

## 6- Circle

### Extra Questions

Q. 1) Radius of a circle is 13cm. The length of the chord is 10 cm. Find the distance of the chord from the centre (3M)

Solution: - In the fig,

length of the chord,  $MN = 10$  cm and centre of the circle is A.

Seg  $AB \perp$  chord  $MN$

Perpendicular drawn from

the centre of the circle on

its chord bisects the chord

$\therefore MB = BN = 5$ cm

Radius of the circle is 13 cm..... (Given)

In right angled triangle  $ABN$ ,

$$AB^2 + BN^2 = AN^2 \quad \text{--- (by pythagoras theorem )}$$

$$AB^2 + (5)^2 = (13)^2$$

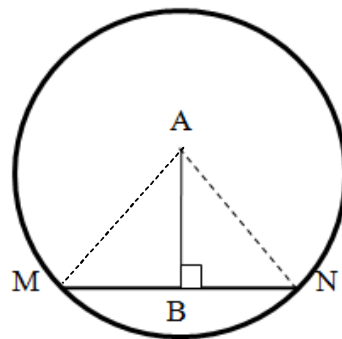
$$AB^2 = (13)^2 - (5)^2$$

$$AB^2 = 169 - 25$$

$$AB^2 = 144$$

$$AB = \sqrt{144} = 12 \text{ cm}$$

$\therefore$  Hence, the distance of the chord from the centre of the circle is 12 cm



Q.2) In the given figure, O is the centre of the circle. AB is chord of the circle which is of 10 cm, here  $OP \perp AB$ . So, what is the length of ?(2M)

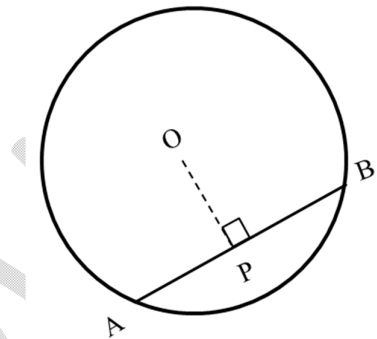
Ans :- given  $OP \perp AB$

P is centre point of the AB

$$\therefore AP = \frac{1}{2}AB$$

$$AP = \frac{1}{2} \times 10 \text{ -- } [\because AB = 10 \text{ cm given}]$$

$$AP = 5 \text{ cm.}$$



Q.3) Radius of the circle is 41 cm. If the distance of circle from centre to chord is 9 cm, then what is length of the chord?(3M)

Ans :- O is the center of the circle

$$\therefore \text{Radius } ON = 41 \text{ cm}$$

$$\therefore OP = 9 \text{ cm}$$

$$OP \perp MN$$

$$MN = PN$$

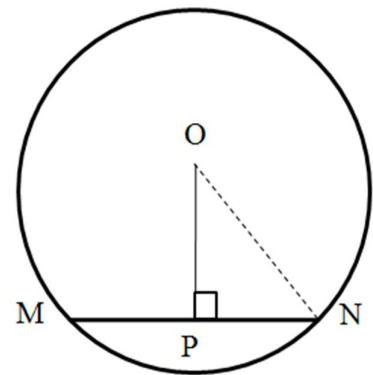
$\Delta OPN$  is right angle triangle

$$OP^2 + PN^2 = ON^2 \quad \text{-- (by Pythagoras theorem)}$$

$$(9)^2 + PN^2 = (41)^2$$

$$PN^2 = (41)^2 - (9)^2$$

$$PN^2 = 1681 - 81 = 1600$$



$$PN = \sqrt{1600}$$

$$PN = 40 \text{ cm}$$

$$\therefore MP = PN = 40 \text{ cm}$$

$$\text{and } MN = MP + PN \quad \dots (\text{M - P - N})$$

$$MN = 40 + 40$$

$$MN = 80 \text{ cm}$$

$\therefore$  length of chord is 80 cm.

Q.4) In the given figure, O is the centre of the circle. If  $AB = 8\text{cm}$  and  $OP = 3\text{cm}$ , then find radius of the circle (3M)

Solution %seg  $OP \perp$  seg AB

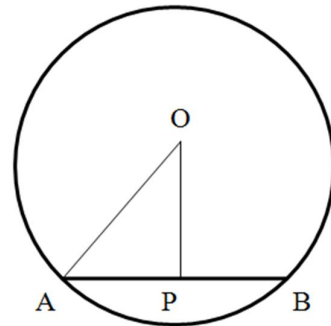
P is the center point of the AB

$$\begin{aligned} AP &= \frac{1}{2}AB \\ &= \frac{1}{2} \times 8 \dots\dots [AB = 8\text{cm given}] \\ &= 4 \text{ cm} \end{aligned}$$

In the  $\Delta OPA$ , To find OA

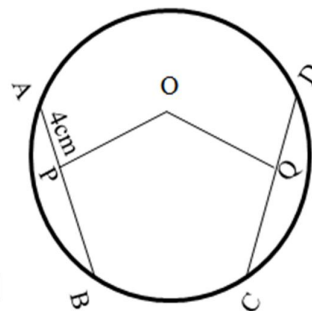
$$\begin{aligned} OA &= \sqrt{OP^2 + AP^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \text{ cm} \end{aligned}$$

$\therefore$  Seg OA = 5 cm



Radius of circle = 5 cm.

Q. 5) In the given figure, O is the center of the circle and  $OP = OQ$ . If  $AP = 4$  cm, then find length of CD(3M)



Solution :  $OP = OQ$

Chord AB and chord CD are at same distance from the center.

$$\overline{AB} = \overline{CD}$$

$$\frac{1}{2}\overline{AB} = \frac{1}{2}\overline{CD}$$

$$\overline{AP} = \frac{1}{2}\overline{CD} \quad \dots \text{(P is the center point of the AB-)}$$

$$4 \text{ cm} = \frac{1}{2}\overline{CD}$$

$$\overline{CD} = 2 \times 4 \text{ cm}$$

$$CD = 8 \text{ cm.}$$

The length of the CD = 8 cm

Q.6) The distance from the center O to the chord AB is 3cm. If the length of the chord AB is 8 cm. then find the diameter of circle. (3M)

Solution:

Given: chord AB = 8 cm

To find : Diameter of circle (d) = ?

Distance from center point of circle to chord-

$$AB = AM + MB$$

$$= AM + AM \quad \dots (\because AM = MB)$$

$$\therefore AB = 2AM$$

$$AM = MB = \frac{8}{2} = 4 \text{ cm}$$

In right angle  $\triangle OMB$   $\angle OMB = 90^\circ$

$$OM = 3 \text{ cm and } MB = 4 \text{ cm}$$

$$OB^2 = OM^2 + MB^2 \quad \dots (\text{by Pythagoras theorem})$$

$$\therefore OB^2 = (3)^2 + (4)^2$$

$$\therefore OB^2 = 9 + 16$$

$$\therefore OB^2 = 25$$

$$\therefore OB = 5 \text{ cm}$$

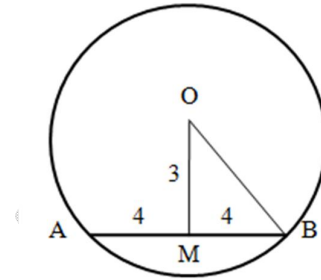
$$OB = \text{Radius of circle} = 5 \text{ cm}$$

$$\text{Diameter of circle} = 2 \times \text{Radius}$$

$$= 2 \times 5$$

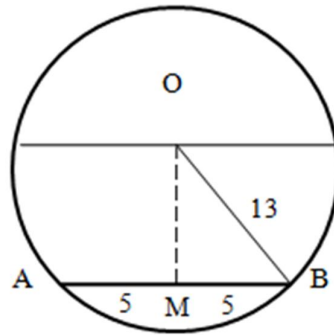
$$= 10 \text{ cm}$$

$\therefore$  The diameter of circle is 10 cm.



Q. 7) Diameter of circle is 26 cm. Length of the chord is 10 cm. Find the distance of the chord from the center. (3M)

Solution:



Radius of circle =  $OB = \frac{26}{2} = 13$ , chord  $AB = 10$  cm

Length of the chord  $AB = 10$  cm

$$\therefore AM = MB = \frac{AB}{2} = \frac{10}{2} = 5 \text{ cm.}$$

In the right angle triangle  $OMB$ ,  $\angle OMB = 90^\circ$

$OB = 13$  cm,  $MB = 5$  cm,  $OM = ?$

$$OB^2 = OM^2 + MB^2 \quad \dots (\text{Pythagoras Theorem})$$

$$(13)^2 = OM^2 + (5)^2$$

$$OM^2 = (13)^2 - (5)^2$$

$$OM^2 = 169 - 25$$

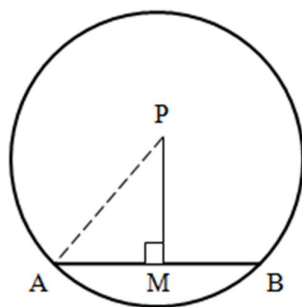
$$OM^2 = 144$$

$$OM = 12 \text{ cm}$$

$\therefore$  The distance of chord from the center is 12 cm.

Q.8) Radius of circle is 25 cm. length of the chord is 48 cm.

find the distance from P(3M)



Solution:

Radius of the circle  $PA = 25$  cm

$PM \perp AB$

$PM$  bisect  $AB$

$AM \cong MB$

$$\therefore AM = MB = \frac{1}{2} AB = \frac{1}{2} \times 48 = 24 \text{ cm}$$

In right angle,  $\Delta PMA$ ,  $\angle PMA = 90^\circ$

According to Pythagoras Theorem ,

$$PA^2 = PM^2 + AM^2$$

$$(25)^2 = PM^2 + (24)^2$$

$$PM^2 = (25)^2 - (24)^2$$

$$PM^2 = (25 + 24)(25 - 24)$$

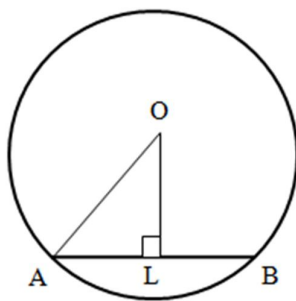
$$PM^2 = (49) - (1)$$

$$PM = 7 \times 1$$

$$PM = 7 \text{ cm}$$

$\therefore$  Distance of the chord from the center is 7 cm.

Q.9) Radius of circle is 13 cm , length of chord is 10 cm. Find the distance of the chord from the center. (3M)



Solution:

AB is the chord, Radius = 13 cm

OA = 13 cm

OL  $\perp$  AB

$$AL = \frac{1}{2}AB = \frac{1}{2} \times 10 = 5 \text{ cm}$$

In right angle triangled.  $\Delta OLA$

$$OA^2 = OL^2 + AL^2$$

$$OL^2 = OA^2 - AL^2$$

$$OL^2 = (13)^2 - (5)^2$$

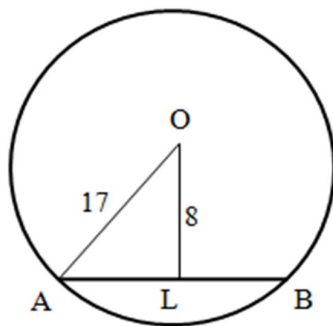
$$OL^2 = 169 - 25$$

$$OL^2 = 144$$

$$\therefore OL = 12 \text{ cm}$$

$\therefore$  The distance of chord from the center is 12 cm

Q. 10) Find the length of the chord where distance of the chord from the center is 8 cm and radius 17 cm(3M)





Solution:

$OL \perp AB$

Radius = 17 cm

If ,  $OL \perp AB$  then  $OL = 8$  cm,

$OA = 17$  cm.

In right angle triangle,  $\Delta OLA$

By Pythagoras theorem

$$OA^2 = OL^2 + AL^2$$

$$AL^2 = OA^2 - OL^2$$

$$= (17)^2 - (8)^2$$

$$= (17 + 8)(17 - 8)$$

$$= 225$$

$$\therefore AL = 15 \text{ cm}$$

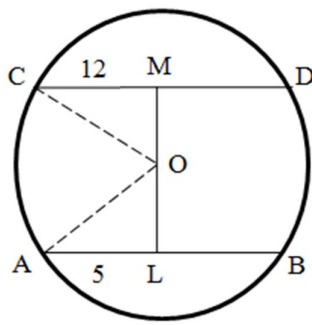
$\therefore$  seg OL bisect the chord AB

$$AB = 2 \times AL$$

$$= 2 \times 15$$

$$= 30 \text{ cm}$$

Q. 11) Seg AB and seg CD are two parallel chord and present at opposite side of the center of circle. If  $AB = 10$  cm,  $CD = 24$  cm. find the distance between AB and CD and also find the radius of the circle. (3M)



**Solution:**

AB and CD are two chords  
of the circle  $AB \parallel CD$ .

$AB = 10$  cm, and  $CD = 24$  cm

$OL \perp AB$  and  $OM \perp CD$

Join the OA and OC

then  $OA = OC = r$  cm ... (radius of circle)

$OL \perp AB$  and  $OM \perp CD$  and  $AB \parallel CD$

$\therefore LM = 17$  cm

$OL = x$   $OM = (17 - x)$  cm

$AL = \frac{1}{2} AB = \left(\frac{1}{2} \times 10\right)$  cm = 5 cm

and

$CM = \frac{1}{2} CD = \left(\frac{1}{2} \times 24\right)$  cm = 12 cm

In right angle triangle,  $\triangle OLA$

$$OA^2 = OL^2 + AL^2$$

$$OA^2 = x^2 + (5)^2 \quad \dots (i)$$

In right angle triangle,  $\Delta OMC$

$$OC^2 = OM^2 + CM^2$$

$$OC^2 = (17 - x)^2 + (12)^2 \quad \dots (ii)$$

From (i) and (ii)

$$x^2 + (5)^2 = (17 - x)^2 + (12)^2$$

$$x^2 + 25 = (x^2 - 34x + 289) + 144$$

$$x^2 + 25 = x^2 - 34x + 433$$

$$34x = 433 - 25$$

$$34x = 408$$

$$x = \frac{408}{34} = 12 \text{ cm}$$

$$x = 12$$

Put value of x in equation (i)

$$OA^2 = (12)^2 + (5)^2$$

$$OA^2 = 144 + 25$$

$$OA^2 = 169$$

$$OA = 13 \text{ cm}$$

$\therefore$  Radius of circle is 13 cm.

Q.12) In the given figure. O is the center of the circle where  $AB = 16 \text{ cm}$ .  $CD = 14 \text{ cm}$  and seg  $OM \perp$  seg  $AB$  and seg  $ON \perp$  seg  $CD$ , if  $OM = 6 \text{ cm}$  then seg  $ON$  (3M)

Solution:

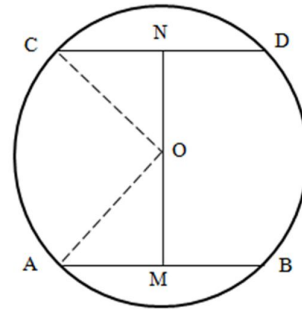
Given :  $AB = 16$  cm

seg  $OM \perp$  seg  $AB$

Solution:

A - M - B

$$\begin{aligned}\therefore AM &= BM = \frac{1}{2}AB \\ &= \frac{1}{2} \times 16 = 8 \text{ cm}\end{aligned}$$



In right angle triangle,  $\Delta OMA$

$$OA^2 = OM^2 + AM^2 \quad \dots (\text{Pythagoras theorem})$$

$$OA^2 = 6^2 + 8^2$$

$$OA^2 = 36 + 64$$

$$OA^2 = 100$$

$$\therefore OA = 10 \text{ cm}$$

$$\therefore OA = OC = 10 \text{ cm}$$

$$CN = \frac{1}{2}CD$$

$$CN = \frac{1}{2} \times 14 = 7 \text{ cm}$$

In right angle triangle,  $\Delta CON$

$$CO^2 = ON^2 + CN^2 \quad \dots (\text{Pythagoras theorem})$$

$$10^2 = ON^2 + 7^2$$

$$100 = ON^2 + 49$$

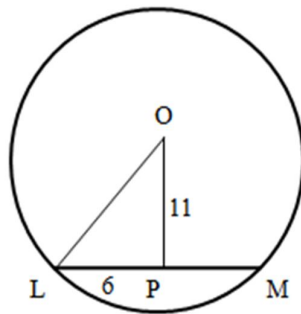
$$ON^2 = 100 - 49$$

$$ON^2 = 51$$

$$\therefore ON = \sqrt{51} \text{ cm}$$

$$\therefore \text{length of seg ON} = \sqrt{51} \text{ cm}$$

Q. 13) The length of the one chord in the circle is 12 cm and distance of chord from the center is 11 cm. Find the radius of the given circle. (3M)



Solution:

$$OP \perp LM$$

P is the center point of the LM.

$$\therefore LP = \frac{1}{2}LM = \frac{1}{2} \times 12 = 6 \text{ cm}$$

In  $\triangle OPL$ ,

$$\text{Radius} = \sqrt{OP^2 + LP^2}$$

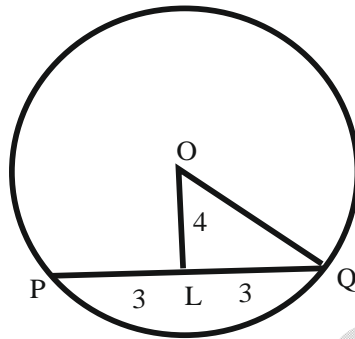
$$OL = \sqrt{(11)^2 + (6)^2}$$

$$= \sqrt{121 + 36}$$

$$= \sqrt{157} \text{ cm}$$

$$\therefore \text{Radius of the circle is } \sqrt{157} \text{ cm}$$

Q. 14) The distance of the chord PQ from the center midpoint is 4 cm. The length of chord PQ is 6 cm , then find the diameter of the circle. (3M)



Solution:

Chord PQ = 6 cm

Diameter of circle (d) = ?

PQ = PL + LQ

PQ = PL + PL ... ( $\because$  LQ = PL)

$\therefore$  PQ = 2PL

$\therefore$  PQ = QL =  $\frac{6}{2}$  = 3 cm

In  $\Delta$  OLQ,  $\angle$  OLQ =  $90^\circ$

OL = 4 cm and LQ = 3 cm

$$OQ^2 = OL^2 + LQ^2$$

$$OQ^2 = (4)^2 + (3)^2$$

$$OQ^2 = 16 + 9$$

$$OQ^2 = 25$$

$$\therefore OQ = 5 \text{ cm}$$

$$\therefore OQ = \text{Radius of circle} = 5 \text{ cm}$$

$$\begin{aligned}\therefore \text{Diameter of circle} &= 2 \times \text{Radius} \\ &= 2 \times 5 \\ &= 10 \text{ cm}\end{aligned}$$

$\therefore$  The diameter of the circle is 10 cm.

Q. 15) In the given figure the chord AB and chord CD are parallel to each other Radius of circle is 5cm AB = 8cm, CD = 6 cm If  $OP \perp AB$   $OQ \perp CD$  find PQ (3M)

Solution:

$$\therefore AP = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$$\therefore CQ = \frac{1}{2}CD = \frac{1}{2} \times 6 = 3 \text{ cm}$$

Join OA and OC

Then  $OA = OC = 5 \text{ cm}$  ... (Given-)

In  $\Delta OPA$

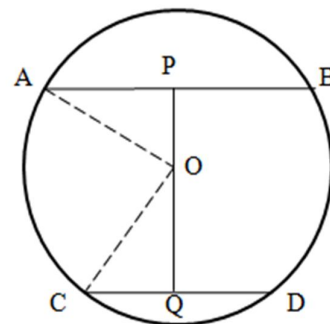
$$\begin{aligned}OP^2 &= OA^2 - AP^2 \\ &= (5)^2 - (4)^2 \\ &= 25 - 16\end{aligned}$$

$$OP^2 = 9$$

$$OP = 3 \text{ cm}$$

In  $\Delta OQC$

$$OQ^2 = OC^2 - CQ^2$$



$$= (5)^2 - (3)^2$$

$$= 25 - 9$$

$$OQ^2 = 16$$

$$OQ = 4 \text{ cm}$$

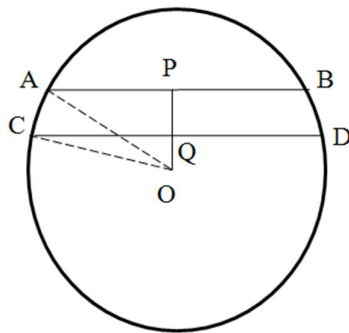
Now,  $OP \perp AB$ ,  $OQ \perp CD$  and  $AB \perp CD$ , Point P, O, Q are congruent.

$$\therefore PQ = OP + OQ = (3 + 4) = 7 \text{ cm}$$

$\therefore$  Then length of PQ is 7 cm.

Q. 16 In the given fig. AB and CD are two chord are at equidistance radius is 5cm.  $AB = 6 \text{ cm}$  and  $CD = 8 \text{ cm}$  If  $OP \perp AB$  and  $OQ \perp CD$  Then find length of PQ (3M)

Solution :



$OP \perp AB$ ,  $OQ \perp CD$  and  $AB \parallel CD$

Point O, Q, P are collinear

$$\therefore AP = \frac{1}{2}AB = \frac{1}{2} \times 6 = 3 \text{ cm}$$

$$\therefore CQ = \frac{1}{2}CD = \frac{1}{2} \times 8 = 4 \text{ cm}$$

Join OA and OC



In  $\Delta OPA$ ,

$$\begin{aligned}OP^2 &= OA^2 - AP^2 \\&= (5)^2 - (3)^2 \\&= 25 - 9\end{aligned}$$

$$OP^2 = 16$$

$$OP = 4 \text{ cm}$$

In  $\Delta OQC$ ,

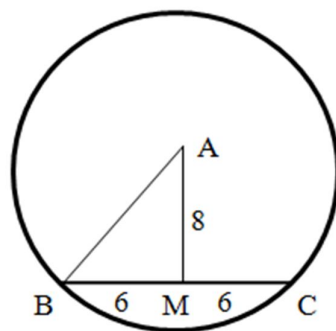
$$\begin{aligned}OQ^2 &= OC^2 - CQ^2 \\&= (5)^2 - (4)^2 \\&= 25 - 16\end{aligned}$$

$$OQ^2 = 9$$

$$OQ = 3 \text{ cm}$$

$$\therefore PQ = OP - OQ = 4 - 3 = 1 \text{ cm}$$

Q. 17) Length of chord for a given circle is 12 cm, distance from the center to midpoint point of chord is 8 cm, find the radius of the circle. (3M)



Solution %  $AM \perp BC$

A is the midpoint of BC

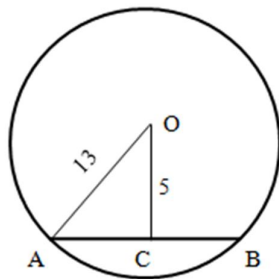
$$BM = MC = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6 \text{ cm}$$

In  $\Delta AMB$

$$\begin{aligned} \text{Radius } AB &= \sqrt{AM^2 + BM^2} \\ &= \sqrt{(8)^2 + (6)^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \text{ cm} \end{aligned}$$

$$AB = 10 \text{ cm} .$$

Q. 18) The length of the the radius of a circle is 13 cm and distance between them is 5 cm. Then what is the length of the chord? (2M)



Solution : In right angled  $\Delta OAC$

$$\begin{aligned} AC &= \sqrt{OA^2 - OC^2} \\ &= \sqrt{(13)^2 - (5)^2} \end{aligned}$$

$$= \sqrt{169 - 25}$$

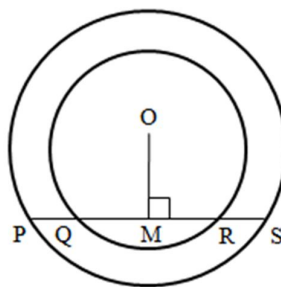
$$= \sqrt{144} \text{ cm}$$

$$\therefore AC = 12 \text{ cm}$$

$$AB = 2 \times AC = 2 \times 12 = 24 \text{ cm}$$

$\therefore$  Length of the chord AB is 24 cm.

Q. 19) In the given figure ,  $OM \perp PS$  If  $PS = 20 \text{ cm}$  and  $QR = 15 \text{ cm}$  then find PQ (3M)



Solution :  $OM \perp PS$

$$QM = MR = \frac{1}{2} QR = \frac{15}{2} \text{ cm}$$

$$PM = MS = \frac{1}{2} PS = \frac{20}{2} = 10 \text{ cm}$$

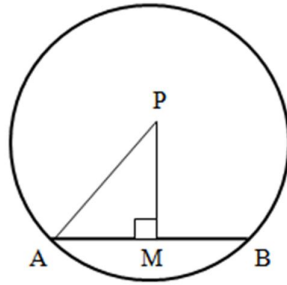
$$\therefore PQ = QM - PM$$

$$= \frac{10-15}{2}$$

$$= \frac{20-15}{2} = \frac{5}{2} = 2.5$$

$$\therefore PQ = 2.5 \text{ cm}$$

Q.20) In a circular pond of water, a ship is at center P which is at distance of 10m from the bridge. The bridge across the pond is of length 40m. find the distance travelled by the ship to reach one end of the pond.(3M)



Solution :

From figure Distance of ship from center of pond bridge,

$$\therefore AB = 40 \text{ m}$$

$$PM \perp AB$$

$$\therefore AM = \frac{1}{2} AB \quad \dots \text{ ( PM bisects chord AB )}$$

$$\therefore AM = \frac{1}{2} \times 40$$

$$\therefore AM = 20 \text{ m}$$

$$\therefore PM = 10 \text{ m}$$

In  $\triangle PMA$

$$PA^2 = PM^2 + AM^2 \dots\dots\dots \text{ (Pythagoras theorem )}$$

$$= 10^2 + 20^2$$

$$= 100 + 400$$

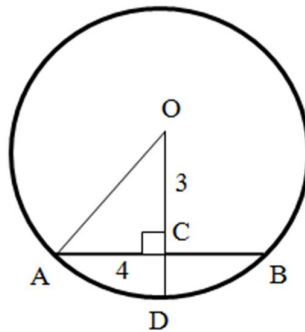
$$PA^2 = 500$$

$$PA = 10\sqrt{5}$$

$$PA = 10\sqrt{5} = \text{Radius of water pond}$$

The ship has to travel  $10\sqrt{5}$  m to reach the end point of the circular pond

Q.21) In the given fig. If  $OA = 5\text{ cm}$ ,  $AB = 8\text{ cm}$   $OD \perp AB$   
then find  $CD = ?$  (3M)



Solution :  $OD \perp AB$  [  $OD$  bisect  $AB$  ]

$$AC = CB = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4\text{ cm}$$

In  $\triangle OCA$

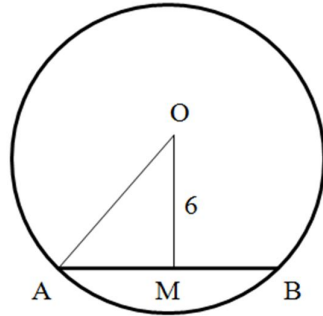
$$OA^2 = OC^2 + AC^2 \dots (\text{Pythagoras theorem})$$

$$\begin{aligned} OC^2 &= AC^2 - OA^2 \\ &= (4)^2 - (5)^2 \\ &= 16 - 25 \\ &= 9 \end{aligned}$$

$$\therefore OC = 3\text{ cm}$$

$$\therefore CD = OD - OC = 5 - 3 = 2\text{ cm}$$

Q. 22) A circle has centre O and chord AB. The distance from center to chord is 6 cm. The radius of the circle is 6 cm less than the length of chord AB. Find the length of the chord AB.(3M)



Solution :

O is the centre of the circle

seg OM  $\perp$  seg AB

OM = 6 cm

Radius OA = (Chord AB – 6)

Let AB = x

Radius OA = x – 6 (By given condition)

Also,  $AM = \frac{1}{2} AB$

$$= \frac{1}{2} x \text{ ( seg OM } \perp \text{ seg AB)}$$

$$= \frac{x}{2}$$

In  $\Delta OMA$ ,  $\angle OMA = 90^\circ$

OA = (x – 6) cm , OM = 6 cm

$$AM = \frac{1}{2} x \text{ cm}$$

By Pythagoras theorem

$$OA^2 = AM^2 + OM^2$$

$$(x - 6)^2 = \left(\frac{x}{2}\right)^2 + (6)^2$$

$$x^2 - 12x + 36 = \frac{x^2}{4} + 36$$

$$4(x^2 - 12x) = x^2$$

$$4x^2 - 48x = x^2$$

$$4x^2 - x^2 = 48x$$

$$4x^2 - x^2 = 48x$$

$$3x^2 = 48x$$

$$3x = 48$$

$$x = \frac{48}{3}$$

$$x = 16$$

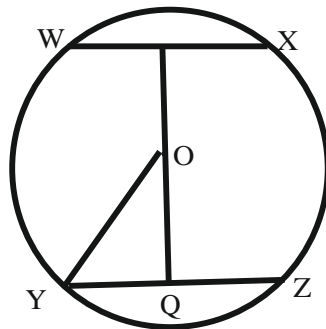
$$\therefore \text{Chord AB} = x = 16 \text{ cm}$$

$$\therefore \text{Radius of circle} = x - 6$$

$$= 16 - 6$$

$$= 10 \text{ cm}$$

Q. 23) The radius of the one circle is 13 cm. In the circle there are two chords each of length 24 cm, then find the distance of chords from the center point of the circle. (3M)



Solution : O is the center of the circle

Radius OY = 13 cm

seg OQ  $\perp$  chord YZ

Y - Q - Z and seg YQ  $\cong$  seg ZQ

$$\therefore YQ = ZQ = \frac{1}{2} YZ = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$\therefore YQ = ZQ = 12 \text{ cm}$$

In right angled triangle ,  $\Delta OYQ$

$$OY^2 = OQ^2 + YQ^2 \dots (\text{Pythagoras theorem})$$

$$13^2 = OQ^2 + 12^2$$

$$169 = OQ^2 + 144$$

$$OQ^2 = 169 - 144$$

$$OQ^2 = 25$$

$$\therefore OQ = 5 \text{ cm}$$

In fig,

Chord YZ = Chord WX ..... [Given]

seg OP  $\perp$  chord WX

$\therefore$  seg OP  $\perp$  seg OQ .... [ congruent side-)

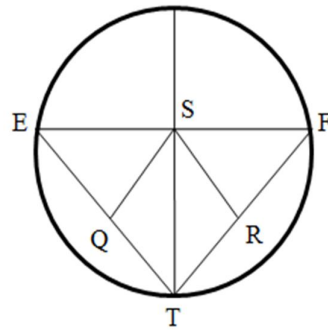
$$\text{seg OP} = 5 \text{ cm}$$

Q.24) A circle with center S having two equidistance chord

ET and chord FT then prove that

ray TS is bisector of  $\angle ETF$  (4M)





**Solution :** A circle with center S

where,

Chord ET = chord FT

seg SQ  $\perp$  chord ET

seg SR  $\perp$  chord FT

To prove : Ray TS is bisector of  $\angle$  ETF

Here, chord ET = chord FT ..... Side (I)

seg SQ  $\perp$  chord ET .....Side (II)

seg SR  $\perp$  chord FT .....Side (III)

$\therefore$  SQ = SR .... [equidistance chord from the center of the circle)

$$TQ = QE = \frac{1}{2} TE \dots\dots (VI)$$

$$TR = RF = \frac{1}{2} TF \dots\dots (V)$$

$$\therefore TQ = QE = TR = RF \dots (I), (V), (VI)$$

In  $\Delta$  SQT and  $\Delta$  SRT

$$SQ = SR \dots\dots\text{from (IV)}$$

$$TQ = TR \quad \dots \text{ from (IV)}$$

$$ST = ST \quad \dots \text{ common side}$$

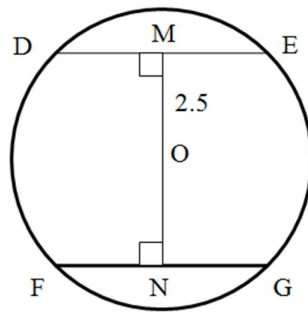
$$\therefore \triangle SQT \cong \triangle SRT \quad \dots [\text{S-S-S test}]$$

$$\therefore \angle SQT \cong \angle SRT \quad \dots \text{ (VIII)}$$

$$\therefore \angle STE \cong \angle STF \quad \dots \text{ from (VIII)} \dots \dots \dots \text{ (IX)}$$

$$\therefore \text{Ray TS is bisector of } \angle ETF$$

Q. 25) In the given figure, point O is the midpoint of the circle.  $DE = FG$ , If  $OM = 2.5$  find the  $ON$  (2M)



Solution :

$$\text{chord } DE \cong \text{chord } FG$$

$$OM \perp DE, ON \perp FG$$

Shown in the figure.

$$OM = 2.5 \text{ cm}$$

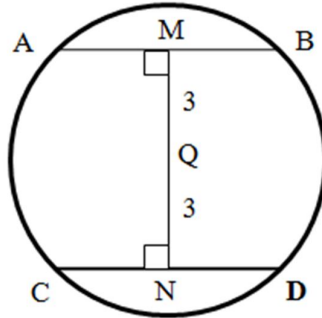
Means Distance of DE from O is 2.5 cm

$$ON = 2.5 \text{ cm}$$

OM and ON (equidistance )

$$ON = 2.5 \text{ cm}$$

Q. 26) In the given figure, point Q is the midpoint of the circle  
 $\cdot$   $QM \perp AB$ ,  $QN \perp CD$ .  $QM = QN = 3$  cm. If  $AM = 2$  cm, then  
 find the length of CD (3M)



Solution :

$$QM \perp AB, QN \perp CD$$

$$QM = QN = 3 \text{ cm}$$

$$\therefore AB = CD \quad \dots \text{(I) [equidistance chord]}$$

$$AM = \frac{1}{2} AB \quad \dots \text{(II)}$$

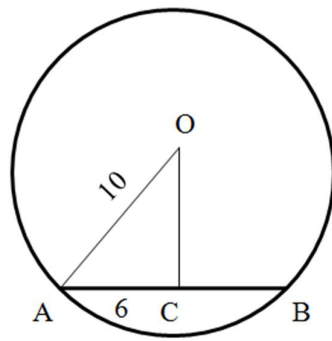
$$AM = 2$$

$$\therefore AM = \frac{1}{2} AB = 2 \quad \dots \text{(II)}$$

$$\therefore AB = 4 \text{ cm} \quad \dots \text{(III)}$$

$$\therefore CD = 4 \text{ cm} \quad \dots \text{from (I) and (III)}$$

Q. 27) In the given figure, O is the center of the circle,  $OA = 10$  cm, chord  $AB \perp OC$ ,  $OC = 8$  cm, then find length of AB (3M)



Solution :

$OC \perp AB$

$AC = CB$

In  $\Delta OAC$ ,

$$AC^2 = OA^2 - OC^2$$

... (by Pythagoras theorem )

$$AC = \sqrt{OA^2 - OC^2}$$

$$= \sqrt{(10)^2 - (8)^2}$$

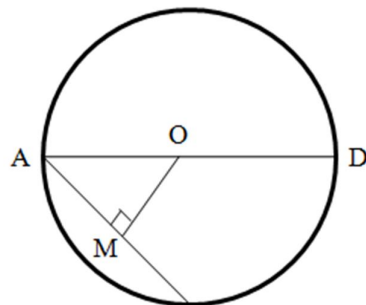
$$= \sqrt{100 - 64}$$

$$= \sqrt{36}$$

$$AC = 6 \text{ cm}$$

$$\therefore AB = 2 \times AC = 2 \times 6 = 12 \text{ cm}$$

Q. 28) A circle has diameter AD and chord AB. If AD = 34 cm and AB = 30 cm ,then find the distance of AB from the center?(4M)



Solution :

Diameter of circle = AD = 34 cm

$$\begin{aligned}\therefore AO = OD &= \frac{1}{2} \times AD \\ &= \frac{1}{2} \times 34 \\ &= 17 \text{ cm}\end{aligned}$$

Chord AB = 30 cm

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 30 = 15 \text{ cm}$$

In  $\triangle OMA$ ,

$$OA^2 = OM^2 + AM^2 \quad \dots \text{(by Pythagoras theorem)}$$

$$(17)^2 = OM^2 + (15)^2$$

$$289 = OM^2 + 225$$

$$OM^2 = 289 - 225$$

$$OM^2 = 64$$

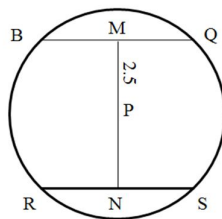
$$OM = 8 \text{ cm}$$

$\therefore$  Distance of AB from the midpoint is 8 cm.

Q. 29) In the given figure. P is the midpoint of the circle.

Chord BQ = 5 cm. chord BQ  $\cong$  chord RS and PM  $\perp$  BQ,

PN  $\perp$  RS , PM = 2.8 cm then find the length of PN.(4M)



Solution:

In the circle, P is the center of the circle.

chord  $BQ \cong$  chord  $RD$

and  $PM \perp BQ$ ,  $PN \perp RS$

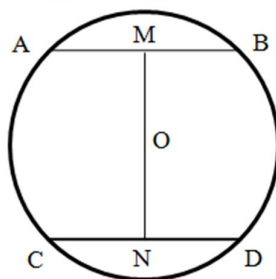
... (as shown in figure-)

$PM = 2.8$  means distance of  $BQ$

from P is 2.8 cm

$\therefore PN = 2.8$  cm ... [congruent chord in a circle are equidistant from the center of circle]

Q. 30) In the circle, O is the center of circle  $OM \perp AB$ ,  $ON \perp CD$ ,  $OM = ON = 3.9$  cm .If chord  $AB = 8.6$  cm ,then find length of chord  $CD$ .(2M)



Solution:

$OM \perp AB$ ,  $ON \perp CD$ ,

$OM = ON = 3.9$  cm

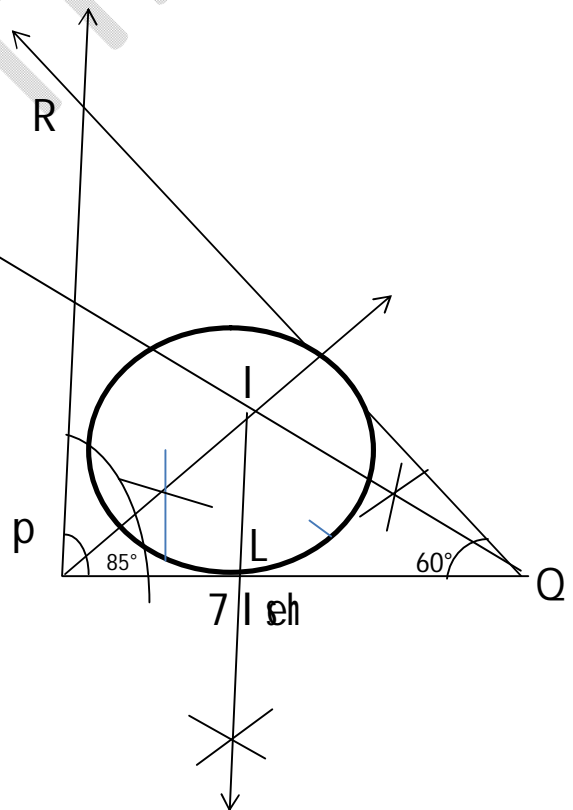
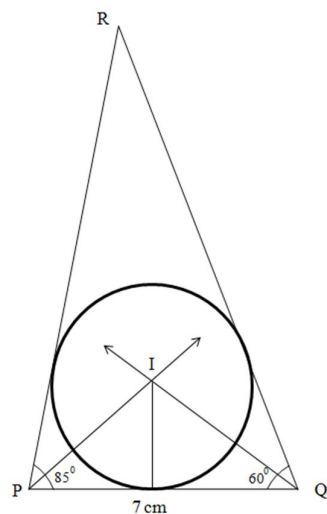
$\therefore AB = CD$

... (congruent chords in a circle are equidistant from the centre of the circle)

$$\therefore AB = 8.6 \text{ cm}$$

$$\therefore CD = 8.6 \text{ cm}$$

Q. 31) Construct  $\Delta PQR$  such that  $\angle P = 85^\circ$ ,  $PQ = 7 \text{ cm}$ ,  $\angle Q = 60^\circ$ . Draw incircle of  $\Delta PQR$ . Draw a rough figure and show all measure on it



**Solution:**

i) Construct  $\Delta PQR$  of given measure.

- ii) Draw bisectors of any two angles of the triangle.
- iii) Do note the point of intersection of angle bisectors as I
- iv) Draw perpendicular IL from point I to the side PQ.
- v) Draw a circle with center I and radius IM.

Q. 32) Distance of chord PQ from midpoint of the circle 1 cm.  
Length of chord is 4 cm, then find the radius of circle.

$$PM = \frac{1}{2} \square \dots (\text{perpendicular drawn from the center of the}$$

chord to chord bisects the chord)

$$\therefore PM = \square$$

In  $\triangle OMP$ ,  $\angle OMP = 90^\circ$

$$\therefore OP^2 = OM^2 + PM^2 \dots (\text{by Pythagoras theorem})$$

$$\therefore OP^2 = \square$$

$$\therefore OP = \square \dots (\text{Take square root of both sides})$$

(3M)

$$\text{Solution : } PM = \frac{1}{2} \square PQ$$

....[perpendicular drawn from the center of the circle to chord bisects the chord]

$$\therefore PM = \frac{1}{2} \times 4 = 2\text{cm}$$

In  $\triangle OMP$ ,  $\angle OMP = 90^\circ$

$$\therefore OP^2 = OM^2 + PM^2 \dots (\text{by Pythagoras theorem})$$



$$\begin{aligned}\therefore OP^2 &= (1)^2 + (2)^2 \\ &= 1 + 4\end{aligned}$$

$$\therefore OP^2 = 5 \quad \dots \text{(Take square root of both sides)}$$

$$\therefore OP = \sqrt{5} \text{ cm}$$

Q. 33) Radius of circle is 61 cm from centre 'O', length of the chord AB is 120 unit. Find the distance of chord from center of circle. (3M)

$$\begin{aligned}AP &= \frac{1}{2} AB \quad \dots [\text{perpendicular drawn on chord bisects chord}] \\ &= \square\end{aligned}$$

In  $\Delta OPA$ ,  $\angle OPA = 90^\circ$

$$\therefore OA^2 = OP^2 + AP^2 \quad \dots \text{(by Pythagoras theorem)}$$

$$\therefore 61^2 = OP^2 + \square$$

$$OP^2 = (61^2 - 60^2) \square$$

$$\therefore OP = \square$$

Solution :

$$AP = \frac{1}{2} AB$$

$\dots$  [perpendicular drawn on chord bisects chord]

$$= \frac{1}{2} \times 120 = 60$$

$$AP = 60 \text{ unit}$$

In  $\Delta OPA$ ,  $\angle OPA = 90^\circ$

$$\therefore OA^2 = OP^2 + AP^2 \quad \dots \text{(by Pythagoras theorem)}$$

$$\therefore 61^2 = OP^2 + 60^2$$

$$OP^2 = (61 - 60) (61 + 60)$$

$$= (1)(121)$$

$$OP^2 = 121$$

$$\therefore OP = 11 \text{ unit,}$$

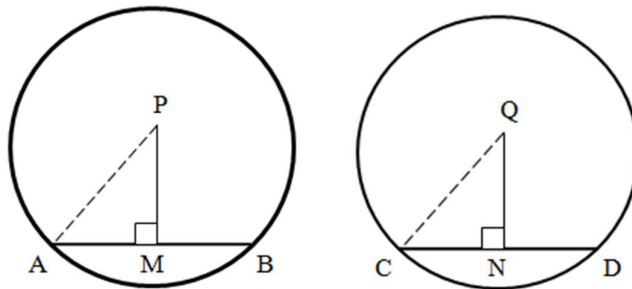
Q. 34) Chord of the circle are congruent and they are equidistance from the center

Side : point P and point q are centers of circles are congruent

seg PM  $\perp$  chord AB    A - M - B

seg QN  $\perp$  chord CD    C - N - D

PM = QN



To prove : chord AB  $\cong$  chord CD

Construction : Draw seg PA and seg QC

$\Delta PMA$  and  $\Delta QNC$  ,

$$\boxed{\phantom{0}} \cong \angle QNC \quad \dots \text{(each } 90^0\text{)}$$

$$\text{seg PM} \cong \text{seg QN} \quad \dots \text{(Given)}$$

$$\text{hypotenuse PA} \cong \text{hypotenuse QC} \quad \dots \text{[congruent radii of circle]}$$

$$\Delta PMA \cong \Delta QNC$$

$$\text{seg AM} \cong \text{seg } \boxed{\phantom{0}}$$

$$\therefore AM = CN$$

$$\therefore AM = \frac{1}{2} \times AB$$

$$CN = \frac{1}{2} \times CD$$

$$\therefore AB = CD$$

$$\therefore \text{chord AB} \cong \text{chord CD}$$

(4M)

Ans :  $\Delta PMA$  and  $\Delta QNC$

$$\boxed{\angle PMA} \cong \angle QNC \quad \dots \text{(each } 90^0\text{)}$$

$$\text{seg PM} \cong \text{seg QN} \quad \dots \text{(Given)}$$

$$\text{hypotenuse PA} \cong \text{hypotenuse QC}$$

.....[congruent radii of circle]

$$\Delta PMA \cong \Delta QNC$$

$$\text{seg AM} \cong \text{seg } \boxed{CN}$$

$$\therefore AM = CN$$

$$\therefore AM = \frac{1}{2} \times AB$$

..... [perpendicular drawn from chord bisect chord]

$$CN = \frac{1}{2} \times CD$$

$$\therefore AB = CD$$

$$\therefore \text{chord } AB \cong \text{chord } CD$$

Q. 35) In the given fig, P is the center of the circle ,chord AB and chord CD intersect in point E at the diameter. If

$\angle AEP \cong \angle DEP$ , then prove  $AB = CD$ .

Side : P is the center of the circle ,chord AB and chord CD intersect in point E of diameter (4M)

$$\angle AEP \cong \angle DEP$$

To prove :  $AB = CD$ -(

Construction : seg  $PM \perp$  chord AB

A - M - B

seg  $PN \perp$  chord CD , C - N - D

$$\angle AEP \cong \angle DEP \quad \dots \text{ (Given)}$$

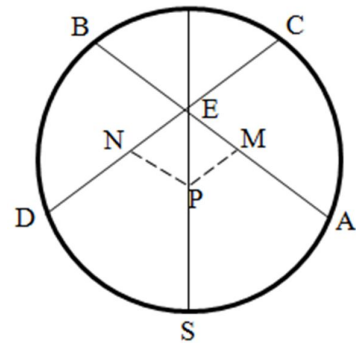
Bisector of seg ES ☐

$$\therefore \text{ ☐ } = PN \quad \dots \text{ (bisector theorem)}$$

$$\text{chord } AB \cong \text{chord } CD \dots$$



$$AB = CD \quad \dots \text{ (length of the congruent segment)}$$



**Solution :**

$$\angle AEP \cong \angle DEP \quad \dots \text{ (Given)}$$

seg ES is the bisector of  $\angle AED$

point P is the bisector of  $\angle AED$

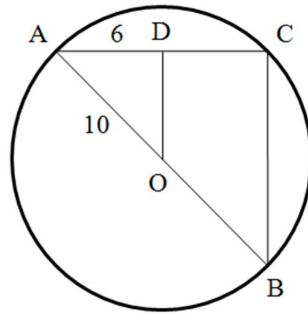
$$\therefore PM = PN \quad \dots \text{ (bisector theorem)}$$

chord  $AB \cong$  chord  $CD \quad \dots \text{ (congruent chords of a circle)}$

are equidistant from the center of the circle)

$$AB = CD \quad \dots \text{ (length of the congruent segment)}$$

Q. 36) In the given figure, Diameter AB and chord AC has end point 'A' length of the AB is 20 cm and  $AC = 12$  cm , find the distance of AC from midpoint of the circle? (4M)



**Solution:**

Given : Diameter AB and chord AC same end points

$$AB = 20 \text{ cm}$$

$$\text{and } AC = 12 \text{ cm}$$

To prove : To find OD

Proof : construct  $OD \perp AC$

$$\therefore AD = DC = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6 \text{ cm}$$

(perpendicular drawn from center of a circle to the chord bisects chord)

$$\therefore OA = OB = \frac{1}{2} AB = \frac{1}{2} \times 20 = 10 \text{ cm}$$

In  $\Delta ODA$ ,

$$\therefore OA^2 = OD^2 + AD^2 \dots (\text{by Pythagoras theorem})$$

$$(10)^2 = OD^2 + (6)^2$$

$$100 = OD^2 + 36$$

$$OD^2 = 100 - 36$$

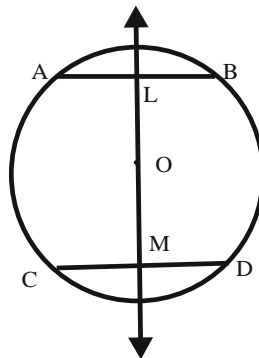
$$OD^2 = 64$$

$$\therefore OD = 8 \text{ cm}$$

The distance of AC from the midpoint of the circle is 8cm.

Q. 37) In the given figure. EF is a line passing through center O of circle. If EF bisects chord AB and CD of the circle.

Prove that  $AB \parallel CD$ . (4M)



Solution : EF is the line passing

though the centre O of  
a circle. EF bisects  
chords AB and CD of the circle.

To prove :  $AB \parallel CD$ .

Proof : EF bisects chord AB

$$\therefore \angle OLB = \angle OLA = 90^\circ \dots (1)$$

[The line drawn through the centre of circle to bisect  
a chord is perpendicular to the chord]

$\therefore$  EF bisects chord CD

$\therefore$  OM bisects chord CD

$$\therefore \angle OMC = \angle OMD = 90^\circ \dots (2)$$

.....(The line drawn through the centre of  
circle to bisect a chord is perpendicular to the chord)

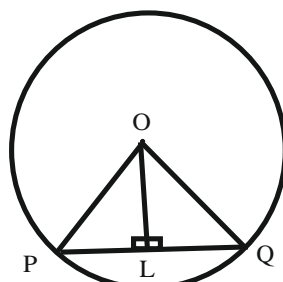
From (1) and (2)

$$\angle OLB = \angle OMC = 90^\circ$$

But these angles form a pair of equal alternate interior angles.

$\therefore AB \parallel CD$ .

**Q. 38) Prove that the perpendicular from the centre of a circle  
to a chord, bisects the chord. (4M)**



Solution : A circle with centre O .

PQ is a chord of this circle.

OL is the perpendicular drawn to chord PQ from centre O.

Chord PQ from centre O

To prove :  $PL = QL$

Construction : Join OP and OQ

Proof : In  $\Delta OLP$  and  $\Delta OLQ$

$$OP = OQ \quad \dots (\text{radii of the same circle})$$

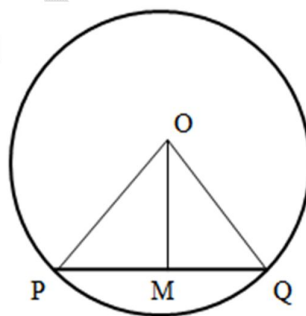
$$OL = OL \quad \dots (\text{common side})$$

$$\angle OLP = \angle OLQ \quad \dots (\text{Each } 90^\circ)$$

$$\Delta OLP \cong \Delta OLQ \quad \dots (\text{By A-S-A test of congruency})$$

$$\therefore PL = QL.$$

Q. 39) Prove that the line drawn through the centre of circle to bisect a chord is perpendicular to the chord. (4M)



Given: A circle with centre O.

PQ is a chord of this circle



M is the mid point of the chord PQ

To prove :  $OM \perp PQ$

Construction : Join OP and OQ

Proof :  $\Delta OMP$  and  $\Delta OMQ$

$OP = OQ$  ... (Radii of same circle)

$OM = OM$  ... (common side)

$MP = MQ$  ... (M is the midpoint of PQ)

$\therefore \Delta OMP \cong \Delta OMQ$  ... (By S-S-S test)

$\therefore \angle OMP = \angle OMQ$

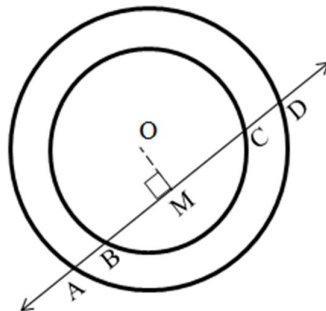
But,

$\angle OMP + \angle OMQ = 180^\circ$  ... (Linear pair axiom)

$\therefore \angle OMP = \angle OMQ = 90^\circ$

$OM \perp PQ$ .

Q. 40 ) If a line intersects two concentric circles ( Circles with same centre ) with centre O at A, B, C and D, prove that  $AB = CD$  (4M)



Solution:

Given : A line intersects two concentric

circles (circles with the same centre)

with centre O at A, B, C and D

To prove :  $AB = CD$

Construction : Draw  $OM \perp BC$

Proof : The perpendicular drawn from the centre of circle to a chord bisects the chord.

$$\therefore AM = DM \quad \dots (1)$$

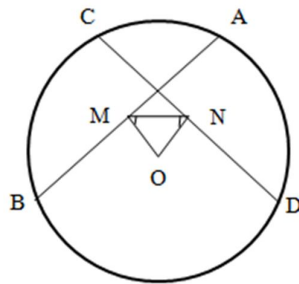
$$BM = CM \quad \dots (2)$$

Subtracting (2) from (1) we get

$$AM - BM = DM - CM$$

$$AB = CD$$

Q. 41) In the given figure. Chord AB and chord CD are two congruent chord passing through center 'O' of the circle. IF  $OM \perp AB$  and  $ON \perp CD$ , then prove that  $\angle OMN = \angle ONM$ . (4M)



Solution:

Given : In the given figure,

AB and CD are congruent

chord of same

length  $OM \perp AB$  and  $ON \perp CD$

To prove:  $\angle OMN = \angle ONM$

Proof : Chord  $AB =$  Chord  $CD$

$OM = ON \dots (1)$  [perpendicular drawn from center of the circle to chord bisect are congruent)

In  $\Delta OMN$

$OM = ON \dots$  from (1)

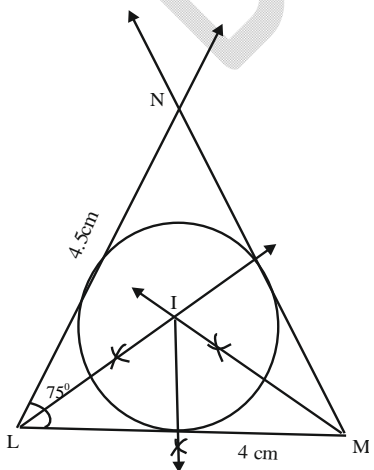
$\therefore \angle OMN = \angle ONM \dots$  (adjacent side of triangle having same angle)

Q.42) Construct  $\Delta LMN$  such that  $LM = 4\text{cm}$ ,  $\angle L = 70^\circ$   
 $LN = 4.5\text{cm}$ , Draw incircle of  $\Delta LMN$  (4M)

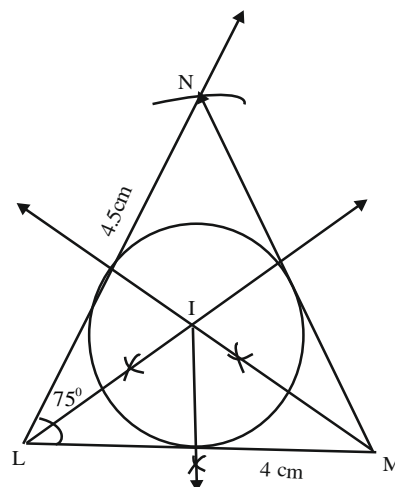
Ans : Draw a rough figure and show all measures init

- 1) Construct  $\Delta LMN$  of given measures
- 2) Draw  $\angle L$  and  $\angle M$  as two bisectors of triangle
- 3) Denote the point of intersection of angle bisectors as I
- 4) Draw perpendicular LM from point I
- 5) Draw a circle with Centre I and radius IA.

Rough diagram



correct diagram



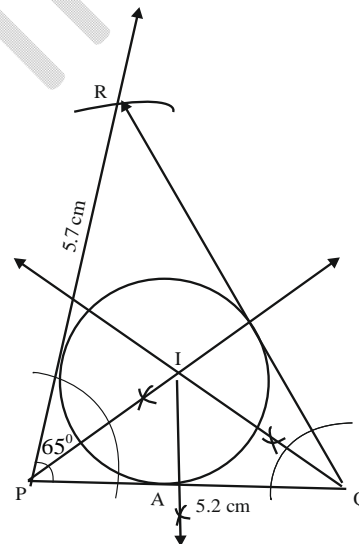
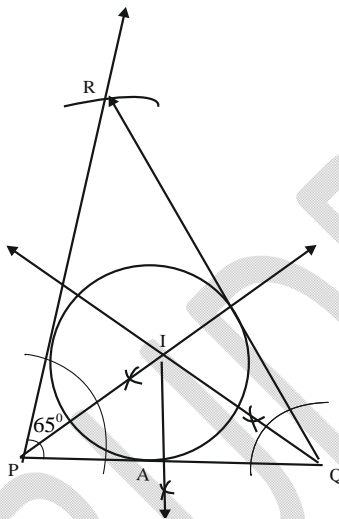
Q.43) Construct right angled triangle  $\Delta ABC$ . Draw incircle of it. (3M)

Ans : Steps:

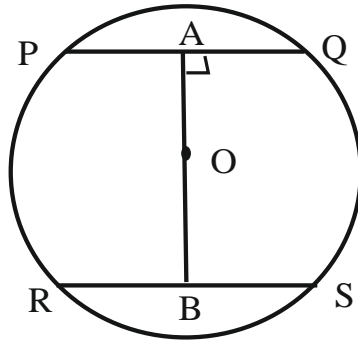
- 1) Construct  $\Delta ABC$  (right angled triangle) of any measure.
- 2) Draw bisectors of any two angles of triangle.
- 3) Denote the point of intersection of angle bisectors as I
- 4) Draw perpendicular IM from the point I

Rough diagram

correct diagram



Q.44) Two chords PQ and RS of a circle are parallel to each other and AB is the perpendicular bisector of PQ. Without using any construction, prove that AB bisects RS(3M)



Given : Two chords PQ and RS of circle are parallel to each other and AB is perpendicular bisector of PQ

To prove : AB bisects RS

Proof : AB is the perpendicular bisector of PQ

$\therefore$  AB passes through the center O

$\therefore$  PQ  $\parallel$  RS

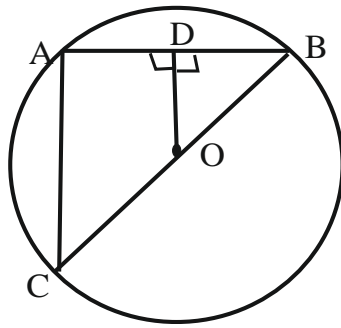
$\therefore$  AB  $\perp$  RS

$\therefore$  AB passed through the center

$\therefore$  AB bisects RS

[perpendicular drawn from the center of circle bisects the chord ]

Q.45) OD is perpendicular to chord AB of circle whose centre is O. If BC is a diameter, prove that CA = 2OD(4M)



Given : OD is perpendicular to chord AB of a circle where centre is O. BC is diameter of the circle.

To prove :  $CA = 2OD$

Proof : D is the midpoint of AB

C the perpendicular drawn from the center of a circle to a chord bisects the chord.

In  $\Delta BAC$ ,

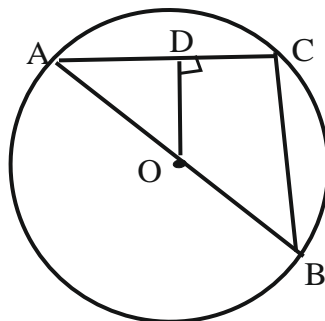
$\therefore$  D is the mid point of AB and O is the mid point of the BC.

$OD \parallel AC$  ..... (By midpoint through)

And  $OD = \frac{1}{2} AC$

$\therefore CA = 2OD$

Q.46) In the figure, diameter AB and chord AC have a common end point A. If the length of AB is 20 cm and AC is 12 cm, how far is AC from the center of the circle? (4M)



Given : Diameter AB and a chord AC have common end point

A.  $AB = 20$  and  $AC = 12$  cm.

To determine : OD

Determination  $\because OD \perp AC$

$$AD = DC = \frac{1}{2} AC$$

$$= \frac{1}{2} \times 12$$

$= 6$  cm [Perpendicular drawn from center of circle to a chord bisects the chord ]

$$OA = OB = \frac{1}{2} AB$$

$$= \frac{1}{2} \times 20$$

$$= 10 \text{ cm}$$

In right triangle ODA,

$$\therefore OA^2 = OD^2 + AD^2 \dots (\text{by Pythagoras theorem})$$

$$(10)^2 = OD^2 + (6)^2$$

$$100 = OD^2 + 36$$

$$OD^2 = 100 - 36$$

$$OD^2 = 64$$

$$\therefore OD = 8 \text{ cm}$$

Hence, AC is 8 cm far from the centre of the circle.

**Q.47)** P is the centre of circle with radius 25 cm length of the chord is 48 cm. Find the distance of P from center.

Solution : In fig.

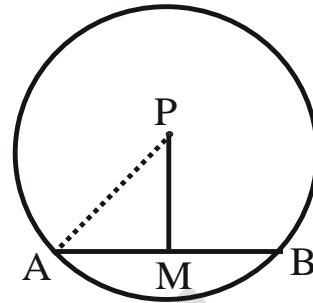
Radius of circle ,  $PA = 25\text{cm}$

$PM \perp AB$  (By construction )

$PM$  bisect  $AB$

$\therefore AM \cong MB$

$$\begin{aligned}\therefore AM &= MB = \frac{1}{2} AB \\ &= \frac{1}{2} \times 48 \\ &= 24 \text{ cm}\end{aligned}$$



In right  $\triangle PMA$  ,  $\angle PMA = 90^\circ$

By Pythagoras theorem,

$$\therefore PA^2 = PM^2 + AM^2$$

$$(25)^2 = PM^2 + (24)^2$$

$$(25)^2 = (24)^2 + (PM)^2$$

$$(25 + 24)(25 - 24) = PM^2$$

$$(49 \times 1) = PM^2$$

$$PM = 7 \times 1$$

$$PM = 7 \text{ cm}$$

$\therefore$  Distance from the centre of the circle is 7 cm.

Q.48) Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre. (4M)



**Solution:**

Given : Radius of circle,  $OB = \frac{26}{2} = 13 \text{ cm}$

Chord  $AB = 24 \text{ cm}$ .

To find :-

Distance from centre,  $OM$

Length of the chord,  $AB = 24 \text{ cm}$ .

$$AM = MB = \frac{AB}{2} = \frac{24}{2} = 12 \text{ cm}$$

(perpendicular drawn from the centre bisects the chord)

In  $\triangle OMB$ ,  $\angle OMB = 90^\circ$

$OB = 13 \text{ cm}$ ,  $MB = 12 \text{ cm}$ ,  $OM = ?$

$$\therefore OB^2 = OM^2 + MB^2 \dots\dots\dots (\text{ by Pythagoras theorem } )$$

$$(OM)^2 = (13)^2 + (12)^2$$

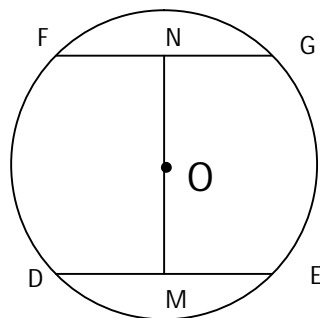
$$(OM)^2 = 169 - 144$$

$$(OM)^2 = 25$$

$$OM = 5 \text{ cm}$$

$\therefore$  Distance of the chord from the centre is  $5 \text{ cm}$ .

**Q.49) In the given figure  $O$  is the centre of the circle  $DE \cong FG$ . If  $OM = 2.5 \text{ cm}$  find  $ON$  (3M)**



Solution : O is the centre of circle -

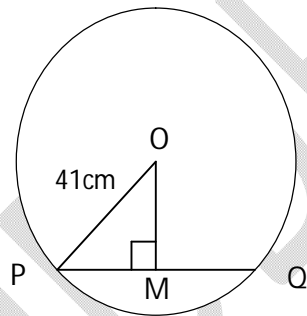
$OM \perp DE$ ,  $ON \perp FG$  and  $DE \cong FG$

$\therefore OM \cong ON$  ..... (congruent chords are equidistant from centre.)

But,  $OM = 2.5$  cm

$\therefore ON = 2.5$  cm

Q.50) Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the chord the centre of the circle. (4M)



Ans : Let seg  $OM \perp$  chord PQ such that P-M-Q ,

$PQ = 80$  units,  $OP = 41$  units

$$\therefore PM = \frac{1}{2} \times 80$$

$$\therefore PM = 40 \text{ units.}$$

In right angled  $\Delta OMP$ ,

By Pythagoras theorem,

$$OP^2 = OM^2 + PM^2$$

$$\therefore 41^2 = OM^2 + 40^2$$

$$\therefore OM^2 = 1681 - 1600$$

$$\therefore OM^2 = 1681 - 1600$$

$$\therefore OM^2 = 81$$

$$\therefore OM = \sqrt{81}$$

$$\therefore OM = 9 \text{ units}$$

$\therefore$  Distance of the chord from the centre is 9 units.

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