5. Quadrilaterals

Extra Questions

Q. 1) □ UVWP is a parallelogram. $\angle V = 65^0$. Find the measure of remaining angles. (3M)

Solution - □ UVWP is a parallelogram.

$\therefore \angle U + \angle V = 180^0$ … (Adjacent angles of a parallelogram)

$\therefore \angle U + 65^0 = 180^0$ ….. (given)

$\therefore \angle U = 180^0 - 65^0$

$\therefore \angle U = 115^0$

$\therefore \angle U = \angle W = 115^0$ … (opposite angles of a parallelogram)

$\therefore \angle V = \angle P = 65^0$… (opposite angles of a parallelogram)

$\therefore \angle V = 65^0, \angle W = 115^0, \angle P = 65^0, \angle U = 115^0$

Q. 2) In parallelogram ratio of opposite angles 2:4 and perimeter 60 cm, then find the length of side. (3M)

Solution: In parallelogram ratio of opposite angles 2:4

$\therefore$ opposite angle $2x$ and $4x$.

Parallelogram opposite angles are congruent.

$2x + 4x + 2x + 4x = 60$

$12x = 60$

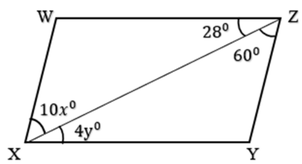
$x = \frac{60}{12}$

$x = 5$

$\therefore 2x = 2 \times 5 = 10$cm and $4x = 4 \times 5 = 20$cm

$\therefore$ In paralellogram sides 10 cm, 20 cm, 10 cm and 20 cm.

Q. 3) In the adjoining figure, □ WXYZ is a parallelogram then find the values of x and y. (3M)



Solution : □WXYZ is a parallelogram.

seg XY ∥ seg WZ and seg XZ is a transversal.

∴ ∠WZX ≅ ∠ZXY …… (Alternate angles)

∴ ∠ZXY ≅ ∠WZX

$4y = 28^0$

$y = \frac{28}{4}$

$y = 7^0$ ……. (I)

Now, ∠WXY ≅ ∠YZW …… (Opposite angles of parallelogram)

∴ ∠WXZ + ∠ZXY = ∠WZX + ∠XZY …. (Opposite angles)

$10x + 4y = 28^0 + 60^0$

$10x + 4 \times 7 = 28^0 + 60^0$

$10x + 28 = 88$

$10x = 88 - 28$

$10x = 60$

$x = \frac{60}{10}$

$x = 6$

$\therefore x = 6^0, y = 7^0$

Q. 4) In parallelogram ratio of opposite angle is 2:3 then find the measure of all angles of parallelogram. (3M)

Solution :

□ PQRS is a parallelogram.

Opposite angles of parallelogram are supplementary.

$\angle$P and $\angle$S are opposite angle.

Let $\angle P = 2x^0, \angle S = 3x^0$

$\angle P + \angle S = 180^0$

$2x + 3x = 180^0$

$5x = 180^0$

$x = \frac{180^0}{5}$

$x = 36^0$

$\therefore \angle P = 2x = 2 \times 36^0 = 72^0$

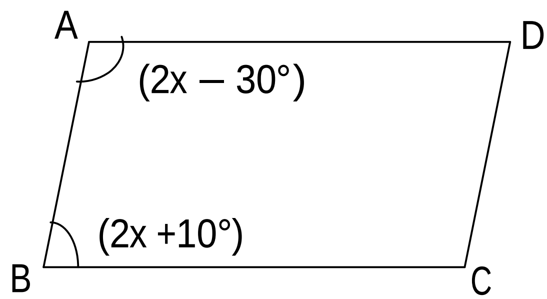
$\therefore \angle S = 3x = 3 \times 36^0 = 108^0$

$\therefore \angle P = \angle R = 72^0$

$\therefore \angle S = \angle Q = 108^0$

$\therefore$ Measures of all angles of parallelogram are $72^0, 108^0, 72^0, 108^0$

Q. 5) In parallelogram ABCD, $\angle A = (2x - 30)^0$, $\angle B = (2x + 10)^0$, then find the value of x then find the measure of $\angle C$ and $\angle D$. (3M)



Solution :

Opposite angle of parallelogram are supplementary opposite angle are ∠ A and ∠ B

$\angle A + \angle B = 180^0$

$(2x - 30) + (2x + 10) = 180$

$4x - 20 = 180$

$4x = 180 + 20$

$4x = 200$

$x = \frac{200}{4}$

$x = 50$

$\angle A = (2x - 30) = 2 \times 50 - 30 = 100 - 30 = 70$

$\angle B = 2x + 10 = 2 \times 50 + 10 = 100 + 10 = 110$

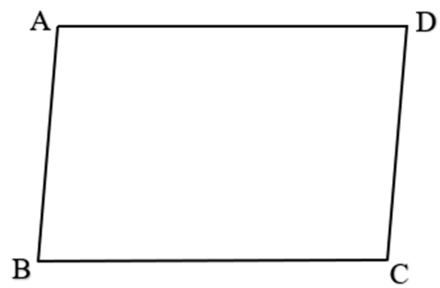
In parallelogram opposite angle are congruent,

$\angle A = \angle C = 70^0$

$\angle B = \angle D = 110^0$

$x = 50^0, \angle A = 70^0, \angle B = 110^0$

Q.6) Perimeter of parallelogram is 160 cm. One of its side is greater than the other side by 30. Find the length of all sides. (3M)



Solution:

Let ◻ ABCD be the given parallelogram.

Let AB = x cm. BC = $(x + 30)$ cm.

Opposite sides of parallelogram are congruent.

AB = CD = x. and BC = AD = $x + 30$

AB + DC + BC + AD = 160

$x + x + (x + 30) + (x + 30) = 160$

$4x + 60 = 160$

$4x = 160 - 60$

$4x = 100$

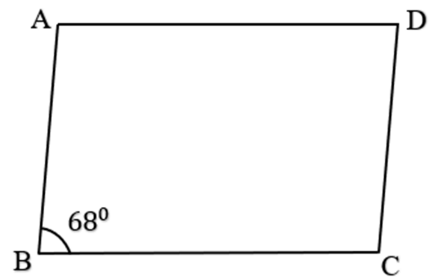
$x = \frac{100}{4}$,

$x = 25$

AB = CD = 25 cm and BC = AD = $x + 30 = 25 + 30 = 55$.

The length of all sides of the parallelogram are 25 cm, 55 cm, 25 cm, 55 cm.

Q. 7) In the adjoining figure, if $\angle B = 68^0$ then find $\angle A, \angle C$ *and* $\angle D$(3M)



Solution :

Opposite angles of a parallelogram are equal .

$\angle B = \angle D \Longrightarrow \angle D = 68^0$ …. ($\because \angle B = 68^0$) ….. (given)

$\angle B$ *and* $\angle C$ are supplementary

$\angle B + \angle C = 180^0$

$\angle C = 180^0 - \angle B$

$\angle C = 180^0 - 68^0$

$\angle C = 112^0$

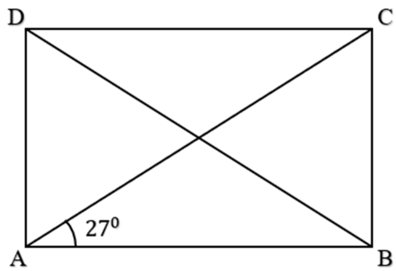
$\angle A$ and $\angle C$ are opposite angle.

$\angle A = \angle C$

$\angle A = 112^0$ ……. ($\because \angle C = 112^0$)

Hence, $\angle A = 112^0, \angle D = 68^0$ *and* $\angle C = 112^0$

Q. 8) In the adjoining figure, is a rectangle whose diagonals AC and BD intersect at O. If $\angle OAB = 27^0$, then find $\angle$OBC. (3M)



Solution : Since the diagonals of a rectangle are equal and bisect each other.

OA = OB

$\Rightarrow \angle OBA = \angle OAB = 27^0$

Each angle of a rectangle measure 90^0

$\angle ABC = 90^0$

$\angle ABO + \angle CBO = 90^0$

$\angle OBA + \angle OBC = 90^0$

$27^0 + \angle OBC = 90^0$

$\angle OBC = 90^0 - 27^0$

$= 63^0$

Q.9) Ratio of angle of rectangle is 3:5:9:13, then find the measure of all angle of rectangle. (3M)

Solution :

Measure of rectangle are $3x$, $5x$, $9x$ and $13x$.

Sum of all angles of a quadrilateral is 360^0

$3x + 5x + 9x + 13x = 360$

$30x = 360$

$x = \frac{360}{30}$

$x = 12^0$

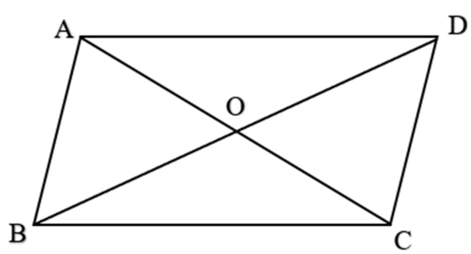
$3x = 3 \times 12 = 36^0$

$5x = 5 \times 12 = 60^0$

$9x = 9 \times 12 = 108^0$

$13x = 13 \times 12 = 156^0$

Q. 10) □ABCD is a parallelogram its diagonal are intersect at point O. $\angle BCD = 140^0$ then $\angle BAD = ?$, $\angle CDA = ?$ If $l(OC) = 6$ cm, then $l(AC) = ?$ (3M)



Solution: In □ABCD is a parallelogram.

$\angle BCD \cong \angle CDA$ ….. (I) (Opposite angle of a parallelogram are congruent)

$\angle BCD = 140^0$ ….. (II)

$\angle BAD = 140^0$ ….. (From I and II) …. (III)

In parallelogram □ ABCD

line BA || line CD and AD is a transversal.

$\angle BAD + \angle CDA = 180^0$ ….. (Interior Angle)

$140^0 + \angle CDA = 180^0$ ……. (From III)

$\therefore \angle CDA = 180^0 - 140^0$

$\therefore \angle CDA = 40^0$

$\therefore l(AC) = 2 \times l(OC)$

….. (Diagonals of a parallelogram bisect each other)

$= 2 \times 6$
$= 12$
$\therefore \angle BAD = 140^0, \angle CDA = 40^0, l(AC) = 12$

Q.11) In a parallelogram LMNO, side *L*MFS 5.4 cm, side MN = $\frac{2}{3}$ times of LM. Find perimeter of □ LMNO(3M)

Solution : □ LMNO is a parallelogram.

LM= 5.4 cm

$\therefore MN = \frac{2}{3} \times LM = \frac{2}{3} \times 5.4 = 3.6cm$

$\therefore LM = ON = 5.4$ cm ….. (opposite side of parallelogram)

$\therefore MN = LO = 3.6$ cm ….. (opposite side of parallelogram)

Perimeter of □ LMNO = 2 (sum of adjacent angle)

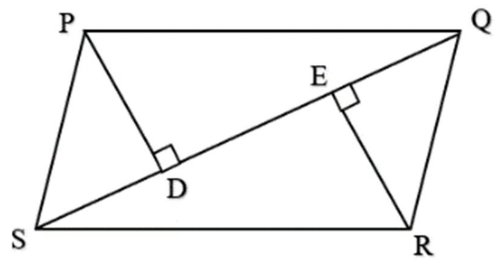
= 2 (5.4 + 3.6)

= 2 (9)

= 18 cm

∴ Perimeter of a parallelogram LMNO is 18 cm.

Q. 12) □ PQRS is a parallelogram, seg PD ⊥ seg QS. seg RE ⊥ seg QS. Prove that seg PD ≅ seg RE. (3M)



Solution :

□ PQRS is a parallelogram.

Seg PS || seg QR, SQ is a transversal.

∴ ∠ PSQ ≅ ∠ SQR …… (Alternate angle)

i.e ∠ PSD ≅ ∠ EQR ….. (I)

seg PS ≅ seg QR ….. (II)

….. (opposite side of parallelogram)

In Δ PSD and Δ RQE,

∠ PSD ≅ ∠ EQR ….. (from I)

seg PS ≅ seg QR …… (from II)

∠ PDS ≅ ∠ REQ ……. (given)

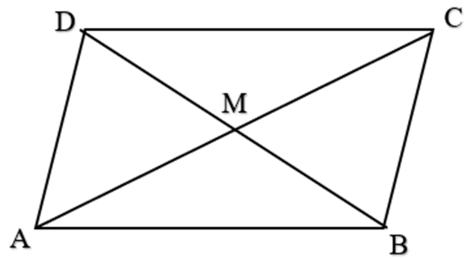
∴ Δ PSD ≅ Δ RQE -------- (By S – A – A test)

∴ seg PD ≅ seg RE.

Q. 13) In the figure seg AD ≅ seg BC, seg DM ≅ seg BM,

∠ADM ≅ ∠ CBM. Prove that □ ABCD is a

parallelogram. (3M)



Solution : In Δ ADM and Δ CDM,

seg AD ≅ seg BC ….. (given)

seg DM ≅ seg BM ….. (given)

∠ADM ≅ ∠CBM ….. (given)

∴Δ ADM ≅ ΔCDM ….. (S. A . S) test

∴seg AM ≅ seg CM ….. (c.s.c.t test)

∴seg DM ≅ seg BM ….. (given)

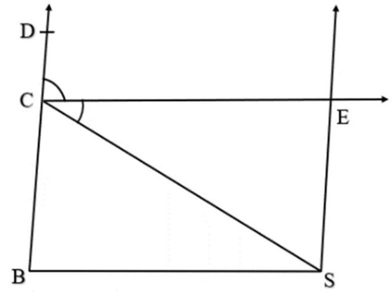
□ ABCD is a parallelogram …… (If the diagonals of a quadrilateral bisect each other, then it is a parallelogram)

Q. 14) In the adjacent figure,

Given : In Δ CBS, seg BC ≅ seg SC , ray CE is the bisector of

∠ DCS, ray SE || ray BC.

To prove : □ is a parallelogram. (4M)



Solution : In Δ CBS ,

seg BC ≅ seg SC …. (given)

∠ B ≅ ∠ CBS ……. (I) (Isosceles triangle theorem)

∠ DCS is an exterior angle of Δ CBS

∴ ∠ B + ∠ CSB = ∠ DCS ….. (Theorem of remote interior angle of a triangle)

∴ ∠ CSB + ∠ CSB = ∠DCS …… (From I)

∴ 2 ∠ CSB = ∠ DCS ……. (II)

Ray CE is the bisector of ∠ DCS …… (given)

∴ ∠ DCE = ∠SCE = $\frac{1}{2}$ ∠ DCS …… (III)

∴ ∠ DCE = 2∠ECS ….. (from II and IV)

2∠ CSB = 2∠ ECS

∠ CSB = ∠ ECS

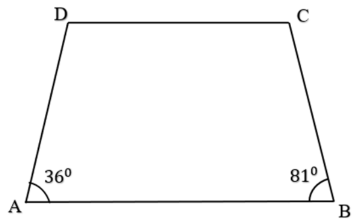
∴ seg CE ∥ seg BS …… (V)

…. (Alternate angle test of parallel lines)

seg SE ∥ seg BC …… (VI) ….. (given)

∴ □ CBSE is a parallelogram. …… (from V and VI)

Q.15) In the adjoining figure ABCD is a trapezium in which AB ∥ CD. If ∠ A = 36^0 and ∠ B = 81^0 then find ∠ C and ∠ D. (4M)



Solution :

AB ∥ CD and AD is a transversal.

ABCD is a trapezium in which AB ∥ CD

∴ ∠ A + ∠ D = 180^0

∠ D = 180^0 – ∠ A

= 180^0 – 36^0

= 144^0

Again AB ∥ CD and BC is a transversal.

∴ ∠B + ∠C = 180^0

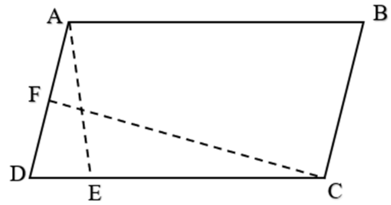
∠ C = 180^0 – ∠ B

= 180^0 – 81^0

= 99^0

∴ The required measure of ∠ C and ∠ D are 144^0 and 99^0 respectively.

Q.16) In the figure, ABCD is a parallelogram, AE ⊥ DC and CF⊥ AD, if AB = 16cm AE = 8 cm and CE = 10 cm. FindAD. (3M)



Solution : We have AE ⊥ DC and AB = 16 cm.

AB = CD …… (Opposite sides of a parallelogram ABCD

CD = 16 cm.

Now, area of a parallelogram ABCD = CD × AE

$= 16 \times 8 \text{ cm}^2$

$= 128 \text{ cm}^2$

Since CF ⊥ AD

∴ Area of a parallelogram ABCD = AD × CF

128 = AD × CF

128 = AD × 10

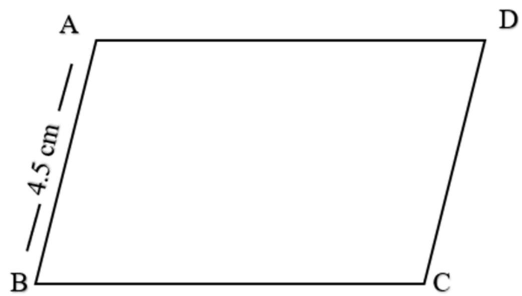
$\frac{128}{10} = AD$

12.8 = AD

∴ AD = 12.8

The required length of AD is 12.8 cm.

Q.17) In the adjoining figure ABCD is a parallelogram. If AB = 4.5 cm. Perimeter is 21 cm, then find the side of The parallelogram. (3M)



Solution : Opposite angle of a parallelogram are equal.

AB = CD = 4.5 cm and BC = AD

Now,

AB + CD + BC + AD = 21 cm

AB + AB + BC + BC = 21 cm

2 [AB + BC] = 21 cm

2 [4.5 + BC] = 21 cm

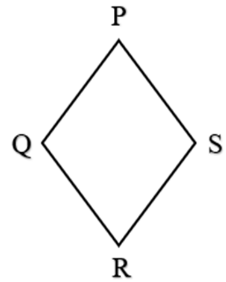
[4.5 + BC] = $\frac{21}{2}$ cm = 11

BC = 11.5 – 4.5

BC = 7 cm

∴ BC = 7 cm, CD = 4.5 cm, and AD = 7 cm.

Q.18 Every rhombus is a parallelogram. (3M)



Given : □ PQRS is a parallelogram ….. (given)

To prove : □ PQRS is a parallelogram

Proof :

Seg PQ = Seg QR = Seg RS = Seg PS ….. (given)

∴ side PQ = side RS

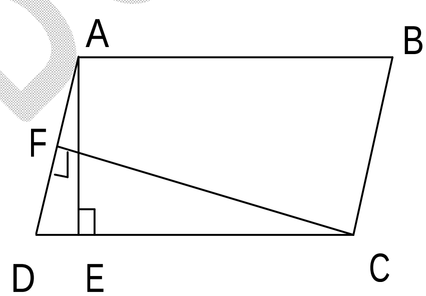
side QR = side PS

□ PQRS is a parallelogram ….. (Opposite sides of parallelogram)

Q.19) In the adjoining figure ABCD is a parallelogram

AE ⊥ DC and CF ⊥ AD. If AB = 18 cm, AE = 10 cm and

CF = 20 cm. Find AD. (3M)



Solution : AE ⊥ DC, AB = 18 cm

AB = CD …. (Opposite of parallelogram)

CD = 18 cm

Now,

Area of parallelogram ABCD = CD × AE

= 18 × 10 cm^2

= 180 cm^2

Now, CF ⊥ AD

Area of parallelogram ABCD = AD × CF

AD × CF = 180

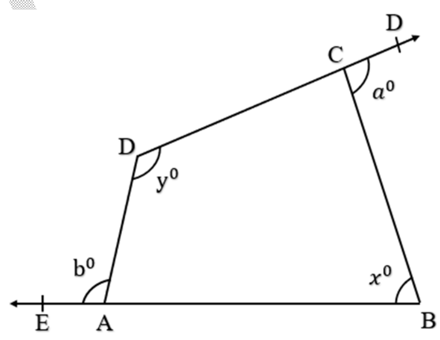
AD × 20 = 180

AD = $\frac{180}{20}$

AD = 9 cm

∴ length of AD is 9 cm.

Q. 20) The sides BA and DC of a quadrilateral are produced as shown in the given figure, prove that $x + y = a + b$ (3M)



Solution : We have ∠ A + b = 180^0 -------- (linear pair)

∠A = $180^0 - b^0$ ….. (I)

Also $\angle C + a^0 = 180^0$ …… (linear pair)

$\angle C = 180^0 - a^0$ ……. (II)

Now, $\angle A + \angle B + \angle C + \angle D = 360^0$

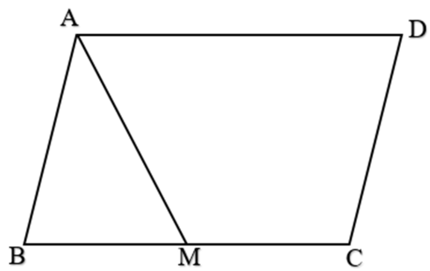
…. (Sum of the angle of a quadrilateral is 360^0)

$\Longrightarrow (180^0 - b^0) + x^0 + (180^0 - a^0) + y^0 = 360^0$

…… (Using I and II)

$\Longrightarrow x + y = a + b$

Q. 21) In the adjoining figure, in the parallelogram ABCD, M midpoint of the side BC then $\angle BAM = \angle DAM$. Then prove that, AD = 2 CD. (3M)



Solution : $\angle AMB = \angle DAM$ ($\because$ AD $\parallel$ BC)

$\angle AMB = \angle BAM$

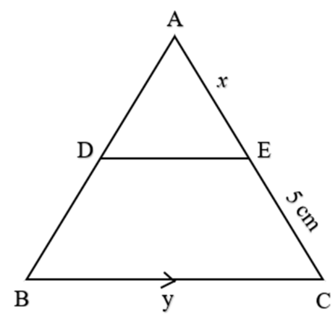
AB = BM …… (Opposite side)

$\Longrightarrow CD = \frac{1}{2}(BC)$

$\Longrightarrow CD = \frac{1}{2}(AD)$

$\therefore 2CD = AD$

Q. 22) In the adjoining figure, midpoint of AD is M and DE ∥ BC and find x and y. (3M)



Solution : DE ∥ BC and midpoint of AD is M.

∴ Midpoint of AC is E.

AE = EC.

x = 5 cm. Now, BE ∥ BC

Now, DE = $\frac{1}{2}$ BC ….. (Multiply by 2.)

$2DE = 2\left(\frac{1}{2}BC\right)$

2DE = BC

2 × 6 cm = BC

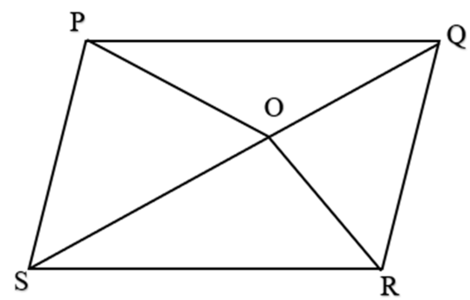
Or

BC = 12 cm, y= 12 cm.

y = 12 cm

x = 5 cm, and y = 12 cm.

Q. 23) □ PQRS is a parallelogram. Diagonal QS as point O then prove that seg OP ≅ seg OR. (4M)



Given :□ PQRS is rhombus.

Point O is any point diagonal QS.

To prove: seg OP ≅ seg OR

Proof : □ PQRS is rhombus and seg QS is diagonal.

∴ ∠ PQS ≅ ∠ RQS

… (Diagonal of rhombus bisect its opposite angle)

i.e ∠ PQO ≅ ∠ RQO

In Δ PQO ≅ Δ RQO

seg PQ ≅ seg RQ ------ [sides of rhombus]

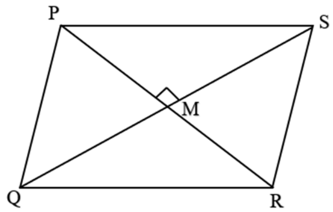
∠ PQO ≅ ∠ RQO ------ [from I]

seg OQ ≅ seg OQ ----- [common side]

Δ PQO ≅ Δ RQO ----- [S. A. S test]

∴ seg OP ≅ seg OR -------- [c.s.c.t]

Q. 24) Length of side of rhombus is 25 cm and one of its diagonal is 30 cm. Find the length of the other diagonal. (3M)



Solution : □ PQRS is rhombus.

Diagonals of rhombus are perpendicular bisectors of each other.

∴ Diagonal PR ⊥ diagonal QS

∴ $\angle PMQ = 90^0$ and PM = RM and QM = SM

$PM = \frac{1}{2} PR = \frac{1}{2} \times 30 = 15$ cm

And PQ = 25 cm

In right angles triangle PMQ

$PQ^2 = PM^2 + MQ^2$ --------(By Pythagoras theorem)

∴ $(25)^2 = (15)^2 + MQ^2$

$625 = 225 + MQ^2$

$MQ^2 = 625 - 225$

$MQ^2 = 400$

MQ = 20 cm

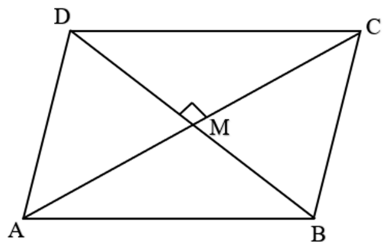
Now QS = $2 \times MQ$ ….. [Diagonals of rhombus are

$= 2 \times 20$ ….. [perpendicular bisectors of each other.]

= 40 cm

∴ The other diagonal of the rhombus is 40 cm.

Q. 25) Diagonals of rhombus ABCD are 6 cm and 8 cm. Find the side of rhombus. (4M)



Solution :

Let in rhombus ABCD.

diagonal = AC = 6cm , and diagonal = BD = 8 cm

Diagonals of rhombus are perpendicular bisectors of each other.

$\therefore \angle AMB = 90^0$ ….. (I)

$AM = \frac{1}{2} AC = \frac{1}{2} \times 6 = 3 \text{ cm}$ ….. (II)

$BM = \frac{1}{2} BD = \frac{1}{2} \times 8 = 4 \text{ cm}$ …. (III)

ΔAMB is a right angled triangle ….. (from (I)

$AB^2 = AM^2 + BM^2$ ….. (By Pythagoras theorem)

$\therefore AB^2 = 3^2 + 4^2$

$\therefore AB^2 = 9 + 16$

$\therefore AB^2 = 25$

AB = 5

$\therefore$ AB = BC = CD = AD = 5 cm ….. (sides of rhombus ABCD)

$\therefore$ side of the given rhombus ABCD is 5 cm.

Q.26) Find the measure of each angle of parallelogram, if one of its angles is 15^0 less than twice the smallest angle.

Solution :

Let the smallest angle be x.

Since the other angle = $(2x - 15^0)$

Thus $(2x - 15^0) + x = 180^0$ .... [∵ x and $(2x - 15^0)$ are the adjacent angle of a parallelogram]

$2x - 15^0 + x = 180^0$

$3x - 15^0 = 180^0$

$3x = 180^0 + 15$

$3x = 195^0$

$x = \frac{195^0}{3} = 65^0$

∴ The smallest angle $= 65^0$

∴ The other angle $= 2x - 15^0 = 2(65^0) - 15^0$

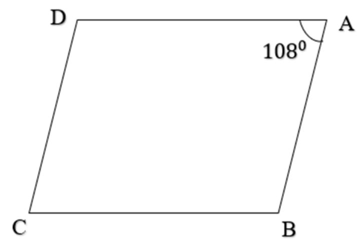
$= 130^0 - 15^0$

$= 115^0$

∴ Thus, the measures of all the angles of parallelogram are

$65^0, 115^0, 65^0$, and 115^0

Q.27) One angle of a quadrilateral is 108^0 and the remaining three angles are equal. Find each of the three equal angles.



Solution : ABCD is a quadrilateral.

$\therefore \angle A + \angle B + \angle C + \angle D = 360^0$

$108^0 + [\angle B + \angle C + \angle D] = 360^0$

$[\angle B + \angle C + \angle D] = 360^0 - 108^0$

$= 252^0$

$\angle D = \angle B = \angle C$

$\angle B + \angle C + \angle D = 252^0$

$\angle B + \angle B + \angle B = 252^0$

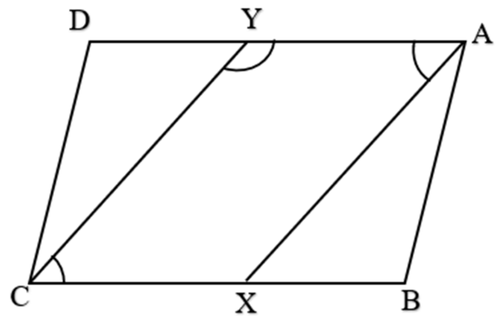
$3\angle B = 252^0$

$\angle B = \frac{252}{3} = 84^0$

$\therefore \angle B = \angle C = \angle D = 84^0$

Thus, the measure of the remaining angle is 84^0

Q. 28) In the figure, AX and CY are respectively the bisectors of opposite angle A and C of a parallelogram ABCD show that AX ∥ CY.



Solution : ABCD is a parallelogram.

∴ Its opposite angles are equal.

$\Rightarrow \angle A = \angle C$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\angle YAX = \angle YCX$

…… (I) (Opposite sides of parallelogram)

Again,

DA ∥ BC …… (Opposite sides of parallelogram)

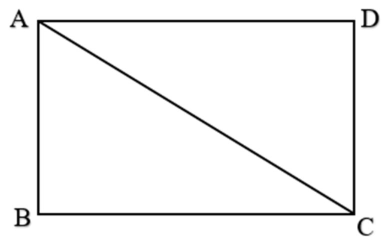
YA ∥ CX

Now, $\angle AYC + \angle YCX = 180^0$ …… (II)

$\angle AYC + \angle YAX = 180^0$ ….. (From I and II)

AX ∥ AY …… (An interior angles on the same side of the transversal are supplementary)

Q. 29) Diagonal of a square is 13 cm. Find the length of its side.



Solution : □ ABCD be the given square.

∴ AB = BC = CD = AD …… (sides of square)

$\angle B = 90^0$ …… (Angle of square)

∴ Δ ABC is a right angled triangle.

$AB^2 + BC^2 = AC^2$ …… (By Pythagoras theorem)

$AB^2 + AB^2 = 13^2$ …… (AB= BC)

$2AB^2 = 13^2$

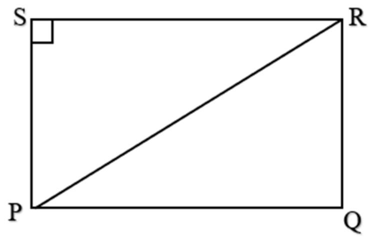
$AB^2 = \frac{13^2}{2}$

$AB = \frac{13}{\sqrt{2}}$

$AB = \frac{13}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{13\sqrt{2}}{2}$

∴ Side of the sqaure is $\frac{13\sqrt{2}}{2}$ cm.

Q.30) Adjacent side of a rectangle are 7 cm, 24 cm respectively. Find the length of its diagonals.



Solution :

Let □ PQRS be the given rectangle

In Δ PQR

PQ = 24 cm, QR = 7 cm, ∠Q = 90^0

∴ $PR^2 = PQ^2 + QR^2$---------- (Pythagoras theorem)

∴ $PR^2 = 24^2 + 7^2$

∴ $PR^2 = 576 + 49$

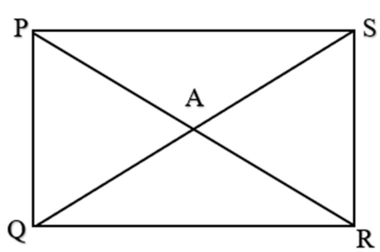
∴ $PR^2 = 625$

PR = 25

∴ Diagonals of a rectangle are congruent

∴ PR = SQ = 25 cm.

Q. 31) Rectangle □ PQRS diagonals are intersect at point O. If PR = 10 cm, then QO = ? If ∠RPS = 40^0 then ∠PRQ = ?



Solution :

Diagonals of a parallelogram bisect each other

∴ Diagonal PR = Diagonal QS

And PA = AR = QA = SA

$l\ (PA) = \frac{1}{2}\ l\ (PR) = \frac{1}{2} \times 10 = 5$

$l\ (QA) = l\ (PA) = 5$ cm.

$\angle PRQ = 40^0, \angle PRQ = ?$

Seg PS ∥ seg QR …… (Opposite side of rectangle and PR is transversal)

$\angle PRS = \angle PRQ$ ….. (pair of alternate angle)

∴ $\angle PRS = \angle PRQ = 40^0$ …… (II)

QA = 5 cm, $\angle PRQ = 40^0$ …… (from (I) and (II))

Q. 32) The measure of the angle of a quadrilateral taken in order are as 1 : 2 : 3 : 4 prove that it is a trapezium.

Solution :

Let measure of $\angle A, \angle B, \angle C, \angle D$ are $x^0, 2x^0, 3x^0$ and $4x^0$ respectively.

Sum of all angles of a quadrilateral is 360^0

$x + 2x + 3x + 4x = 360$

$10x = 360$

$x = \frac{360}{10}$

$x = 36$

$\therefore \angle A = x^0 = 36^0$

$\angle B = 2x = 2 \times 36 = 72^0$

$\angle C = 3x = 3 \times 36 = 108^0$

$\angle D = 4x = 4 \times 36 = 144^0$

$\angle A + \angle D = 36^0 + 144^0 = 180^0$ ….. (I)

$\angle B + \angle C = 108^0 + 144^0 = 180^0$ ….. (II)

∴ seg PQ ∥ seg SR … (Test of interior angle of parallel lines)

$\angle A + \angle B = 36^0 + 72^0 = 108^0 \neq 180^0$ …… (III)

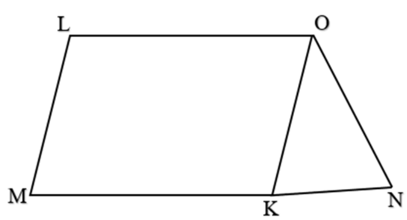
$\angle C + \angle D = 108^0 + 144^0 = 252^0 \neq 180^0$ …… (IV)

Seg AD and seg BC are not parallel

∴ One pair of opposite angles are parallel.

Hence, □ ABCD is a trapezium.

Q. 33) In a trapezium □ LMNO. side LO ∥ side MN , side LM ≅ side ON, side MN > side LO. Prove that $\angle LMN \cong \angle ONM$



Solution :

Given : In trapezium □ LMNO side LO ∥ side MN

side LM ≅ side ON, side MN > side LO.

To prove : ∠ LMN ≅ ∠ONM

Construction : Draw the segment parallel to side through point O. which intersect side MN in point K.

Proof : In □ LMNO

Seg LO ∥ side MK ……. (Given)

Seg LM ∥ side OK ….. (Construction)

∴ □ LMKO is a parallelogram

∴ ∠ LMK ≅ ∠ OKN …… (I) …. (Corresponding angle)

Seg LM ≅ Seg OK …… (II) … (opposite side)

Seg LM ≅ Seg ON …. (III) ….. (Given)

Seg OK ≅ Seg ON …… (from (II) and (III))

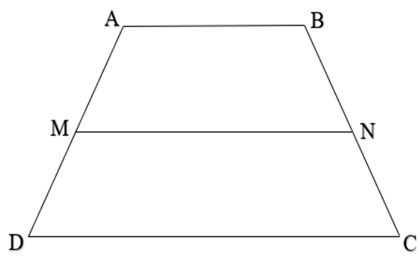
∴ ∠ OKN ≅ ∠ ONK … (IV) … (Theorem of isosceles triangle)

∴ ∠ LMK ≅ ∠ ONK …… (From (I) and (IV)

∴ ∠ LMN ≅ ∠ ONM …… (M – K – N)

Hence, base angles of an isosceles trapezium are congruent.

Q. 34) In trapezium ABCD. Side AB ∥ side CD.Points M and N are the midpoints of seg AD and seg BC respectively. AB = 16 cm and DC = 18 cm. Find MN.



Solution :

In trapezium ABCD. Seg AB ∥ seg CD

Point M is the midpoint of seg AD

Point N is the midpoint of seg BC

$\therefore$ MN = $\frac{1}{2}$ (AB + CD)

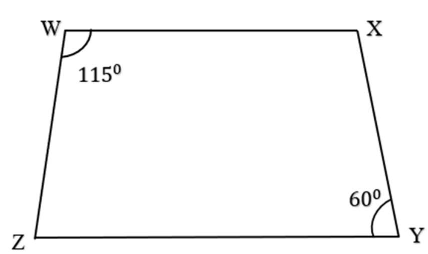
= $\frac{1}{2}$ (16 + 18)

= $\frac{1}{2}$ (34)

= 17 cm.

$\therefore$ MN = 17 cm.

Q. 35) In □ WXYZ, side WX ∥ YZ , ∠W = 115^0, ∠Y = 60^0 ,then find the measure of ∠X and ∠Z.



Solution :

In □ WXYZ

Side WX ∥ side YZ, XY is transversal ….. (given)

$\therefore \angle Y + \angle X = 180^0$ ….. (interior angle)

$\therefore 60^0 + \angle X = 180^0$

$\angle X = 180^0 - 60^0$

$\angle X = 120^0$

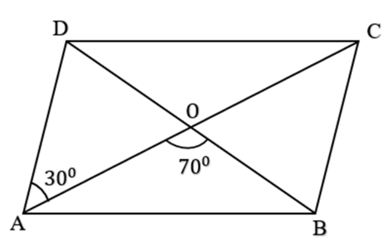
$\therefore \angle W + \angle Z = 180^0$ …… (interior angle)

$115^0 + \angle Z = 180^0$

$\angle Z = 180^0 - 115^0$

$\angle Z = 65^0$

Q.36) The diagonal AC and BD of a parallelogram ABCD intersect each other at the point O, such that $\angle DAC = 30^0$ and $\angle AOB = 70^0$ then $\angle DBC$ = ?



Solution :

$\angle AOC = \angle DAC = 30^0$ …… (alternate interior angle)

$\Rightarrow \angle OCB = 30$

$\angle AOB + \angle BOC = 180^0$

$70^0 + \angle BOC = 180^0$

$\angle BOC = 180^0 - 70^0$

$\angle BOC = 110^0$

In Δ OBC

$\angle BOC + \angle OCB + \angle OBC = 180^0$

$110^0 + 30^0 + \angle OBC = 180^0$

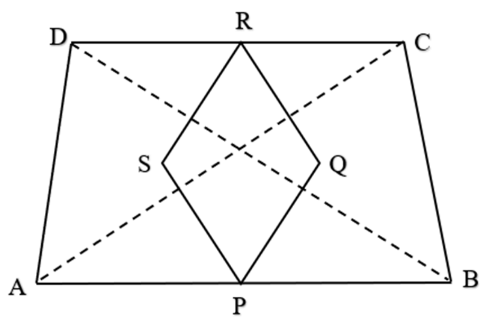
$140° + \angle OBC = 180^0$

$\angle OBC = 180^0 - 140^0$

$\therefore \angle OBC = 180^0 - 140^0$

$\therefore \angle OBC = 40^0$

Q. 37) In the adjoining figure, ABCD is a trapezium in which AB || DC and AD = BC. If P, Q, R, S be respectively the midpoints of BA, BD, CD, CA then show that PQRS is a rhombus.



Solution :

In Δ BDS , Q and R are the midpoints of BD and CD respectively.

$\therefore$ QR $\parallel$ BC and QR $= \frac{1}{2}$BC

Similarly, PS $\parallel$ BC and PS $= \frac{1}{2}$BC

$\therefore$ PS $\parallel$ QR and PS $\parallel$ QR ...... (each equal to $\frac{1}{2}$BC)

PQRS is a parallelogram

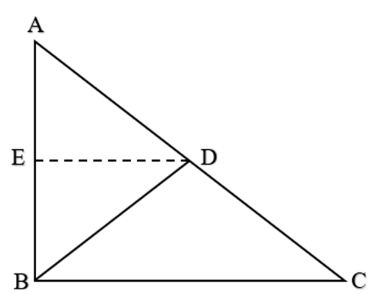
In Δ ACD, S and R are the midpoints of AC and CD respectively.

$\therefore$ SR $\parallel$ AD and SR $= \frac{1}{2}$AD $= \frac{1}{2}$ BC ..... ($\because$ AD = BC)

$\therefore$ PS = QR = SR = PQ

Hence PQRS is a rhombus.

Q. 38) Let ABC be a triangle, right angled at B and D be the midpoint of AC. Show that DA = DB = DC



Solution : Through D, draw DE $\parallel$ BC, meeting AB at E

Now, $\angle$ AED $= \angle$ ABC $= 90^0$ ..... (Corresponding angle)

$\therefore \angle$ BED $= \angle$ AED $= 90^0$ ... ($\angle$ AED + $\angle$ BED $= 180^0$)

Now, in Δ ABC , it is given that D is the midpoint of AC and DE ∥ BC ...... (by construction)

∵ E must be the midpoint of AB ..... (By converse of midpoint theorem)

∴ AE = BE

Now, in Δ AED and ΔBED, we have,

AE = BE ..... (proved)

ED = ED ...... (Common side)

$\angle$ AED = $\angle$ BED ..... (each equal to 90^0)

Δ AED ≅ Δ BED

∴ DA = DB

But, DA = DC ..... (∵ D is the midpoint of AC)

Hence, DA = DB = DC

Q. 39) Measure of angle of □ ABCD are in the ratio 3 : 4 : 5 : 6. Show that □ ABCD is a trapezium.

Solution : measure of $\angle$ A, $\angle$ B , $\angle$ C and $\angle$ D are $3x^0, 4x^0, 5x^0$ and $6x^0$ respectively.

Sum of angle of quadrilateral are 360^0

$3x^0 + 4x^0 + 5x^0 + 6x^0 = 360^0$

$18x^0 = 360$

$x = \frac{360}{18}$

$x = 20$

$\angle A = 3 \times 20 = 60$

$\angle B = 4 \times 20 = 80^0$

$\angle C = 5 \times 20 = 100^0$

$\angle D = 6 \times 20 = 120^0$

Now,

$\angle A + \angle D = 60^0 + 120^0 = 180^0$ …… (I) …. (Pair of interior angle)

$\angle C + \angle B = 100^0 + 80^0 = 180^0$ …… (II) ….. (Pair of interior angle)

∴ seg AB ∥ seg CD …… (III) ….. (Test of interior angle)

But,

$\angle A + \angle B = 60 + 80 = 140^0 \neq 180^0$ ….. (IV)

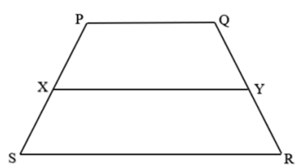
$\angle C + \angle D = 100^0 + 120^0 = 220^0 \neq 180^0$ …… (V)

Seg BC is seg AD are not similar ….. (VI)

….. (From (IV) and (V))

∴ □ ABCD is trapezium ….. (from (III) and (VI))

Q. 40) In trapezium PQRS, side PQ ∥ side RS, seg PS and seg QR are the midpoints of X and Y, PQ = 15 cm SR = 20 cm then find XY.



Solution : In trapezium PQRS,

Seg PQ ∥ seg RS

Point X is the midpoint of seg PS

Point Y is the midpoint of seg QR.

$\therefore XY = \frac{1}{2}(PQ + RS)$

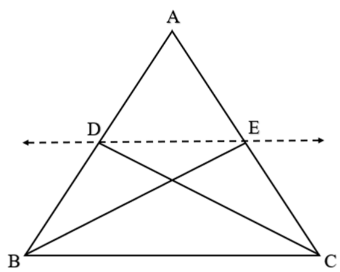
$= \frac{1}{2}(15 + 20)$

$= \frac{1}{2}\, 35$

XY = 11.5 cm

∴ XY = 11.5 cm

Q. 41) In Δ ABC median of ABC is seg CD and seg BE then prove that ED = $\frac{1}{2}$ BC



Solution : In Δ ABC ,

Point D is the midpoint of seg AB ….. (seg CD is the median)

Point E is the midpoint of seg AC ….. (Seg BE is the median)

∴ DE = $\frac{1}{2}$ BC ….. (By midpoint theorem)

Hence, its proved.

Q. 42) Δ LMN, seg LM and LN are midpoint of D and E respectively, then DE = 5.8 cm. Find MN.

Solution : In Δ LMN,

Point D is the midpoint of seg LM and point E is the midpoint of seg LN.

∴ DE = $\frac{1}{2}$MN …… (By midpoint theorem)

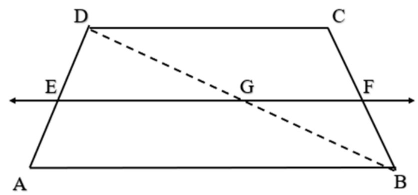
MN = 2DE

= 2 × 5.8

= 11.6 unit

∴ MN = 11.6 unit.

Q. 43) In □ ABCD, AB ∥ CD, point E is the midpoint of side AD. A line passing through point E and parallel to AB intersects side BC at point F. Prove that, F is the midpoint of seg BC.



Solution :

Given : □ ABCD. AB ∥ CD, point E is the midpoint of side AD. A line passing through point E and parallel to AB intersects side BC at point F.

To prove : point F is the midpoint of seg BC.

Construction : Draw diagonal BD.

Proof : In Δ ABD,

Seg EG ∥ seg AB

Point E is midpoint of seg AD

∴ Point G is the midpoint of seg BD ….. (Converse of midpoint theorem)

Side EF ∥ side AB. But AB ∥ CD ….. (Given)

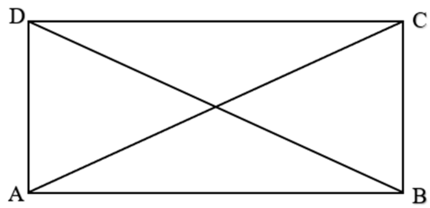
∴ Seg EF ∥ seg CD

In Δ BDC seg EF ∥ seg CD

Point G is midpoint of seg BD

∴ Point F is the midpoint of seg BC ….. (Converse of midpoint theorem)

Q. 44) If the diagonals of a parallelogram are equal then show that it is a rectangle.



Solution : A parallelogram ABCD such that AC = BD.

In Δ ABC and Δ DCB

AC = DB …… (Given)

AB = DC …… (Opposite side of parallelogram)

BC = CB …… (Common)

Δ ABC ≅ Δ DCB …… (By S-S-S Test)

There corresponding parts are equal.

∠ ABC ≅ ∠ DCB ….. (I)

∴ AB ∥ DC and BC is a transversal ….. (∵ ABCD is a parallelogram)

$\angle ABC + \angle DCB = 180^0$ …… (interior opposite angles are supplementary)

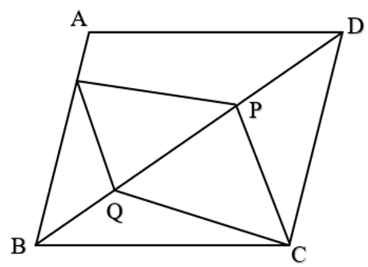
From (1) and (2) we have,

$\angle ABC = \angle DCB = 90^0$

i.e. ABCD is parallelogram having an angle equal to 90^0

∴ ABCD is a rectangle.

Q. 45) In parallelogram ABCD, two points P and Q taken on diagonal BD such that DP = BQ .Show that Δ APD ≅ Δ CQB.



Solution : Parallelogram ABCD. BD is a diagonal and P and Q such that

PD = QB …… (Given)

To Prove : Δ APD ≅ Δ CQB.

Proof:

Δ APD ≅ Δ CQB

∴ AD ∥ BC and BD is transversal

….. (∵ ABCD is parallelogram)

∴ ∠ ADB = ∠ CBD ….. (interior alternate angle)

⇒ ∠ ADP = ∠ CBQ

Now Δ APD and Δ CQB we have

AD = CB ….. (Opposite side of parallelogram)

PD = QB ….. (given)

∴ ∠ CBQ = ∠ ADP ….. (Proved)

∴ Δ APD ≅ Δ CQB ….. (SAS test)

Q. 46) ABCD is rectangle in which diagonal AC bisect ∠A as well as ∠C. Show that ABCD is a square.

Solution : AB ∥ DC and transversal AC intersect them

∴ ∠ACD = ∠CAB ….. (Alternat interior angle)

But, ∠CAB = ∠CAD ….. (AC bisects ∠A)

∴ ∠ACD = ∠CAD

∴ AD = CD ….. (sides opposite to equal angles of a triangle are equal)

∴ ABCD is a square.

Q. 47) Prove that each angle of a rectangle is a right angle.

Solution :

Given : ABCD is a rectangle with $\angle A = 90^0$

Prove : $\angle A = \angle B = \angle C = \angle D = 90^0$

Proof : ABCD is rectangle

∴ ABCD is a parallelogram

∴ AD ∥ BC ….. (Opposite side of a parallelogram are parallel and a transversal AB intersect them)

∴ $\angle A + \angle B = 180^0$

$90^0 + \angle B = 180^0$ …… (∵ ∴ $\angle A = 90^0$ given)

$\angle B = 180^0 - 90^0$

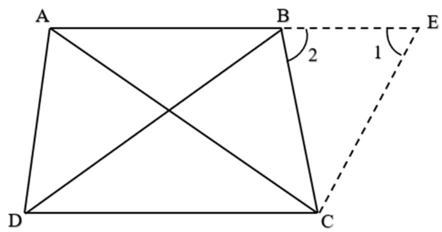
$\angle B = 90$

∴ ABCD is a parallelogram and opposite angles of a parallelogram are equal.

∴ $\angle C = \angle A = 90^0$
and $\angle D = \angle B = 90^0$
Hence $\angle A = \angle B = \angle C = \angle D = 90^0$

Q. 48) ABCD is a trapezium in which AB ∥ CD and AD = BC.

Show that ∠ A = ∠ B



Solution :

Construction : Through C. Draw CE ∥ DA to intersect AB produced at E

Proof : AB ∥ CD …… (given)

and AD ∥ CE …… (by construction)

∴ AECD is a parallelogram …… (A quadrilateral is a parallelogram if both the pairs of opposite sides are parallel)

∴ AD = EC …. (Opposite sides of a parallelogram are equal)

But AD = BC ….. (Given)

∴ BC = EC

∴ ∠ 1 = ∠ 2

… (Angles opposite to equal sides of a triangle are equal)

$\angle B + \angle 2 = 180^0$ ….. (I) …. (linear pair)

∵ AD ∥ EC …. (by construction)

and AE intersects them

$\angle A + \angle 1 = 180^0$ ….. (II)

From (I) and (II)

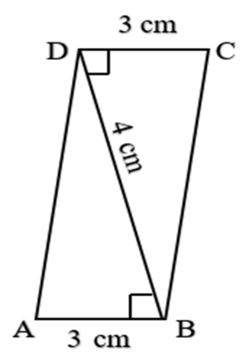
$\angle B + \angle 2 = \angle A + \angle 1$

But $\angle 1 = \angle 2$ ….. (Proved above)

$\angle B = \angle A$

$\Rightarrow \angle A = \angle B$

Q. 49) ABCD is a quadrilateral and BD is one of its diagonal as shown in figure. Show that ABCD is parallelogram and find its area.



Solution :

Given : ABCD is a quadrilateral and BD is one of its diagonal

Prove : ABCD is parallelogram and to determine its area.

Proof : $\angle ABD = \angle BDC$ ….. (given)

But these angles form a linear pair of equal alternate interior angles for line AB, DC and a transversal BD.

$\therefore AB \parallel DC$

Also, AD = DC (3 cm) ….. (given)

Hence quadrilateral ABCD is a parallelogram A quadrilateral is a parallelogram ,if its one pair of opposite sides are parallel and equal.

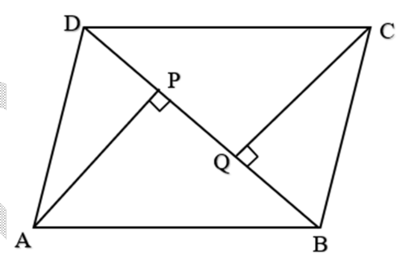
Now,

Area of parallelogram ABCD = Base × corresponding angle

$= 3 \times 4$

$= 12\ cm^2$

Q. 50) ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively. Show that Δ APB ≅ Δ CQD



Solution :

Given : ABCD is a parallelogram and AP and PQ are perpendiculars from vertices A and C on diagonal BD respectively.

Prove : Δ APB ≅ Δ CQD

Proof : In Δ APB and Δ CQD

AB = CD …. (Opposite sides of parallelogram ABCD)

∠ ABP = ∠ CDQ ….. (∵ AB ∥ DC and transversal BD intersect them)

∠ APB = ∠ CQD ….. (Each 90^0)

∴ Δ APB ≅ Δ CQD ….. (By AAS test)