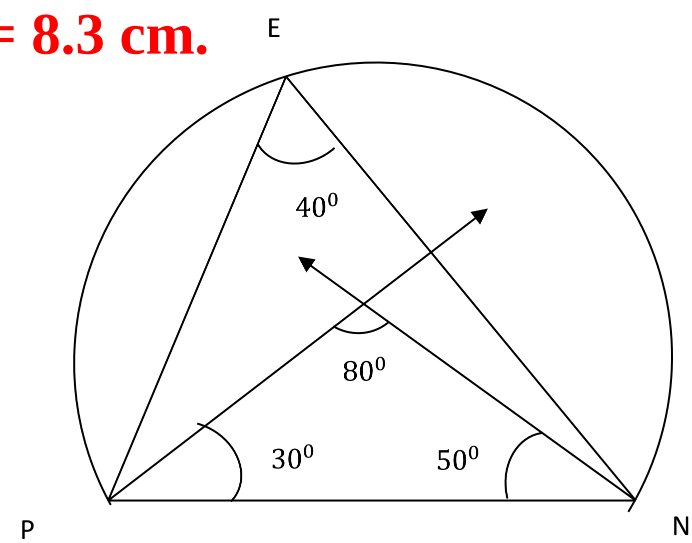


ΔABC is tringle of the given size

Q. 39

Construct an arc PEN which inscribes an angle of 40^0 with PN. Length of PN = 8.3 cm.

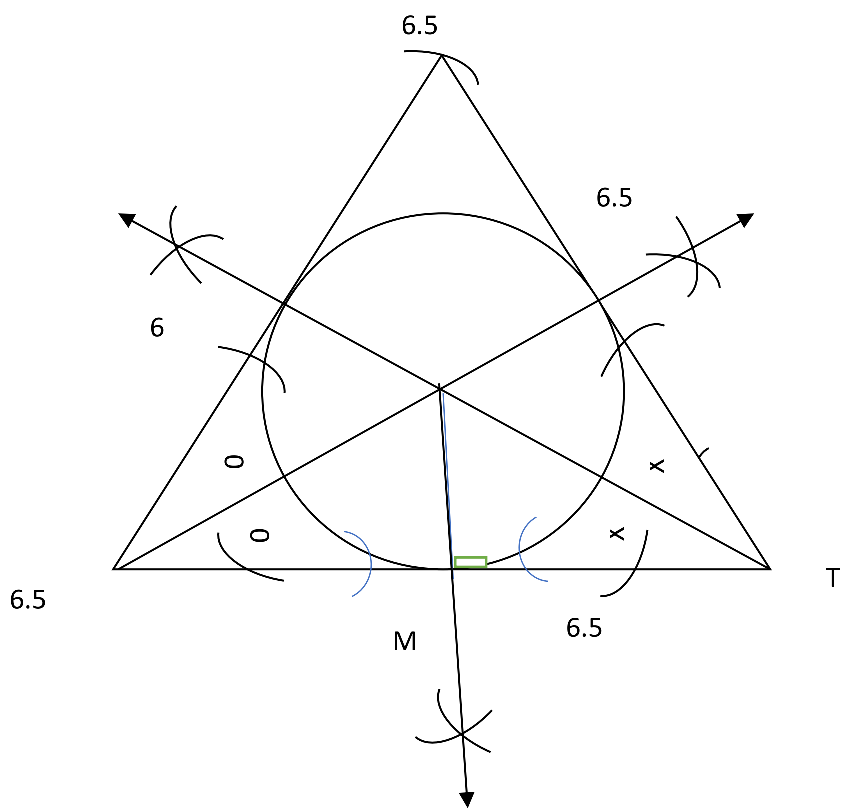
Solution:



$\angle PEN = 40^0$ is inscribed angle with arc PEN

Q. 40

In ΔRST, RS = 6 cm, ST = 7 cm, and RT = 6.5 cm, then construct ΔRST and find inscribed circle.



Construction:

i) Draw ΔRST

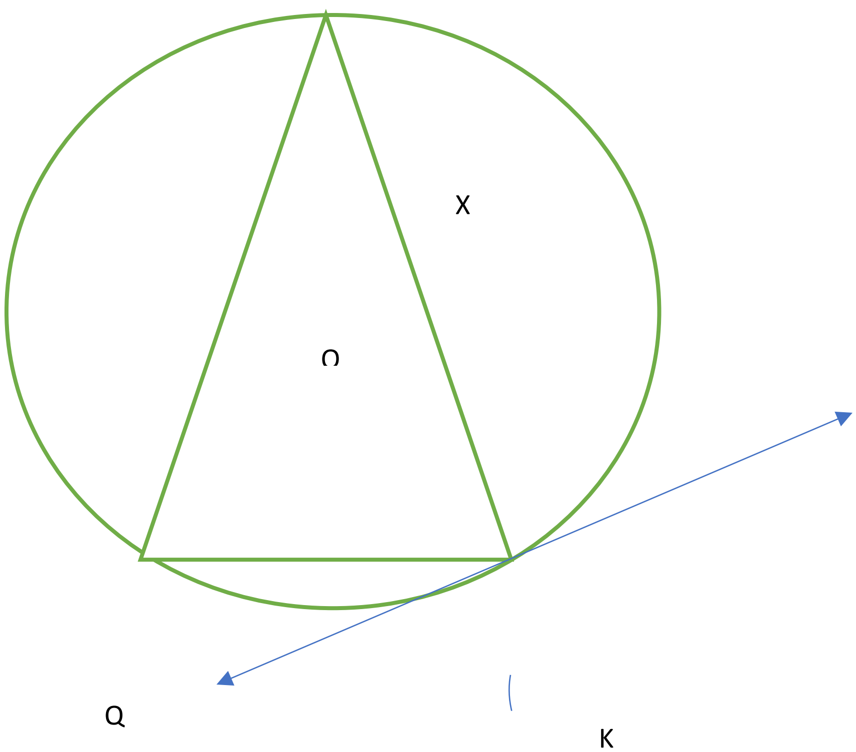
ii) Find angle bisectors of angles ∠ S, ∠ T Bisectors meet at point I

iii) Draw IM $\perp$ ST

iv) With Centre I draw circle with radius IM

Q. 41

Draw a circle with Centre 3.5 cm. Take any point k on the circle. Draw tangent to the circle through the point k, without using Centre of the circle.



Construction

i) Draw a circle with Centre 3.5 cm

ii) Take any point k on the circle

iii) Draw chord KQ

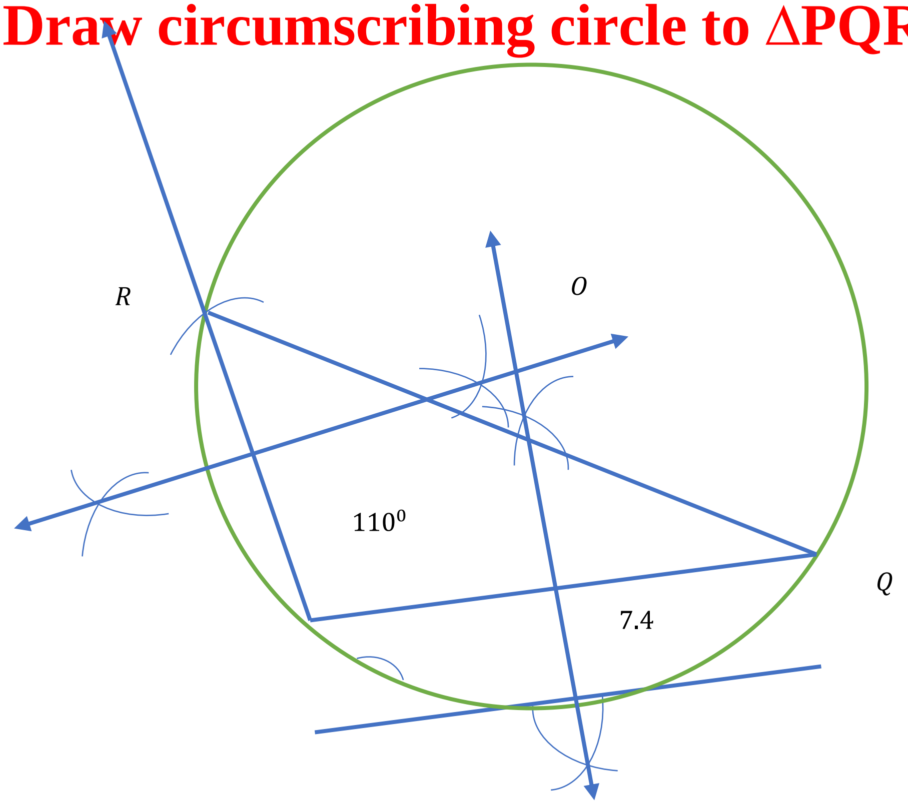
iv) Take any point P on the major circle

v) Draw an angle similar to $\angle KPQ$

vi) Line KL is the tangent to the circle at point K.

Q. 42

Draw ΔPQR, where PQ = 7.4 cm, $\angle P = 110^0$, RP = 6 cm, Draw circumscribing circle to ΔPQR



Q. 43

In ΔDEF, DE = EF = 5.8 cm, $\angle$ DEF = 65^0, then construct inscribed circle of the triangle.

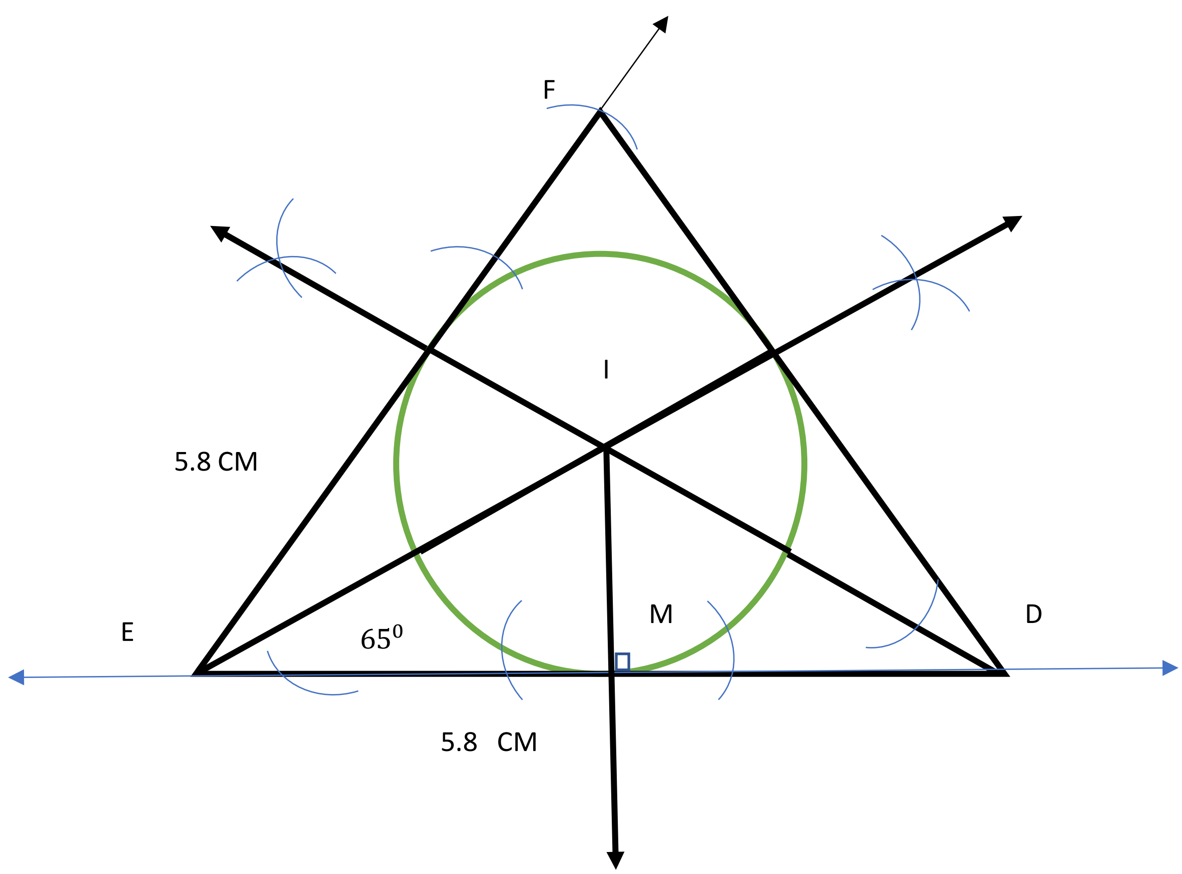
Solution:

i) Draw ΔDEF

ii) Find the angle bisectors of $\angle E$ and $\angle D$ they intersect each other at point I

iii) Line LM$\perp$ $seg\ ED$

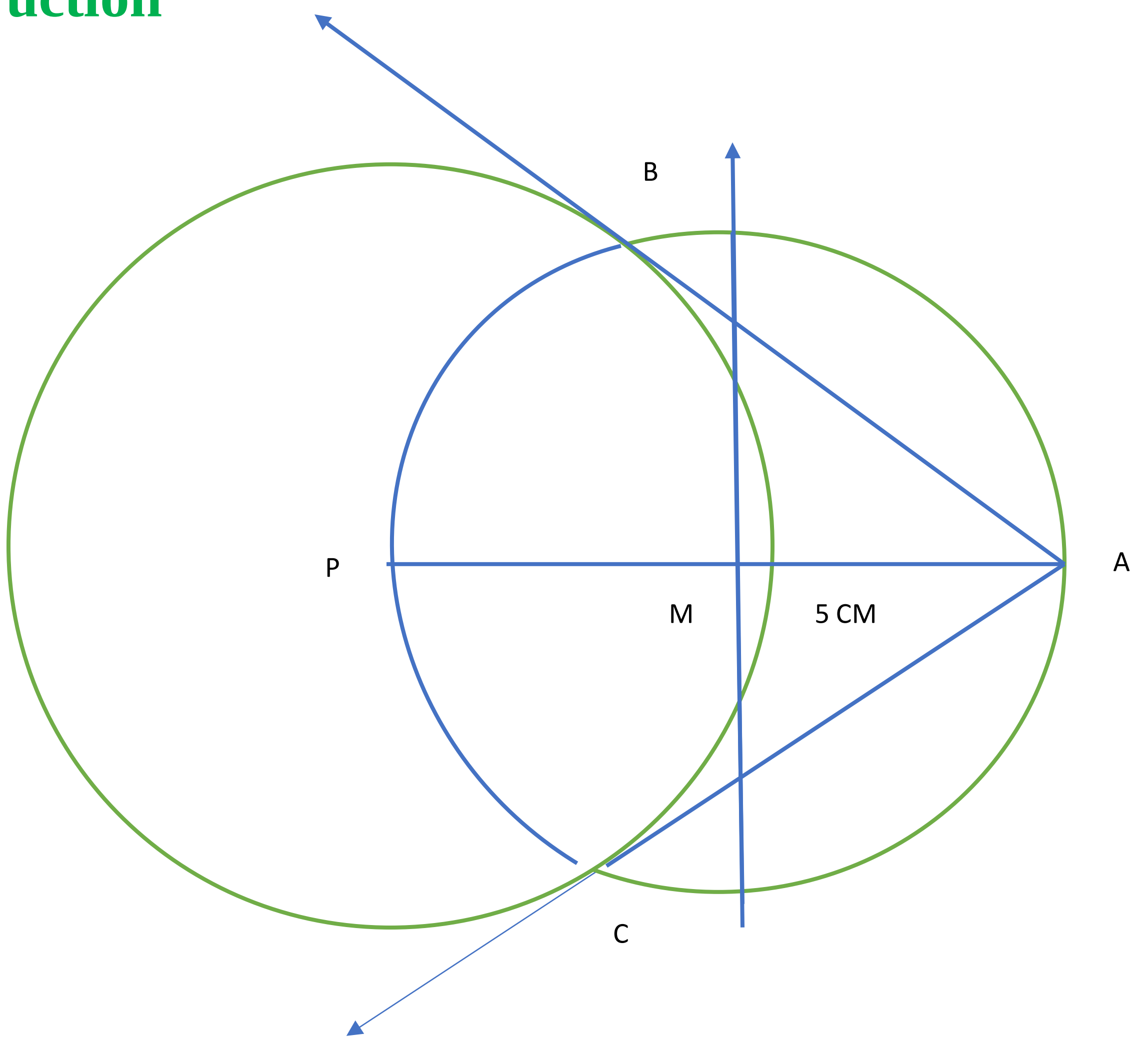
iv) Draw a circle with radius IM and Centre I



Q. 44

Draw a circle with center P and radius 3.1 cm. Draw tangent through the point A. Point A is at the distance of 5 cm from P.

Construction



i) Draw a circle with radius 3.1 cm and center P

ii) Take a point P such that *d* (P, A) = 5 cm

iii) Draw perpendicular bisector of segment PA, which meets segment PA @ M.

iv) Take center M and draw circle with radius MP.

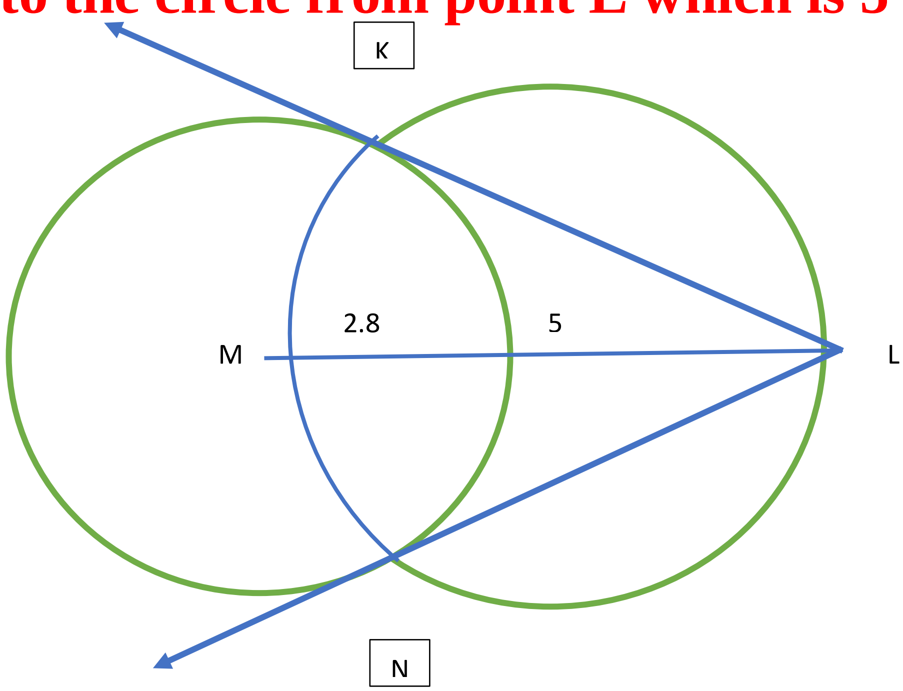
v) This circle intersects first circle at points B and C

vi) Join AB and AC

vii) Measure length of AB and AC

Q. 45

Draw a circle with center M with radius 2.8 cm. Draw tangents to the circle from point L which is 5 cm from point L.



Construction:

i) Take center M and radius 2.8 cm

ii) Take a point L at distance of 5 cm from point M

iii) Construct perpendicular bisector of ML, which intersects line ML at point O

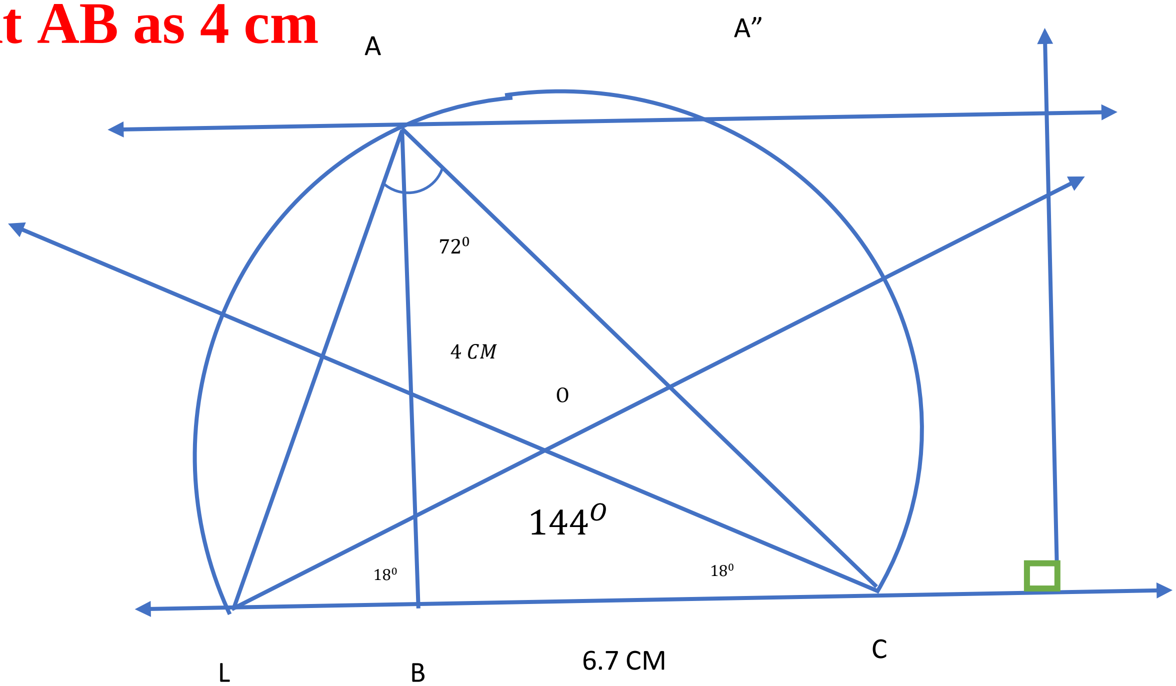
iv) Take center O and draw a circle with radius OM

v) Points K and N are the cutting points of the circle.

vi) Draw line KL & NL. These are the two tangents.

Q46

Draw ΔLAC such that LC = 6.7 cm, $\angle LAC = 72^0$ and height AB as 4 cm



i) Line LC = 6.7 CM

ii) Draw angles $\angle L = \angle C = 18^0$ $(18^0 - 72^0)$ which intersect each other at point O

iii) $\angle$L = 6.7 cm

iv) *With centre O draw a circle with radius OL*

v) Draw parallel line to BC at a distance of 4 cm. The line will cut the arc at points A and A"

vi) Join LA and LC

vii) ΔLAC is the triangle required

Q. 47

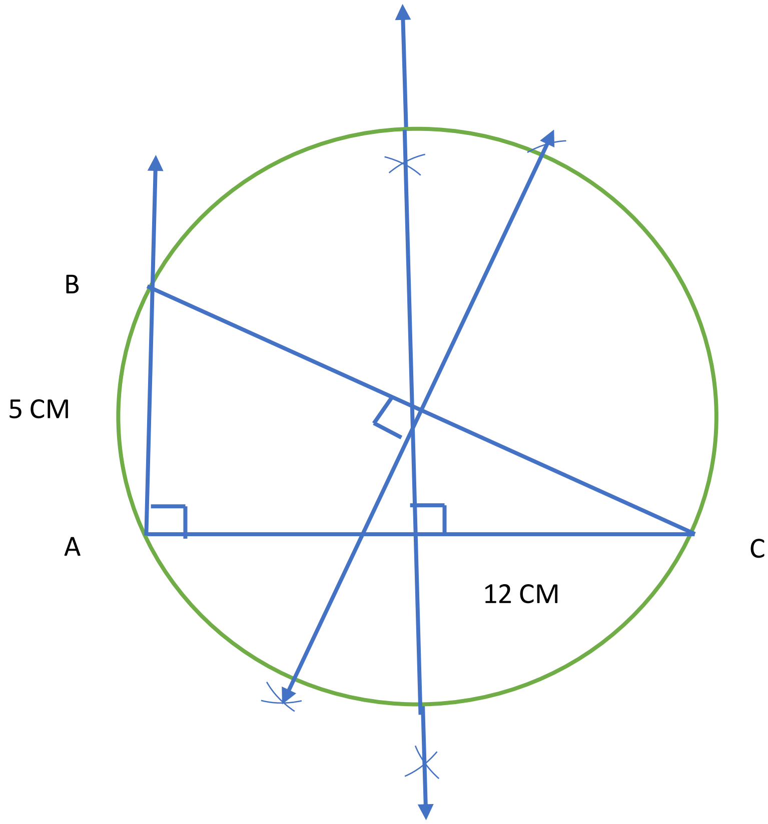
In ΔABC, AB = 5 cm, AC = 12 cm, $\angle$BAC = 90^0. Construct ΔABC and draw its circumscribing circle.

Solution:

i) Draw ΔABC

ii) Draw perpendicular bisectors of side AC & BC. They meet at the point O

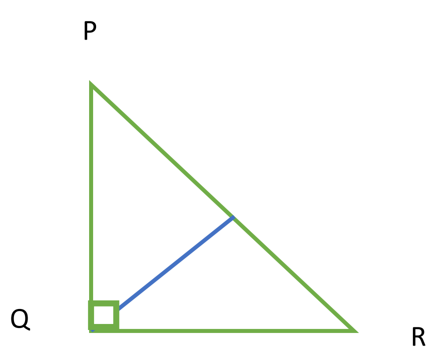
iii) With Centre C draw the circle with radius OB



Q. 48

In $\Delta PQR, \angle Q = 90^0$ Line QM is median. Draw the circumscribing circle of ΔPQR

Construction:



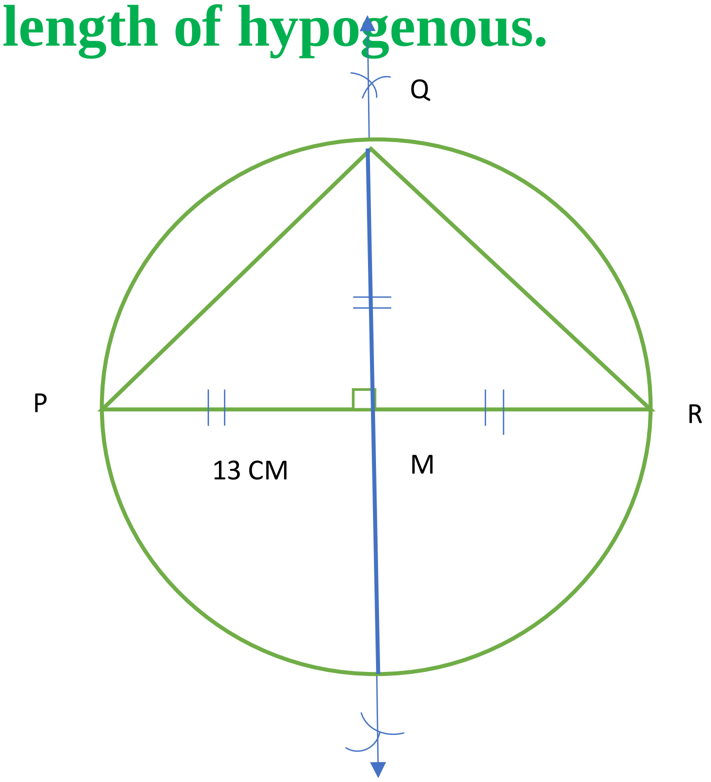
In ΔPQR , $\angle Q = 90^0$

$$PQ^2 + QR^2 = PR^2$$

$$169 = PR^2$$

$$PR = 13$$

In right angled triangle, median on the hypogenous is half of the length of hypogenous.



i) Draw Line PR = 13 cm

ii) Draw perpendicular bisector of line PR

iii) Take point M as Centre of the circle and draw circle with radius of 6.5 cm. The circle intersects perpendicular bisector at point P.

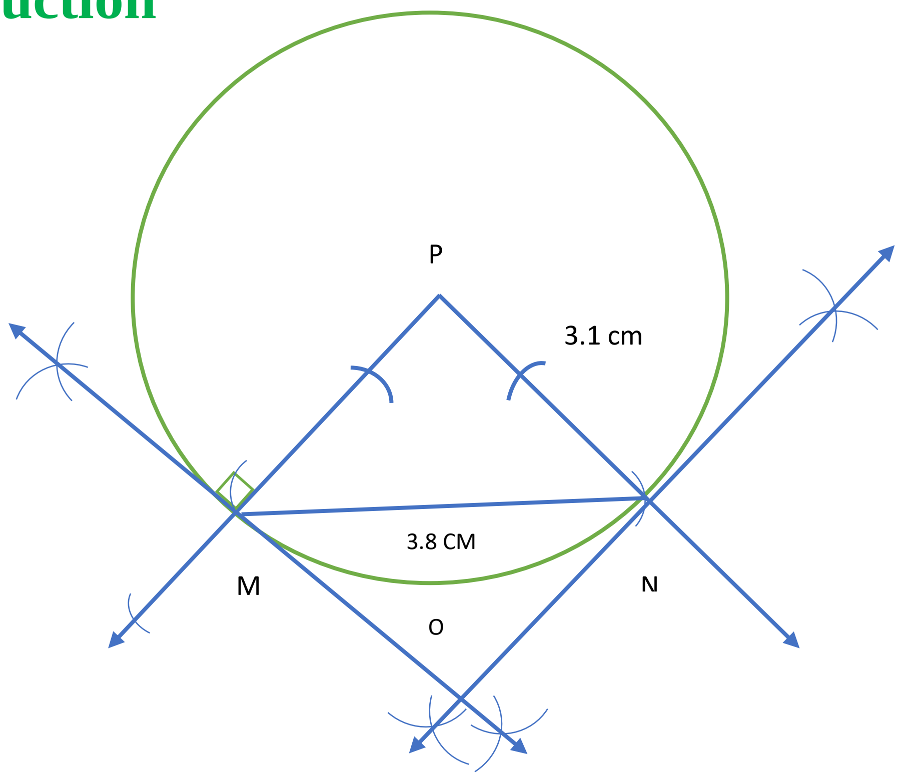
iv) Join PQ and QR

v) ΔPQR is the triangle required.

Q. 49

With centre P and radius 3.1 cm, draw a circle. Draw one chord of length 3.8 cm on the circle. Draw tangents through points M and N.

Construction



In above figure line MO and NO are the tangents through points M and N

Q. 50

In Δ APK, PK = 7.5 cm, $\angle 90^0$, AD $\perp$ PK , AD = 3.2 cm, then construct ΔAPK