CHAPTER – 6

TRIGONOMETRY

LONG QUESTIONS AND ANSWERS

Q. 1

Eliminate θ, if x = *c* sec θ, y = *d* tan θ

SOLUTION:

If x = *c* sec θ, then $\frac{x}{c}$ = sec θ … (1)

Similarly, if y = *d* tan θ, then $\frac{y}{d}$ = tan θ … (2)

$\sec^2\theta = 1 + \tan^2\theta$ … (3)

By putting values from (1) & (2)

$$\left(\frac{x}{c}\right)^2 - \left(\frac{y}{d}\right)^2 = 1$$

$$\therefore \frac{x^2}{c^2} - \frac{y^2}{d^2} = 1$$

Q. 2

If θ is an acute angle, and sin θ = $\frac{8}{17}$, then find cos θ

SOLUTION:

θ is an acute angle

Hence, θ lies in first quadrant

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \left[\frac{8}{17}\right]^2$$

$$= 1 - \left[\frac{64}{289}\right]$$

$$= \left[\frac{289 - 64}{289}\right]$$

$$= \frac{225}{289}$$

$$\cos\theta = \sqrt{\frac{225}{289}} = \frac{15}{17}$$

Q. 3

If θ is an acute angle, and $\sin\theta = \frac{7}{25}$, then find cos θ

SOLUTION:

θ is an acute angle

Hence, θ lies in first quadrant

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \left[\frac{7}{25}\right]^2$$

$$= 1 - \left[\frac{49}{625}\right]$$

$$= \left[\frac{625 - 49}{625}\right]$$

$$= \frac{576}{625}$$

$$\cos\theta = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

Q. 4

Eliminate θ, if x = *p* sec θ, y = *q* tan θ

SOLUTION:

If x = *p* sec θ, then $\frac{x}{p} = \sec\theta$ **… (1)**

Similarly, if y = *q* tan θ, then $\frac{y}{q} = \tan\theta$ **… (2)**

$\sec^2\theta = 1 + \tan^2\theta$ **… (3)**

By putting values from (1) & (2)

$$\left[\frac{x}{p}\right]^2 - \left[\frac{y}{q}\right]^2 = 1$$

$$\frac{x^2}{p^2} - \frac{y^2}{q^2} = 1$$

Q. 5

Prove that

cosec A – cot A = $\frac{sin\ A}{1+cos\ A}$

SOLUTION:

LHS = cosec A – cot A = $\frac{1}{sinA} - \frac{cosA}{sinA}$

$$= \frac{1 - cosA}{sinA}$$

$$= \left(\frac{1 - cos\ A}{sin\ A}\right) \times \frac{(1 + cos\ A)}{(1 + cos\ A)}$$

$$= \frac{1 - cos^2 A}{sin\ A\ (1 + cos\ A)}$$

$$= \frac{sin^2\ A}{sin\ A\ (1 + cos\ A)}$$

$$= \frac{sin\ A}{(1 + cos\ A)}$$

= RHS

Thus proved that cosec A – cot A = $\frac{sinA}{cosA}$

Q. 6

Prove that

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

SOLUTION:

$$\text{LHS} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A) \times (1 + \sin A)}{(1 - \sin A) \times (1 + \sin A)}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \text{RHS}$$

Thus proved that $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

Q. 7

Prove that

$$\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$$

SOLUTION:

$$\text{LHS} = \sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sqrt{\frac{(1 - \sin A) \times (1 - \sin A)}{(1 + \sin A) \times (1 - \sin A)}}$$

$$= \sqrt{\frac{(1 - \sin \mathrm{A})^2}{\cos^2 A}}$$

$$= \frac{1 - \sin A}{\cos A}$$

$$= \sec A - \tan A$$

$$= \text{RHS}$$

Thus proved that $\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$

Q. 8

Prove that

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$$

SOLUTION:

$$\text{LHS} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{(1 - \cos A) \times (1 - \cos A)}{(1 + \cos A) \times (1 - \cos A)}}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}}$$

$$= \frac{1 - \cos A}{\sin A}$$

= cosec A + cot A

= RHS

Thus proved that $\sqrt{\frac{1 - \cos A}{1 + \cos A}}$ = cosec A – cot A

Q. 9

Prove that

$$\frac{1 + \sin\theta}{1 - \sin\theta} = (\sec\theta + \tan\theta)^2$$

SOLUTION:

$$\text{LHS} = \frac{1 + \sin\theta}{1 - \sin\theta} = \frac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{(1 + \sin\theta)^2}{1 - \sin^2 A}$$

$$= \frac{(1 + \sin\theta)^2}{\cos^2 A}$$

$$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta + \tan\theta)^2$$

= RHS

Thus proved that $\frac{1 + \sin\theta}{1 - \sin\theta} = (\sec\theta + \tan\theta)^2$

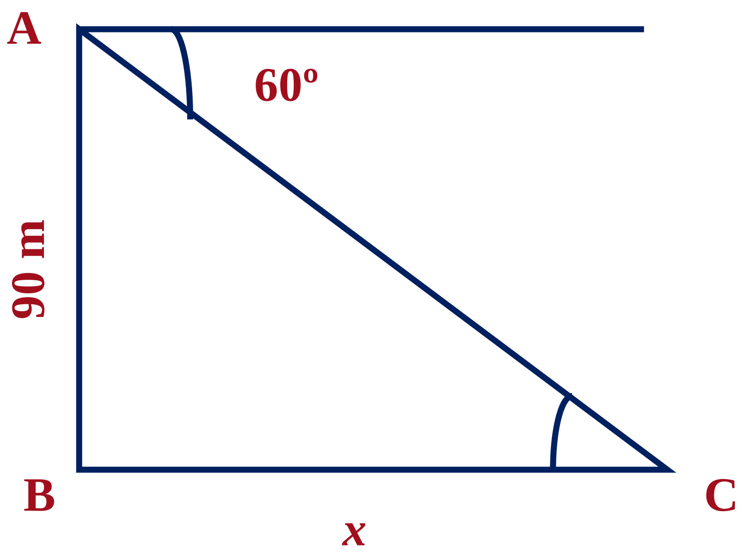
Q. 10

From the top of a lighthouse, an observer looking at the ship makes and angle of depression of 60º. If the height of light house is 90 meters, find how far the ship is from lighthouse $(\sqrt{3} = 1.73)$

SOLUTION:

Let AB be the lighthouse and C be the position of the ship from the light house

Let *x* be the distance of ship from the lighthouse



In the right ΔABC,

$$\tan 60^0 = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{90}{BC}$$

$$x = \frac{90}{\sqrt{3}}$$

$$x = \frac{9 \times 10}{\sqrt{3}}$$

$$x = 30\sqrt{3}$$

$$x = 30 \times 1.73$$

$$x = 30 \times 1.73$$

$$x = 51.9$$

Thus the ship is 51.9 meters from lighthouse

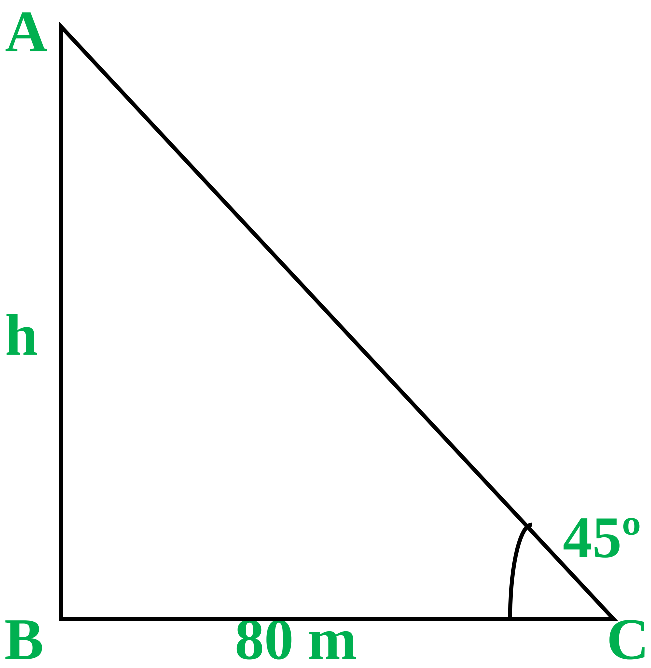
Q. 11

A person is standing at a distance of 80 meters from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.

SOLUTION:

Let AB be the church and C be the position of the person from the church.

Suppose the height of the church be *h* metres.



In right angle ΔABC,

$\tan 45^0 = \frac{AB}{BC}$

$\therefore 1 = \frac{h}{80}$

$\therefore 1 = \frac{h}{\sqrt{3}}$

$\therefore 1 = \frac{h}{80}$

$\therefore$ h = 80

Ans.: Thus the height of the church is 80 meters.

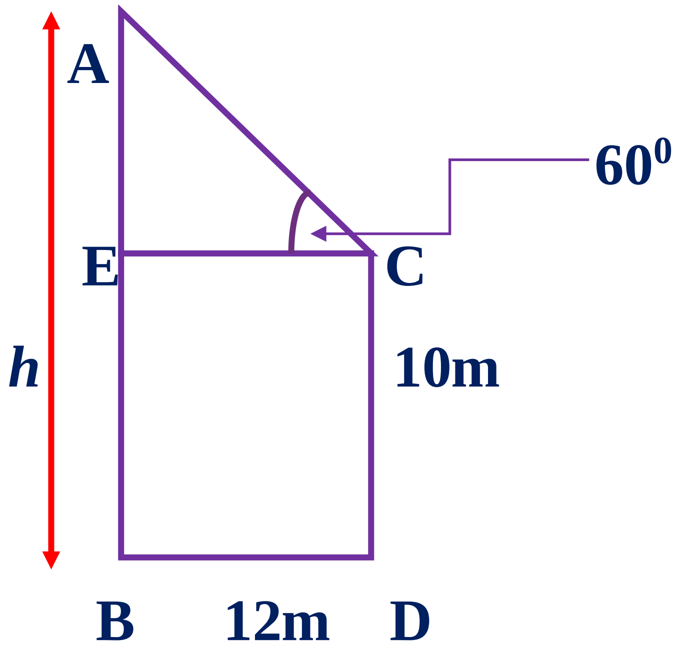
Q. 12

Two buildings are facing each other on a road of width 12 meter. From the top of first building, which is 10 meter high, the angle of elevation of the top of the second is found to be 60^0. What is the height of second building?

SOLUTION:

Let AB and CD be the two buildings standing on the road

Suppose the height of the second building be *h* meter



Here CD = 10 m, BD = 12m and CE ⊥ AB

AE = AB – EB = (h – 10) m … (since EB = CD)

CE = BD = 12m

In right Δ AEC

$$\tan 60^0 = \frac{AE}{CE}$$

$$\sqrt{3} = \frac{h - 10}{12}$$

$$\therefore h - 10 = 12\sqrt{3}$$

Ans.: Thus the height of the second building is

$(10 + 12\sqrt{3})$m

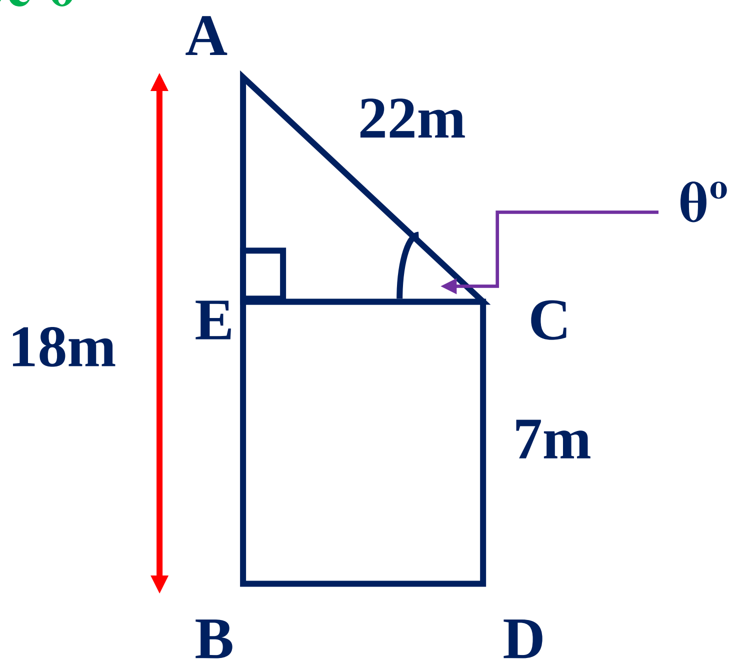
Q. 13

Two poles of height 18 meter and 7 meter are erected on a ground. The length of the wire fastened at their tops is 22 meters. Find the angle made by wire with horizontal.

SOLUTION:

Let AB and CD be the two poles standing on the ground.

Suppose, the angle made by wire with horizontal be θ



Here AB = 18m, CD = 7m

Length of the wire fastened at their tops = AC = 22

AB – EB = 18 – 7 = 11 … (EB = CD)

CE = BD = 12

In right Δ AEC

$$\sin \theta^0 = \frac{AE}{AC}$$

$$= \frac{11}{22}$$

$$= \frac{1}{2} \sin 30$$

$\therefore\ \theta = 30^0$

Ans.: Thus the angle made by the wire with the horizontal is 30^0

Q. 14

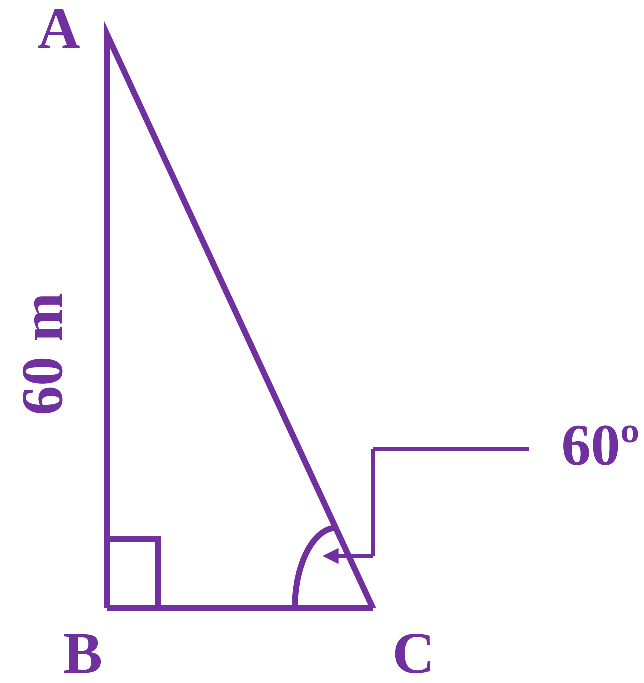
A kite is flying at a height of 60 meters above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60^0 with ground.

Assuming that the string is straight, find the length of the string ($\sqrt{3} = 1.73$)

SOLUTION:

Let AB be the height of kite above the ground and C be the position of the string attached to the kite which is tied at the ground.

Suppose the length of the string be *x* meters



Here AB = 60 and ∠ ACB = 60^0

$$\sin 60 = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{60}{x}$$

$$x = \frac{120}{\sqrt{3}}$$

$$= 40\sqrt{3}$$

$$= 40 \times 1.73$$

$$= 69.2$$

Ans.: Thus length of string is 69.2 meters.

Q. 15

If tan $\theta = \frac{3}{4}$ find the values of sec θ and cos θ

SOLUTION:

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\sec^2\theta = 1 + \left(\frac{3}{4}\right)^2$$

$$\sec^2\theta = 1 + \frac{9}{16}$$

$$= \frac{16 + 9}{16}$$

$$= \frac{25}{16}$$

$$\text{Sec } \theta = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Q. 16

If $cot = \frac{40}{9}$ find the values of cosec θ and sin θ

SOLUTION:

$$\text{cosec}^2 \theta = 1 + \cot^2\theta$$

$$\text{cosec}^2 \theta = 1 + \left(\frac{40}{9}\right)^2$$

$$\text{cosec}^2 \theta = 1 + \frac{1600}{81}$$

$$\text{cosec } \theta = \sqrt{\frac{1681}{81}}$$

$$\sin \theta = \sqrt{\frac{81}{1681}}$$

Q. 17

If $5 \sec \theta - 12 \text{ cosec } \theta = 0$

Find the values of sec θ and cos θ, sin θ

SOLUTION:

$$5\ sec\ \theta\ - 12\ cosec\ \theta = 0$$

$$5\ sec\ \theta = 12 cosec\theta$$

$$\frac{5}{cos\ \theta} = \frac{12}{sin\ \theta}$$

$$\frac{sin\ \theta}{cos\ \theta} = \frac{12}{5}$$

$$tan\ \theta\ = \frac{12}{5}$$

$$\sec^2\theta\ = 1 + \left(\frac{12}{5}\right)^2$$

$$\sec^2\theta = 1 + \frac{144}{25}$$

$$\sec^2\theta = \frac{25\ +\ 144}{25}$$

$$\sec^2\theta = \frac{169}{25}$$

$$sec\ \theta = \frac{13}{5}$$

$$cos\ \theta = \frac{1}{sec\ \theta} = \frac{5}{13}$$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$$

$$\sin^2\theta = 1 - \frac{25}{169}$$

$$\sin^2\theta = \frac{169 - 25}{169}$$

$$\sin^2\theta = \frac{144}{169}$$

$$\sin\theta = \sqrt{\frac{144}{169}}$$

$$\sin\theta = \frac{12}{13}$$

Q. 18

If $\tan\theta = 1$, find the values of $\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta}$

SOLUTION:

Given $\tan\theta = 1$

we know that, $\tan 45^0 = 1 \;\therefore \theta = 45^0$

Now, $\sin 45^0 = \frac{1}{\sqrt{2}}$

$$\cos 45^0 = \frac{1}{\sqrt{2}}$$

$$\sec 45^0 = \sqrt{2}$$

$$\text{cosec } 45^0 = \sqrt{2}$$

$$\therefore \frac{\sin\theta + \cos\theta}{\sec\theta + \text{cosec}\,\theta} = \frac{\sin 45 + \cos 45}{\sec 45 + \text{cosec}\,45}$$

$$= \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{\frac{2}{\sqrt{2}}}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}\sqrt{2}}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Q. 19

Prove that

$$\left(\frac{1}{\sin\theta} + \frac{1}{\tan\theta}\right)^2 = \frac{1 + \cos\theta}{1 + \sin\theta}$$

SOLUTION:

$$\text{LHS} = \left(\frac{1}{\sin\theta} + \frac{1}{\tan\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}}\right)^2$$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 + \cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta}$$

= RHS

Thus proved that $\left(\frac{1}{\sin\theta} + \frac{1}{\tan\theta}\right)^2 = \frac{1 + \cos\theta}{1 + \sin\theta}$

Q. 20

Prove that

$\sec^2\theta + \mathrm{cosec}^2\theta = \sec^2\theta \times \mathrm{cosec}^2\theta$

SOLUTION:

LHS = $\sec^2\theta + \mathrm{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\ \sin^2\theta}$$

$$= \frac{1}{\sin^2\theta\ \cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$$

$= \sec^2\theta \times \mathrm{cosec}^2\theta$

= RHS

Thus proved that $\sec^2\theta + \mathrm{cosec}^2\theta = \sec^2\theta \times \mathrm{cosec}^2\theta$

Q. 21

Prove that

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$$

SOLUTION:

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta\,(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{\cancel{(\sin\theta - \cos\theta)}(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{\sin\theta\cos\theta\,\cancel{(\sin\theta - \cos\theta)}}$$

$$= \frac{(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \tan\theta + 1 + \cot\theta$$

$$= 1 + \tan\theta + \cot\theta$$

$$= \text{RHS}$$

Thus proved that

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$$

Q. 22

If $\sec\alpha=\frac{2}{\sqrt{3}}$ then find value of $\frac{1-\operatorname{cosec}\alpha}{1+\operatorname{cosec}\alpha}$

SOLUTION:

If $\sec\alpha=\frac{2}{\sqrt{3}}$ then $\cos\alpha=\frac{\sqrt{3}}{2}$

$\alpha = 360^0 - 30^0$

$\alpha = 330^0$ or -30^0

$\operatorname{cosec}\alpha = \operatorname{cosec}(-30^0)$

$\operatorname{cosec}\alpha = -2$

$$\frac{1-\operatorname{cosec}\alpha}{1+\operatorname{cosec}\alpha}=\frac{1+2}{1-2}$$

$$=\frac{3}{-1}$$

$$=-3$$

$$\therefore \frac{1-\operatorname{cosec}\alpha}{1+\operatorname{cosec}\alpha}=-3$$

Q. 23

The terminal sides of an angle is passing through the point (-1,$\sqrt{3}$) then write the trigonometric ratio. (find sin θ, cos θ only)

SOLUTION:

The terminal side of the passes through (-1,$\sqrt{3}$)

Then x = -1, $y = \sqrt{3}$

r = $\sqrt{x^2 + y^2}$

$= \sqrt{(-1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

= 2

Suppose the measure of the angle is θ

Then $\sin\theta = \frac{\sqrt{3}}{-1}$

Q. 24

If cot θ = $\frac{-7}{24}$; then find the values of cosec θ and tan θ

SOLUTION:

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$= 1 + \left(\frac{-7}{24}\right)^2$$

$$= 1 + \left(\frac{49}{576}\right)$$

$$= \frac{576 + 49}{576}$$

$$= \frac{625}{576}$$

$$\operatorname{cosec}\theta = \frac{25}{24}$$

$$\tan\theta = \frac{1}{\cot\theta} = -\frac{24}{7}$$

Q. 25

Prove that

$(sin\ x + cosec\ x)^2 + (cos\ x + sec\ x)^2 =$

$tan^2x + cot^2x + 7$

SOLUTION:

LHS = $sin^2x + cosec^2x + 2\ sin\ x\ cosec\ x +$

$cos^2x + sec^2x + 2\ cos\ x\ sec\ x$

$= sin^2x + cos^2x + 2\ sin\ x\ cosec\ x +$

$cosec^2x + sec^2x + 2\ cos\ x\ secx$

$= 1 + 2(1)\ cosec^2x + sec^2x + 2(1)$

$= 1 + 4\ cosec^2x + sec^2x$

$= 5 + 1 + cot^2x + 1 + tan^2x$

$= 7 + cot^2x + tan^2x$

= RHS

Thus proved that

$(sin\ x + cosec\ x)^2 + (cos\ x + sec\ x)^2 =$

$tan^2x + cot^2x + 7$

Q. 26

If sec $x = -\frac{13}{12}$ then find value of tan x & cot x

SOLUTION:

$\sec^2\theta = 1 + \tan^2\theta$

$\therefore \left(\frac{-13}{12}\right)^2 = 1 + \tan^2\theta$

$\therefore \frac{169}{144} = 1 + \tan^2\theta$

$\therefore \tan^2\theta = \frac{169}{144} - 1$

$\therefore \tan^2\theta = \frac{169 - 144}{144}$

$\therefore \tan^2\theta = \frac{25}{144}$

$\therefore \tan\theta = \frac{5}{12}$ AND $\cot\theta = \frac{12}{5}$

Q. 27

Prove that

$$(1 + \tan\theta)^2 + (1 + \cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2$$

SOLUTION:

LHS = $(1 + \tan\theta)^2 + (1 + \cot\theta)^2$

$= (1 + \tan^2\theta + 2\tan\theta) + (1 + \cot^2\theta + 2\cot\theta)$

$= \sec^2\theta + 2\tan\theta + \operatorname{cosec}^2\theta + 2\cot\theta$

$= \sec^2\theta + \operatorname{cosec}^2\theta + 2(\tan\theta + \cot\theta)$

$= \sec^2\theta + \operatorname{cosec}^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \sec^2\theta + \operatorname{cosec}^2\theta + 2\left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta\sin\theta}\right)$

$= \sec^2\theta + \operatorname{cosec}^2\theta + 2\left(\frac{1}{\cos\theta\sin\theta}\right)$

$= \sec^2\theta + \operatorname{cosec}^2\theta + 2\sec\theta\operatorname{cosec}\theta$

$= (\sec\theta + \operatorname{cosec}\theta)^2$

= RHS

Thus proved that

$(1 + \tan\theta)^2 + (1 + \cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2$

Q. 28

Prove that

$\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

SOLUTION:

LHS = $\sec\theta\,(1-\sin\theta)(\sec\theta\ +\tan\theta)$

$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1}{\cos\theta}\ +\ \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1\ +\ \sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\cos^2\theta}(1+\sin\theta)(1-\sin\theta)$$

$$= \frac{1\ -\ \sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta}$$

= 1

= RHS

Thus proved that $\sec\theta\,(1-\sin\theta)(\sec\theta+\tan\theta)=1$

Q. 29

Prove that

$\cot^2\theta-\tan^2\theta=\mathrm{cosec}^2\theta-\sec^2\theta$

SOLUTION:

LHS = $\cot^2\theta - \tan^2\theta$

$= (\operatorname{cosec}^2\theta - 1) - (\sec^2\theta - 1)$

$= \operatorname{cosec}^2\theta - 1 - \sec^2\theta + 1$

$= \operatorname{cosec}^2\theta - \sec^2\theta$

= RHS

Thus proved that $\cot^2\theta - \tan^2\theta = \operatorname{cosec}^2\theta - \sec^2\theta$

Q. 30

Prove that

$$\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = 2\, cosec^2\theta$$

$$\text{LHS} = \frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta}$$

$$= \frac{(1 + \sin\theta) + (1 - \sin\theta)}{(1 - \sin\theta)\,(1 + \sin\theta)}$$

$$= \frac{1 + \sin\theta + 1 - \sin\theta}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{2}{\sin^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$$= 2\,\mathrm{cosec}^2\theta$$

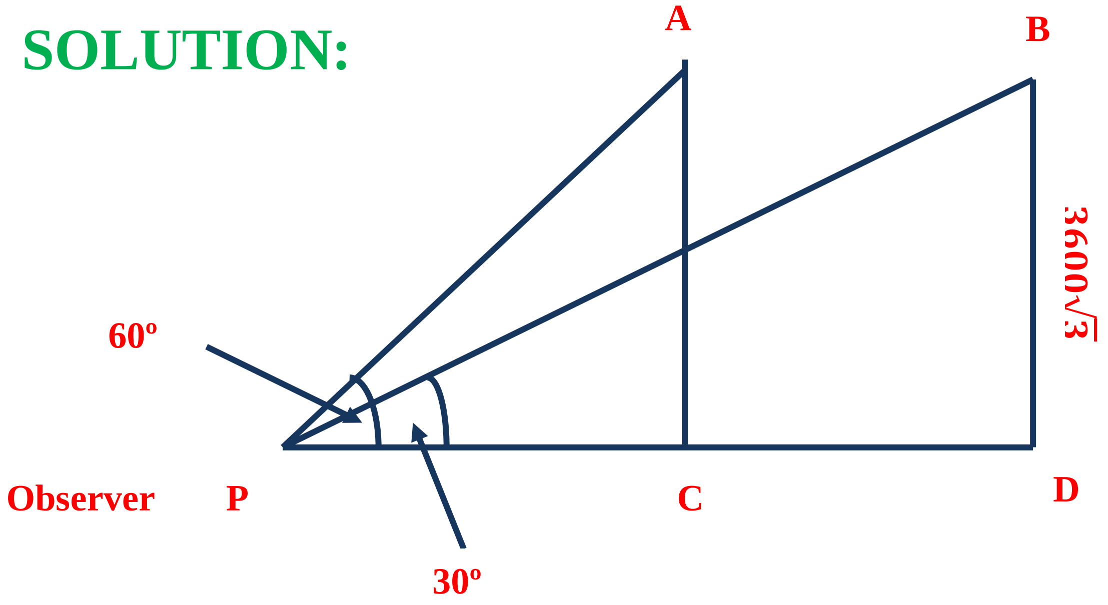
$$= \text{RHS}$$

Thus proved that $\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = 2\sec^2\theta$

Q. 31

While take off of a Jet aeroplane pilot makes angle of elevation of 60^0. After 30 seconds of takeoff the angle of elevation is 30^o, then find the speed of Jet plane at a constant height of $3600\sqrt{3}$ meters.

SOLUTION:



Observer standing at P observes the plane at the angle of elevation is 60 degrees

After 30 seconds the plane reaches at B point and the angle of elevation at that point is 30°

$AC \perp PD$ and $BD \perp PD$

Height $AC = BD = 3600\sqrt{3}$ meter

In Δ APC $\cot 60^0 = \frac{PC}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{PC}{AC}$$

$$PC = 3600\sqrt{3} \times \frac{1}{\sqrt{3}}$$

$$PC = 3600 \text{ meters}$$

In Δ BPD, $\cot 30^0 = \frac{PD}{BD}$

$$\sqrt{3} = \frac{PD}{BD}$$

$$PD = \sqrt{3}\ BD$$

$$PD = \sqrt{3} \times 3600\sqrt{3}$$

$$= 10800 \text{ meter}$$

The distance travelled by the plane within 30 seconds = 10800 – 3600 = 7200

Speed of Jet plane = $\frac{\text{DISTACE}}{\text{TIME}}$

$= \frac{7200}{30}$

= 240 meters / second

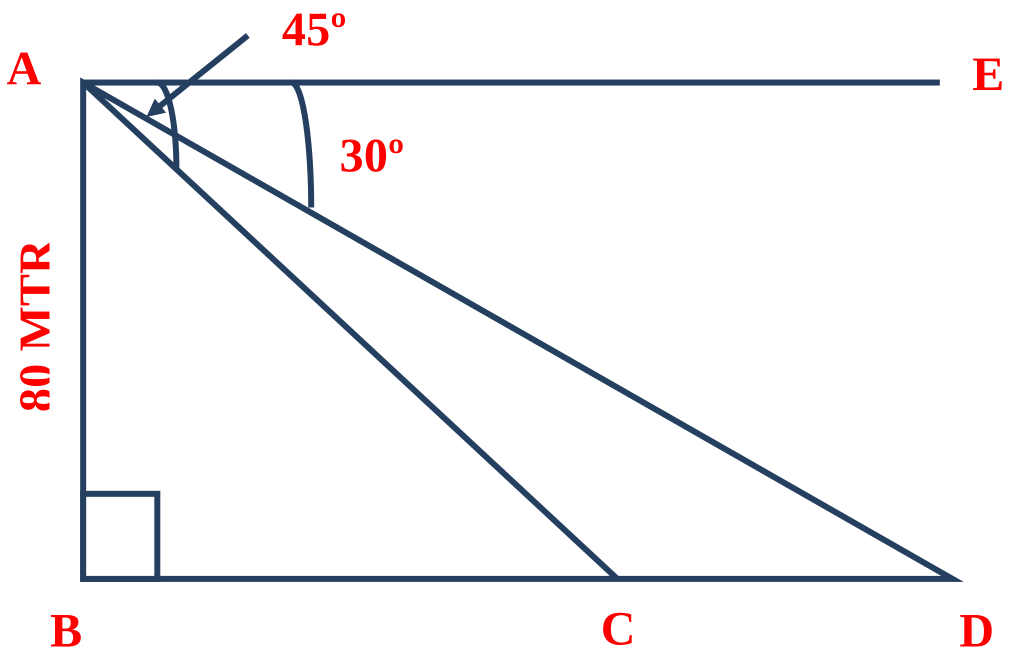
Speed of Jet plane per Hour = 240 x $\frac{18}{5}$ **km per hr**

= 860 km / hr

Q. 32

From the top of a lighthouse of 80 meters height, an observer looking at two ships at the same side of lighthouse, making angles of depression 45^0 and 30^0, then find the distance between the ships (assume the ships and base of lighthouse in one line).

SOLUTION:



AB is the lighthouse of 80 meter height

C and D are two ships. AE is the horizontal line

Angle of depression $\angle EAC = 45^{\circ}$ $\angle EAD = 30^{\circ}$

$\angle BCA \cong \angle EAC$ and $\angle BDA \cong \angle EAD$

$\angle BCA = 45^{\circ}$ and $\angle BDA = 30^{\circ}$

In ΔABC $\angle ABC = 90^{\circ}$ and $\angle BCA = 45^{\circ}$

Now, $\tan C = \frac{AB}{BC}$

$\therefore \tan 45^{\circ} = \frac{AB}{BC}$

$\therefore\ 1 = \frac{80}{BC}$

BC = 80 meter

In ΔABD, $\angle ABD = 90^\circ$, $\angle ADB$

$\tan D = \frac{AB}{BD}$

$\tan 30^\circ = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{80}{BD}$

BD = $80\sqrt{3}$ metre

CD = BD – BC

CD = $80\sqrt{3} - 80$

CD = $80(\sqrt{3} - 1)$

Ans.: Distance between two ships is $80(\sqrt{3} - 1)$

Q. 33

If α is an acute angle and if $3\sin\alpha - 4\cos\alpha = 0$, then find the values of $\tan\alpha$, $\sec\alpha$

SOLUTION:

α is an acute angle hence in the first quadrant

$3\sin\alpha = 4\cos\alpha$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{4}{3}$$

$$\tan\alpha = \frac{4}{3}$$

$$\sec^2\alpha = 1 + \tan^2\alpha$$

$$\sec^2\alpha = 1 + \left(\frac{4}{3}\right)^2$$

$$= 1 + \frac{16}{9}$$

$$= \frac{16+9}{9}$$

$$= \frac{25}{9}$$

$$\sec\alpha = \sqrt{\frac{25}{9}}$$

$$= +\frac{5}{3}$$

Q. 34

Eliminate θ if x = r cos θ, y = r sin θ

SOLUTION:

x = r cos θ, y = r sin θ

$x^2 = r^2\cos^2\theta,\ y^2 = r^2\sin^2\theta$

$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$x^2 + y^2 = r^2\,(\cos^2\theta + \sin^2\theta)$

$x^2 + y^2 = r^2$

Q. 35

If 8 sin x – cos x = 4, then find the possible values of sin x

SOLUTION:

8 sin x – 4 = cos x

$(8 \sin x - 4)^2 = \cos^2 x$

$64 \sin^2 x - 64 \sin x + 16 = \cos^2 x$

$64 \sin^2 x - 64 \sin x + 16 = 1 - \sin^2 x$

$65 \sin^2 x - 64 \sin x + 15 = 0$

$65 \sin^2 x - 25 \sin x - 39 \sin x + 15 = 0$

$5 \sin x\,(13 \sin x - 5) - 3\,(5 \sin x - 3) = 0$

$13 \sin x - 5 = 0$ or $5 \sin x - 3 = 0$

$13 \sin x = 5$ or $5 \sin x = 3$

$\sin x = \frac{13}{9}$ or $\sin x = \frac{3}{5}$

Q. 36

Prove that

$$\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2 \operatorname{cosec} A$$

SOLUTION:

$$\text{LHS} = \frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1}$$

$$= \frac{\tan A(\sec A + 1) + \tan A\,(\sec A - 1)}{(\sec A - 1)(\sec A + 1)}$$

$$= \frac{\tan A \sec A + \tan A + \tan A \sec A - \tan A}{(\sec^2 A - 1)}$$

$$= \frac{2 \tan A \sec A}{(\tan^2 A)}$$

$$= \frac{2 \sec A}{(\tan A)}$$

$$= \frac{2 \cos A}{\cos A(\sin A)}$$

$$= 2 \operatorname{cosec} A$$

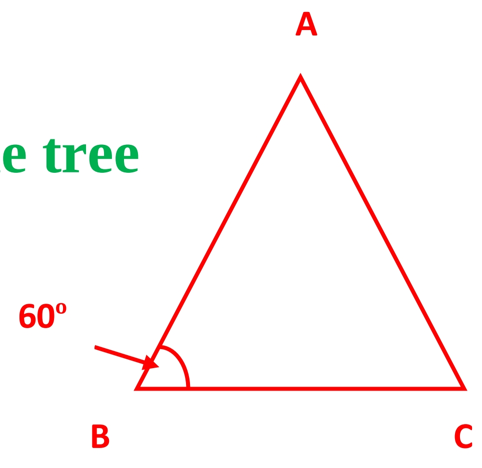
= RHS

Thus proved that $\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2 \operatorname{cosec} A$

Q. 37

A boy is standing 60 meters away from a tree, if he makes an angle of elevation of 60° while observing the tip of the tree, find the height of the tree. ($\sqrt{3}$ = 1.73)

SOLUTION:



Suppose AB = h, height of the tree

Angle of elevation 60°

C is observer (boy)

$$\frac{AB}{CB} = \tan 60^0$$

AB = CB tan60^0

= 60 tan60^0

= 60 $\sqrt{3}$

= 60×1.73

AB = 103.80 meter

Height of the tree is 103.8 meter

Q. 38

Terminal side of an angle is passing through the point (-1, $\sqrt{3}$) then write the trigonometric ratios.

SOLUTION:

x = -1 and y = $\sqrt{3}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$= \sqrt{1 + 3}$

$= 2$

Suppose the measure of the angle is θ

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\cos\theta = \frac{x}{r} = -\frac{1}{2}$

Q. 39

PROVE THAT

$cosec^4\alpha\,(1 + cos^2\alpha)(1 - cos^2\alpha) - 2\,cot^2\alpha = 1$

SOLUTION:

LHS = $cosec^4\alpha\,(1 + cos^2\alpha)(1 - cos^2\alpha) - 2\,cot^2\alpha$

$$= \frac{1}{sin^4\alpha}(1 + cos^2\alpha)sin^2\alpha - 2cot^2\alpha$$

$$= \frac{1}{sin^2\alpha}(1 + cos^2\alpha) - 2cot^2\alpha$$

$$= \frac{1}{sin^2\alpha}(1 + cos^2\alpha) - 2\frac{cos^2\alpha}{sin^2\alpha}$$

$$= \frac{1 - cos^2\alpha}{sin^2\alpha}$$

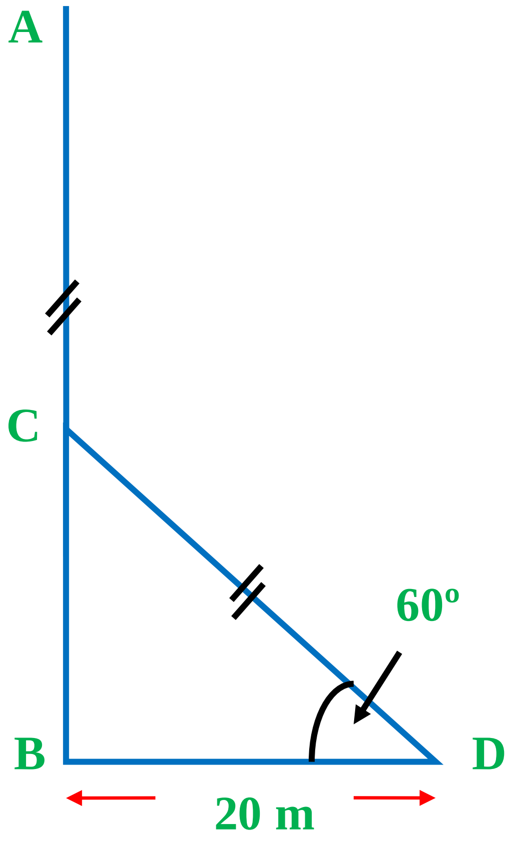
= RHS

Thus proved that

$$cosec^4\alpha\ (1 + cos^2\alpha)(1 - cos^2\alpha)\ - 2\ cot^2\alpha = 1$$

Q. 40

A storm broke a tree and the treetop rested 20 meter from the base of the tree, making an angle of 60º with the horizontal. Find the height of the tree.



SOLUTION:

AB = height of tree

Suppose the tree broken at point C and its top touches the ground at D

AC is broken part of the tree which takes position CD such that $\angle CDB = 60^0$

AC = CD … (i)

BD = 20 m

in right ΔCDB

$\tan 60 = \frac{BC}{BD}$ by definition

$\therefore \sqrt{3} = \frac{BC}{20}$

$\therefore BC = 20\sqrt{3}$

Also, $\cos 60 = \frac{BC}{CD}$

$\therefore \frac{1}{2} = \frac{BC}{CD}$ (by definition)

$\therefore \frac{1}{2} = \frac{20}{CD}$

$\therefore CD = 40$ m

AC = 40 m …. From (i)

Now, AB = AC + BC (A – C – B)

$= 40 + 20\sqrt{3}$

$= 40 + 34.6$

$= 74.6$

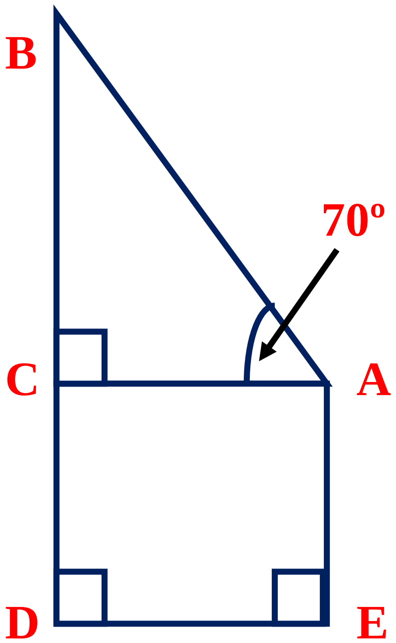
Ans.: The height of the tree is 74.6 m

Q. 41

A ladder on the platform of a bridge van can be elevated at an angle of 70º to the maximum. The length of ladder can be extended up to 20 m above the ground, find the maximum height from the ground up to which the ladder can reach. (sin 70 = 0.94)

SOLUTION:

Let AB represent the length of the ladder and AE represent the height of the platform.



AB = length of ladder

AE = height of platform

Draw seg AC ⊥ seg BD

Angle of elevation = ∠ BAC

AB = 20 m

AE = 2 m

In right angle ΔABC

$$\sin 70 = \frac{BC}{AB}$$

$$\therefore\ 0.94 = \frac{BC}{AB}$$

$\therefore$ BC = 0.94 x 20

= 18.8 m

ACDE is rectangle

CD = AE = 2 m

Now BD = BC + CD

= 18.8 + 2

= 20.8

Ans.: The maximum height from the ground up to which the ladder can reach is 20.8 meters.

Q. 42

Find the value of $\dfrac{\sin^2 63 + \sin^2 27}{\cos^2 27 + \cos^2 73}$

SOLUTION:

$$\frac{sin^2 63 + sin^2 27}{cos^2 27 + cos^2 73} = \frac{[sin\,(90 - 27)]^2 + sin^2 27}{[cos\,(90 - 73)]^2 + cos^2 73}$$

$$= \frac{cos^2 27 + sin^2 27}{sin^2 73 + cos^2 73}$$

$$= \frac{1}{1}$$

$$= 1$$

Q. 43

Find the value of:

$$\sin 25 \cos 65 + \cos 25 \sin 65$$

SOLUTION:

$$\sin 25 \cos 65 + \cos 25 \sin 65$$

$$= \sin 25 \cos(90 - 25) + \cos 25 \sin(90 - 25)$$

$$= \sin 25 \sin 25 + \cos 25 \cos 25$$

$$= \sin^2 25 + \cos^2 25$$

$$= 1$$

Q. 44

Find the value of: tan 48 tan 23 tan 42 tan 67

SOLUTION:

tan 48 tan 23 tan 42 tan 67

= tan (90 – 42) tan (90 – 67) tan 42 tan 67

= cot 42 cot67 tan 42 tan 67

= 1 x 1

= 1

Q. 45

Find the value of $\frac{\sin 18}{\cos 72}$

SOLUTION:

$$\frac{\sin 18}{\cos 72} = \frac{\sin (90-72)}{\cos 72}$$

$$= \frac{\cos 72}{\cos 72}$$

$$= 1$$

Q. 46

Find the value of $\frac{\tan 26}{\cot 64}$

SOLUTION:

$$\frac{\tan 26}{\cot 64} = \frac{\tan(90 - 64)}{\cot 64}$$

$$= \frac{\cot 64}{\cot 64}$$

$$= 1$$

Q. 47

Find the value of cos 48 – sin 42

SOLUTION:

cos (90 – 42) – sin 42 = sin 42 – sin 42

= 0

Q. 48

Find the value of cosec 31 – sec 59

SOLUTION:

cosec 31 – sec 59 = cosec (90 – 59) – sec59

= cosec (90 – 59) – sec 59

= sec 59 – sec 59

= 0

Q. 49

Find the value of angle A if Sec 4A = cosec (A – 20)

SOLUTION:

sec 4A = cosec (A – 20)

cosec (90 – 4A) = cosec (A – 20)

(90 – 4A) = (A – 20)

110 = 5A

A = 22

Q. 50

Find the value of angle A if $\tan 2A = \cot(A - 18)$

SOLUTION:

$$\tan 2A = \cot(A - 18)$$

$$\cot(90 - 2A) = \cot(A - 18)$$

90 – 2A = A – 18

3A = 90 + 18

3A = 108

A = 36

Q. 51

Prove that

$$(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$$

SOLUTION:

$$\text{LHS} = (\sec\theta + \tan\theta)(1 - \sin\theta)$$

$$= \left[\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right](1 - \sin\theta)$$

$$= \left[\frac{1 + \sin\theta}{\cos\theta}\right](1 - \sin\theta)$$

$$= \frac{1 - \sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta}$$

$= \cos\theta$

$= \text{RHS}$

Thus proved that

$(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

Q. 52

Prove That

$$\frac{1 + \sin A}{\cos A} = \frac{1 + \sin A + \cos A}{1 + \sin A - \cos A}$$

SOLUTION:

$1 - sin^2\theta = \cos^2\theta$

$(1 - \sin\theta)(1 + \sin\theta) = \cos A \cos A$

$$\frac{1 + \sin A}{\cos A} = \frac{\cos A}{1 - \sin A}$$

Now each ratio $= \frac{1 + \sin A + \cos A}{1 + \sin A - \cos A}$

Hence proved,

$$\frac{1+\sin A}{\cos A}=\frac{1+\sin A+\cos A}{1+\sin A-\cos A}$$