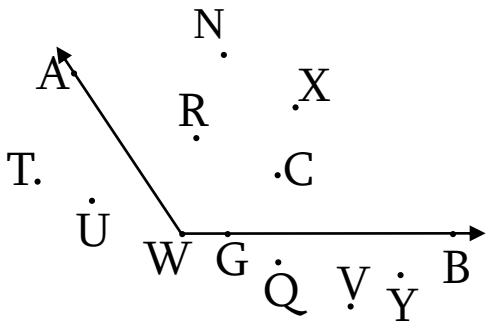
4. Angles and Pairs of Angles

Practice Set 15

1. Observe the figure and complete the table for ∠AWB.

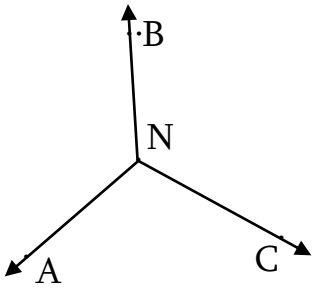


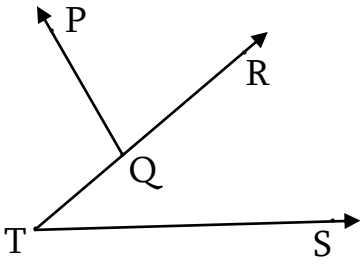
Points in the interior	
Points in the exterior	
Points on the arms of the angles	

Ans.

Points in the interior	R, N, C, X
Points in the exterior	T,U,Q,V,Y
Points on the arms of the angles	A, W, G, B

2. Name the pairs of adjacent angles in the figures below.





Ans. ∠ BNA and ∠ BNC, ∠ CNB and ∠ CNA, ∠ ANB and ∠ ANC, ∠ PQR and ∠ PQT are the pairs of adjacent angles.

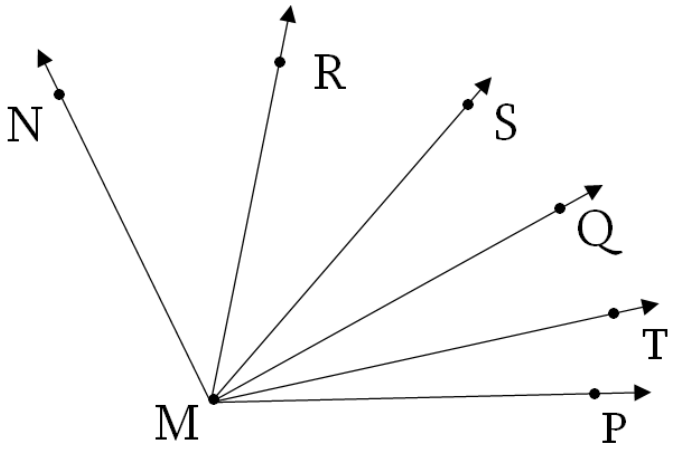
3. Are the following pairs adjacent angles? If not, state the reason.

(i) ∠PMQ and ∠RMQ

(ii) ∠RMQ and ∠SMR

(iii) ∠RMS and ∠RMT

(iv) ∠SMT and ∠RMS



Ans.

(i) ∠PMQ and ∠RMQ are adjacent angles.

(ii) ∠RMQ and ∠SMR are not adjacent angles because their interiors are not separate.

(iii) ∠RMS and ∠RMT are not adjacent angles because their interiors are not separate.

(iv) ∠SMT and ∠RMS are adjacent angles.

Practice Set 16

1. The measures of some angles are given below. Write the measures of their complementary angles.

(i) 40^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore\ 40 + x = 90$

$\therefore\ x = 90 - 40$

$\therefore\ x = 50$

$\therefore$ The measure of the complement of an angle of measure 40^0 is 50^0.

(ii) 63^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore\ 63 + x = 90$

$\therefore\ x = 90 - 63$

$\therefore\ x = 27$

$\therefore$ The measure of the complement of an angle of measure 63^0 is 27^0.

(iii) 45^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore\ 45 + x = 90$

$\therefore\ x = 90 - 45$

$\therefore\ x = 45$

$\therefore$ The measure of the complement of an angle of measure 45^0 is 45^0.

(iv) 55^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore 55 + x = 90$

$\therefore \quad x = 90 - 55$

$\therefore \quad x = 35$

$\therefore$ The measure of the complement of an angle of measure 55^0 is 35^0 .

(v) 20^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore 20 + x = 90$

$\therefore \quad x = 90 - 20$

$\therefore \quad x = 70$

$\therefore$ The measure of the complement of an angle of measure 20^0 is 70^0.

(vi) 90^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be x^0.

$\therefore\ 90 + x = 90$

$\therefore\ x = 90 - 90$

$\therefore\ x = 0$

$\therefore$ The measure of the complement of an angle of measure 90^0 is 0^0.

(vii) x^0

Solution: The sum of the complementary angle is 90^0.

Suppose the measure of the complementary angle be y^0.

$\therefore\ x + y = 90$

$\therefore\ y = 90 - x$

$\therefore$ The measure of the complement of an angle of measure x^0 is $(90 - x)^0$.

2. $(y - 20)^\circ$ and $(y + 30)^\circ$ are the measures of complementary angles. Find the measure of each angle.

Solution: $(y - 20)^\circ$ and $(y + 30)^\circ$ are the measures of complementary angles.

The sum of the measures of the complementary angles is 90^0.

$\therefore (y - 20)^0 + (y + 30)^0 = 90^0$

$\therefore y - 20 + y + 30 = 90$

$\therefore 2y + 10 = 90$

$\therefore 2y = 90 - 10$

$\therefore 2y = 80$

$\therefore y = 40$

$\therefore (y - 20)^0 = (40 - 20)^0 = 20^0$

And $(y + 30)^0 = (40 + 30)^0 = 70^0$

$\therefore$ The measures of the angles $(y - 20)^0$, $(y + 30)^0$ are 20^0 and 70^0 respectively.

Practice Set 17

1. Write the measures of the supplements of the angles given below.

(i) 15^0

Solution: The sum of the measures of two supplementary angles is 180^0 .

Suppose the supplementary angle measure x^0.

$\therefore\ 15 + x = 180$

$\therefore 15 + x - 15 = 180 - 15$ …(Subtracting 15 from both sides)

$\therefore\ x^0 = 165^0$

$\therefore$ The measure of the supplement of an angle of 15^0 is 165^0.

(ii) 85^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure be x^0.

$\therefore\ 85 + x = 180$

$\therefore 85 + x - 85 = 180 - 85$ …(Subtracting 85 from both sides)

$\therefore\ x = 95^0$

$\therefore$ The measure of the supplement of an angle of 85^0 is 95^0.

(iii) 120^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure x^0.

$\therefore 120 + x = 180$

$\therefore 120 + x - 120 = 180 - 120$

…(Subtracting 120 from both sides)

$\therefore\ x = 60$

$\therefore$ The measure of the supplement of an angle of 120^0 is 60^0.

(iv) 37^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure x^0.

$\therefore\ 37 + x = 180$

$\therefore 37 + x - 37 = 180 - 37$…(Subtracting 37 from both sides)

$\therefore\ x = 143$

$\therefore$ The measure of the supplement of an angle of 37^0 is 143^0.

(v) 108^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure x^0.

$\therefore\ 108 + x = 180$

$\therefore 108 + x - 108 = 180 - 108$

…(Subtracting 108 from both sides)

$\therefore\ x = 72$

$\therefore$ The measure of the supplement of an angle of 108^0 is 72^0.

(vi) 0^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure x^0.

$\therefore\ 0 + x = 180$

$\therefore\ x = 180$

$\therefore$ The measure of the supplement of an angle of 0^0 is 180^0.

(vii) a^0

Solution: The sum of the measures of two supplementary angles is 180^0.

Suppose the supplementary angle measure x^0.

$\therefore\ a + x = 180$

$\therefore a + x - a = 180 - a$…(Subtracting 'a' from both sides)

$\therefore\ x = (180 - a)$

$\therefore$ The measure of the supplement of an angle of 0^0 is $(180 - a)^0$

2. The measures of some angles are given below. Use them to make pairs of complementary and supplementary angles.

$m\angle B = 60^0$, $m\angle N = 30^0$, $m\angle Y = 90^0$, $m\angle J = 150^0$,

$m\angle D = 75^0$, $m\angle E = 0^0$, $m\angle F = 15^0$, $m\angle G = 120^0$.

Ans.

Pairs of complementary angles :

The sum of the measures of two complementary angles is 90^0.

(i) $\therefore m\angle B + m\angle N = 60^0 + 30^0 = 90^0$

$m\angle B$, $m\angle N$

(ii) $\therefore m\angle Y + m\angle E = 90^0 + 0^0 = 90^0$

$m\angle Y$, $m\angle E$

Pairs of supplementary angles :

The sum of the measures of two supplementary angles is 180^0.

(i) $\therefore m\angle B + m\angle G = 60^0 + 120^0 = 180^0$

$m\angle B$, $m\angle G$

(ii) $\therefore m\angle N + m\angle J = 30^0 + 150^0 = 180^0$

$m\angle N$, $m\angle J$

3. In ΔXYZ, m∠Y = 90°. What kind of a pair do ∠X and ∠Z make?

The sum of the measures of the angles of a triangle is 180^0.

∴ In ΔXYZ,

$m\angle Y + m\angle X + m\angle Z = 180^0$

$\therefore 90^0 + m\angle X + m\angle Z = 180^0$

$\therefore 90^0 + m\angle X + m\angle Z - 90^o = 180^0 - 90^0$

$\therefore m\angle X + m\angle Z = 90^0$

∴ ∠X and ∠Z are complementary angles.

i.e. ∠X and ∠Z are complements of each other.

4. The difference between the measures of the two angles of a complementary pair is 40°. Find the measures of the two angles.

Solution: Suppose the measure of the smaller angle of the complementary pair be x^0.

Given that the difference between the measures of the two angles is 40^0.

$\therefore$ the measure of the greater complementary angle is $(x + 40)^0$.

The sum of the measures of two complementary angle is 90^0.

$\therefore\ x + x + 40 = 90$

$\therefore\quad 2x + 40 = 90$

$\therefore\qquad 2x = 90 - 40$

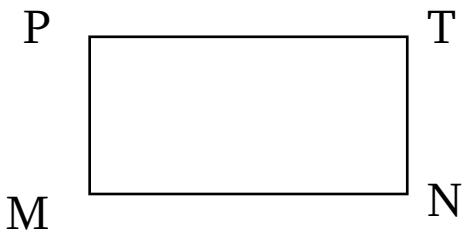
$\therefore\qquad 2x = 50$

$\therefore\qquad x = \frac{50}{2}$

$\therefore\qquad x = 25$

$\therefore$ The measures of the two angles of a complementary pair are 25^0 and 65^0 respectively.

5. □ PTNM is a rectangle.
Write the names of the
pairs of supplementary angles.



Ans. The pairs of supplementary angles in □PTNM:

(i) ∠ P and ∠ M (ii) ∠ T and ∠ N (iii) ∠ P and ∠ T

(iv) ∠ M and ∠ N (v) ∠ P and ∠ N (vi) ∠ M and ∠ T.

6. If m∠A = 70°, what is the measure of the supplement of the complement of ∠A?

Solution: Given : m∠A = 70°.

Suppose the measure of the complementary angle be x^0.

$\therefore \quad 70 + x = 90$

$\therefore x = 90 - 70$

$\therefore x = 20$

The complementary angle of 70^0 is 20^0.

To find the supplementary angle of 20^0.

Suppose the measure of the supplementary angle be y^0.

$\therefore 20 + y = 180$

$\therefore$ y = 180 – 20

$\therefore$ y = 160

$\therefore$ The measure of the supplement of $\angle A$ is 160^0.

7. If $\angle A$ and $\angle B$ are supplementary angles and $m\angle B = (x + 20)^0$, then what would be $m\angle A$?

Solution:

Suppose the measure of $\angle A$ be y^0

The sum of supplementary angles is 180^0.

$\therefore$ $x + 20 + y = 180$

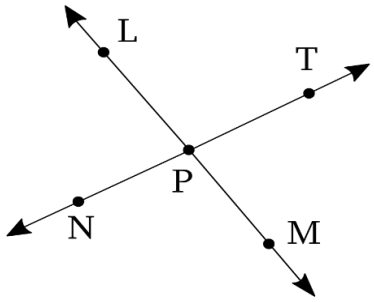
$\therefore$ $y = 180 – 20 – x$

$\therefore$ $y = 160 – x$

$\therefore$ The measure of the required angle A is $(160 - x)^0$.

Practice Set 18

(1) Name the pairs of opposite rays in the given figure alongside.



Ans. Opposite rays:

(a) Ray PM and ray PL,

(b) Ray PT and ray PN.

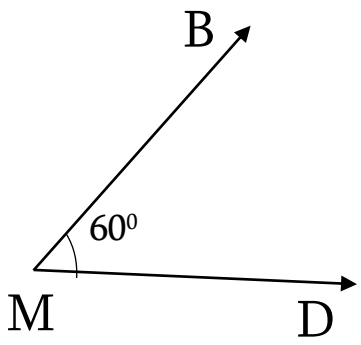
(2) Are the ray PM and PT opposite rays? Give reasons for your answer.

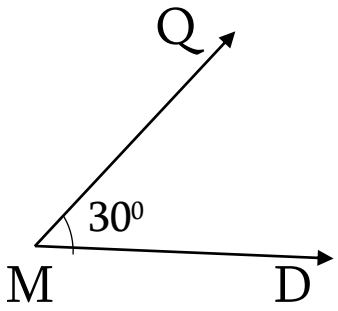
Ans. Ray PM and ray PT are not opposite rays because the ray PM and ray PT have a common origin but they do not form a straight line.

◘ Draw the pairs of angles as described below. If that is not possible, say why.

(i) Complementary angles that are not adjacent.

Ans.





Here, $m\angle BMD = 60^o, m\angle QMD = 30^o$.

$m\angle BMD + m\angle QMD = 60^o + 30^o = 90^o$.

Therefore, ∠BMD and ∠QMD are complementary angles.

The interior of the ∠QMD is contained in the interior of ∠BMD. Therefore, ∠QMD and ∠BMD are not the adjacent angles.

(ii) Angles in a linear pair which are not supplementary.

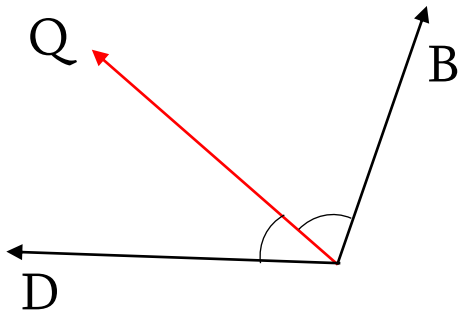
Ans. If two angles are in a linear pair the sum of their measures is 180^o and hence they are supplementary. Angles in a linear pair which are not supplementary, it is not possible.

(iii) Complementary angles that do not form a linear pair.

Ans. Complementary angles are angles that sum to 90^0 linear pair are angles that form a straight line, the angle measure of straight line is 180^0. So complementary angles that do not form a linear pair. Angles in a linear pair are always supplementary angles.

(iv) Adjacent angles which are not in a linear pair.

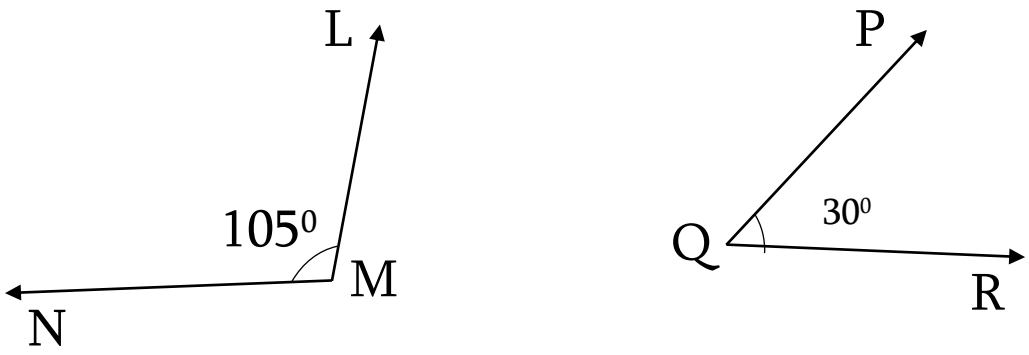
Ans. .



∠QMD & ∠QMB are adjacent angles.

(v) Angles which are neither complementary nor adjacent.

Ans.



(vi) Angles in a linear pair which are complementary.

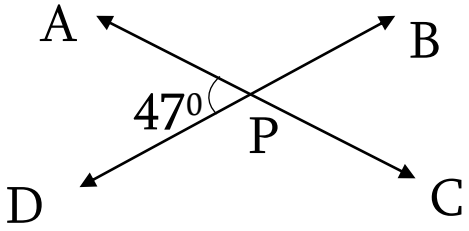
Ans. If two angles are in linear pair then the sum of their measures is 180^{o}. Therefore, the sum of their measures is not equal to 90^{o}. Hence the two angles in linear pair are not complementary.

Practice Set 20

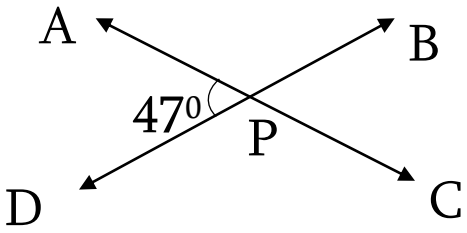
1. Lines AC and BD intersect at point P.

   $m\angle APD = 47°$. Find the measures

   of $\angle APB$, $\angle BPC$, $\angle CPD$.



Solution:



Given : $m\angle APD = 47^o$

$m\angle APD = m\angle BPC = 47^o$ …(Vertically opposite angles)

$\angle APD$ and $\angle APB$ are in a linear pair.

$\therefore m\angle APD + m\angle APB = 180^o$

$\therefore\ m\angle APB = 180^o - m\angle APD$

$= 180^o - 47^o$

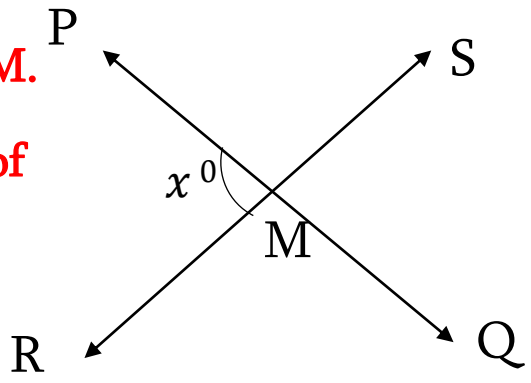
$\therefore\ m\angle APB = 133^o$

$\therefore .m\angle CPD = m\angle APB = 133^o$. …(Vertically opposite angles)

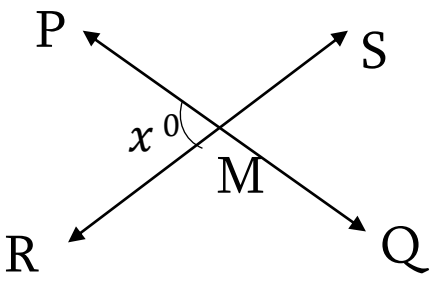
$\therefore$ The measure of angles are $m\angle APB = 133^o$, $m\angle BPC = 47^o$,

$m\angle CPD = 133^o$.

2. Lines PQ and RS intersect at point M. m∠PMR = $x°$ What are the measures of ∠PMS, ∠SMQ and ∠QMR?



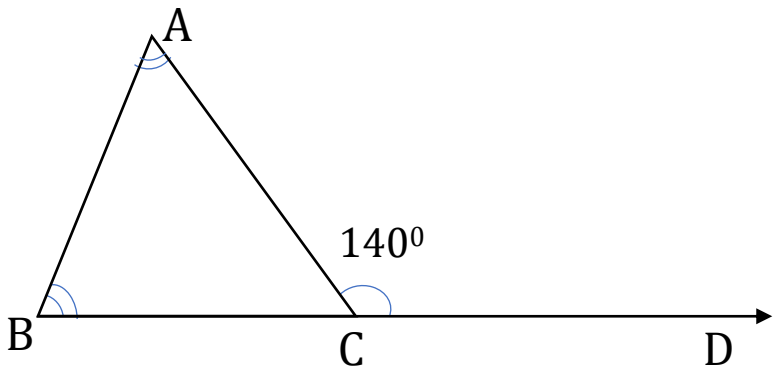
Solution: The angles, ∠PMR,∠PMS, ∠SMQ and ∠QMR are shown in the diagram below.



Statement number	Statement	Reason
1	$\angle PMR \cong \angle SMQ$	They are vertically opposite angles.
2.	$m\angle PMR = m\angle SMQ$	Statement 1, Measures of congruent angles
3.	$m\angle SMQ = x^o$	Given, $m\angle PMR = x^o$, and statement 2.
4.	$\angle PMR$ and $\angle PMS$ are in a linear pair	Given, RS is a line and M lies on RS.
5.	$m\angle PMR + m\angle PMS = 180^o$	Sum of the angles in a linear pair is 180^o.
6	$m\angle PMS = 180^o - m\angle PMR$	Simplify Statement 5
7.	$m\angle PMS = 180^o - x^o$.	Statement 3, given , $m\angle PMR = x^o$
8.	$\angle PMS \cong \angle RMQ$	they are vertically opposite angles.
9.	$m\angle PMS = m\angle RMQ$	Statement 8, Measures of congruent angles.
10	$m\angle RMQ = 180^o - x^o$.	Statements 7 and 9.
11.	$m\angle PMS = 180^o - x^o$, $m\angle SMQ = x^o$, $m\angle RMQ = 180^o - x^o$.	Statements 7, 3 and 10.

Practice Set 21

1. ∠ACD is an exterior angle of ΔABC. The measures of ∠A and ∠B are equal. If $m\angle ACD = 140°$, find the measures of the angles ∠A and ∠B.

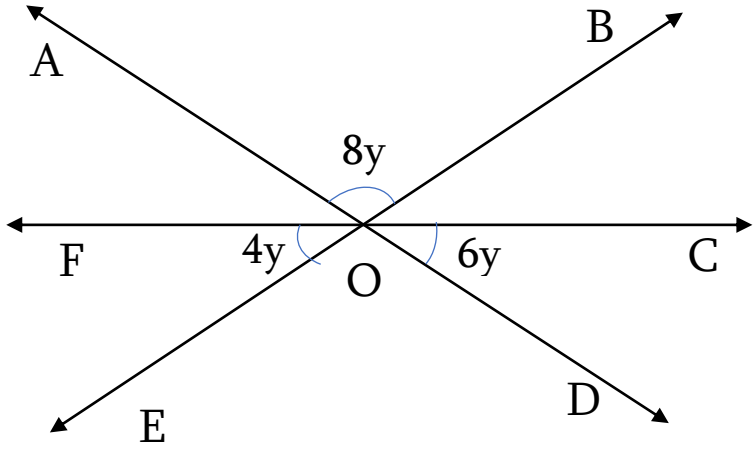


Solution:

Statement Number	Statement	Reason
1.	∠ACB and ∠ACD are in a linear pair.	Given, ∠ACD is an exterior angle of ΔABC.
2.	$m\angle ACB + m\angle ACD = 180^o$	Sum of the measures of the angles in a linear pair is 180^o.
3.	$m\angle ACB = 180^o - m\angle ACD$	Statement 2 simplified.
4.	$m\angle ACB = 180^o - 140^o$	Statement 3, given $m\angle ACD = 140^o$

5.	$m\angle ACB = 40^o$	Statement 4 simplified.
6.	$m\angle ABC + m\angle BAC + m\angle ACB = 180^o$	Sum of the measures of the angles of a triangle = 180^o.
7.	$m\angle ABC + m\angle BAC + 40^0 = 180^o$	i. Given, $m\angle ABC = m\angle BAC$ ii. statement 5.
8.	$2m\angle ABC + 40^o = 180^o$	Simplifying statement 7
9.	$m\angle ABC = \frac{180^o - 40^o}{2}$	Simplifying statement 8
10.	$m\angle ABC = 70^o$	Simplifying the statement 9.
11.	$m\angle BAC = 70^o$	i. statement 10 ii. Given, $m\angle ABC = m\angle BAC$
12.	measure of $\angle A = m\angle BAC = 70^o$, measure of $\angle B = m\angle ABC = 70^o$.	Statements 10 and 11.

2. Using the measures of the angles given in the figure alongside, find the measures of the remaining three angles.



Solution:

Given:

(i) $\overleftrightarrow{AD}$, $\overleftrightarrow{FC}$, and $\overleftrightarrow{EB}$ intersect at point O

(ii) $m\angle AOB = 8y^0$

(iii) $m\angle FOE = 4y^0$

(iv) $m\angle DOC = 6y^0$

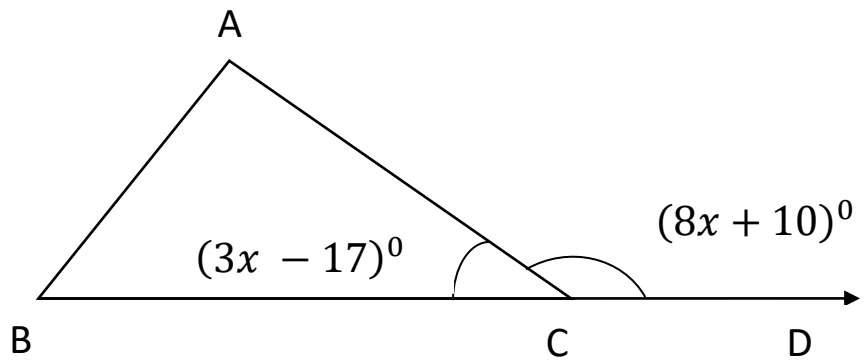
To find m ∠AOF, m∠EOD, and m∠BOC

Statement Number	Statement	Reason
1.	∠EOD and ∠AOB are vertically opposite angles.	Given, $\overleftrightarrow{AD}$ and $\overleftrightarrow{EB}$ intersect at O.
2.	∠EOD≅∠AOB	1. Statement 1 2. Vertically opposite angles are congruent.
3.	m∠EOD = m∠AOB	Measures of congruent angles
4.	m∠EOD = $8y^0$.	Given, m∠AOB = $8y^0$.
5.	∠FOE and ∠BOC are vertically opposite angles.	Given, $\overleftrightarrow{FC}$ and $\overleftrightarrow{EB}$ intersect at point O.
6.	∠ BOC ≅ ∠ FOE	1. Statement 5 2. Vertically opposite angles are congruent.
7.	m∠ BOC = m∠ FOE	Measures of congruent angles
8.	m∠ BOC = $4y^0$.	Given, m∠FOE = $4y^0$.
9.	∠ AOF and ∠ DOC are vertically opposite angles.	Given, $\overleftrightarrow{AD}$ and $\overleftrightarrow{FC}$ intersect at O.

10.	$\angle AOF \cong \angle DOC$	1. Statement 8 2. Vertically opposite angles are congruent.
11.	$m\angle AOF = m\angle DOC$	Measures of congruent angles
12.	$m\angle AOF = 6y^0$.	Given, $m\angle DOC = 6y^0$.
13.	$m\angle AOF = 6y^0$, $m\angle EOD = 8y^0$ and $m\angle BOC = 4y^0$.	Statements 12, 4 and 8 respectively.

3. In the isosceles triangle ABC, ∠A and ∠B are equal. ∠ACD is an exterior angle of ΔABC. The measures of ∠ACB and ∠ACD are $(3x - 17)^0$ and $(8x + 10)^0$ respectively. Find the measures of ∠ACB and ∠ACD. Also find the measures of ∠A and ∠B.

Solution:



Given: $m\angle ACB = (3x - 17)^0$

$m\angle ACD = (8x + 10)^0$

$m\angle ACB$ and $m\angle ACD$ form a linear pair.

$\therefore m\angle ACB + m\angle ACD = 180^0$

$\therefore\ 3x - 17 + 8x + 10 = 180$

$\therefore\ 11x - 7 = 180$

$\therefore 11x = 180 + 7$

$\therefore 11x = 187$

$\therefore\ x = \frac{187}{11}$

$\therefore\ x = 17$

$m\angle ACB = (3x - 17)^0$

$=[\ (3 \times 17) - 17]^0$

$= (51 - 17)^0 = 34^0$

$m\angle ACD = (8x + 10)^0$

$= [(8 \times 17) + 10]^0$

$= (136 + 10)^0 = 146^0$

From property of the exterior angle of a triangle.

$m\angle A + \angle B = m\angle ACD$

$\therefore$ m$\angle$ A + $\angle$ B = 146^0

$\therefore$ 2m$\angle$ A = 146^0 .....(m$\angle$ A = m$\angle$ B)

$\therefore$ $m\angle A = \frac{146}{2}$

$\therefore$ m$\angle$ A = 73^0

$\therefore$ m$\angle$ A = m$\angle$ B = 73^0

$\therefore$ The measures of angles are m $\angle$ACB = 34^0, m$\angle$ACD = 146^0,

m $\angle$A = 73^0, m $\angle$B = 73^0.

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