4. Ratio and proportion
Extra Questions

- From the following pairs of numbers find the reduced form of ratio of first number to second number. (1 mark)

Q. 1) 84, 96

Ans : Ratio = $\frac{84}{96} = \frac{12 \times 7}{12 \times 8} = \frac{7}{8} = 7:8$

Q. 2) 54, 72

Ans : Ratio = $\frac{54}{72} = \frac{18 \times 3}{18 \times 4} = \frac{3}{4} = 3:4$

- Express the following percentages as ratios the reduced from. (2 mark)

Q.3) 225 %

Ans : $\frac{225}{100}$

$= \frac{45 \times 5}{20 \times 5}$

$= \frac{45}{20} = \frac{5 \times 9}{5 \times 4}$

$= \frac{9}{4}$

$= 9 : 4$

Q. 4) 25 : 100

Ans : $\frac{25}{100}$

$= \frac{5 \times 5}{20 \times 5}$

$= \frac{5}{20}$

$= \frac{1}{4}$

$= 1:4$

Q. 5) 0. 32%

Ans : 0. 32%

$= \frac{0\,.\,32}{100}$

$= \frac{0.32 \times 100}{100 \times 100}$ ..... (Multiply numerator and denominator by 100)

$= \frac{32}{10000}$

$= \frac{8 \times \cancel{4}}{25 \times 25 \times 4 \times \cancel{4}}$

$= \frac{\cancel{8}2}{25 \times 25}$ (∵divide 8 by 4)

$= \frac{2}{25 \times 25}$

$= \frac{2}{625}$

=2 : 625

- Find the reduced form of the ratio of the first quantity to second quantity. (2 Mark)

Q. 6) Rs. 17, Rs. 25.60 paise

Ans : Rs. 17 = 17 × 100 = 1700p

Rs. 25.60p

= 25 × 100 + 60

= 2500 + 60 = 2560 p

Rs. 17 : Rs. 25.60 p

= 1700 : 2560

$= \frac{1700}{2560} = \frac{20 \times 85}{20 \times 128} = \frac{85}{128}$

= 85 : 128

Here first and second quantity are different. For multiplication both the quantities should be same. So let's convert rupees in to paise.

∴ 15 Rs = 15 × 100 = 1500

and 13 Rs - = 13 × 100 = 1300

Q. 7) 4.2 kilogram, 1700 gram

Ans : Ratio of 4.2 kilogram, 1700 gram $= \frac{4.2 \times 1000 \text{ gm}}{1700 \text{ gm}}$

$[\because 1\text{kg} = 1000\text{gm}] = \frac{4200 \text{ gm}}{1700 \text{ gm}}$

$= \frac{50 \times 84 \text{ gm}}{50 \times 34 \text{ gm}}$

$= \frac{84}{34} = \frac{42}{17}$

$= 42 : 17$

Q 8) 9 Minutes , 40 seconds; 3 minutes ,8 seconds

Ans : Ratio of 9 minutes 40 ;seconds, 3 minutes

8 seconds $= \frac{9 \text{ minutes } 40 \text{ seconds}}{3 \text{ minutes } 8 \text{ seconds}}$

$\because$ 1 minute = 60 seconds

$= \frac{(9 \times 60) + 40 \text{ seconds}}{(3 \times 60) + 8 \text{ seconds}}$

$= \frac{540 + 40}{180 + 8}$

$= \frac{580}{188}$

$= \frac{2 \times 290}{2 \times 94}$

$= \frac{290}{94}$

$= 290 : 94$

➢ convert the following ratios into percentage.

Q 9) 14 : 20

Ans : Let 14 : 20 = x %

$$\therefore \frac{14}{20} = \frac{x}{100}$$

$$\therefore x = \frac{14}{20} \times 100$$

$$\therefore x = 14 \times 5$$

$$\therefore x = 70\ \%$$

$$\therefore 14 : 20 = 70\ \%$$

Q. 10) $\frac{7}{8}$

Ans : Let $\frac{7}{8} = x$ %

$$\therefore \frac{7}{8} = \frac{x}{100}$$

$$\therefore x = \frac{7}{8} \times 100$$

$$\therefore x = 7 \times 12.5$$

$$\therefore x = 87.5\ \%$$

➢ Compare the following pairs of ratios.

Q. 11) $\frac{9}{13}, \frac{11}{15}$

Ans : $9 \times 15, 13 \times 11$

$135 < 143$

$\therefore \frac{9}{13} < \frac{11}{15}$

Q 12) $\frac{3\sqrt{3}}{2\sqrt{2}}, \frac{2\sqrt{2}}{3\sqrt{3}}$

Ans : $3\sqrt{3} \times 3\sqrt{3} ; 2\sqrt{2} \times 2\sqrt{2}$

$9 \times (\sqrt{3} \times \sqrt{3}) ; 2 \times (\sqrt{2} \times \sqrt{2})$

$9 \times 3 ; 2 \times 2$

$27 ; 4$

$27 > 4$

$\therefore \frac{3\sqrt{3}}{2\sqrt{2}} > \frac{2\sqrt{2}}{3\sqrt{3}}$

Q 13) $\frac{\sqrt{12}}{\sqrt{9}}, \frac{\sqrt{24}}{\sqrt{18}}$

$\sqrt{12} \times \sqrt{18} ; \sqrt{9} \times \sqrt{24}$

$\sqrt{216} ; \sqrt{216}$

$\therefore \sqrt{216} = \sqrt{216}$

$\therefore \frac{\sqrt{12}}{\sqrt{9}} = \frac{\sqrt{24}}{\sqrt{18}}$

Q 14) The ratio of two numbers is 13 : 6 and their difference is 70. Find those numbers. (3 marks)

Ans : The ratio of two numbers is 13 : 6

Let the common multiple of the given ratio be x.

First number $= 13x$

Second number $= 6x$

According to the given condition, $13x - 6x = 70$

$$7x = 70$$

$$x = \frac{70}{7}$$

$$x = 10$$

First number $= 13 \times x = 13 \times 10 = 130$

Second number $= 6x = 6 \times 10 = 60$

Q . 15) The ratio of two numbers is 11 : 13 and their sum is 264. Find these numbers. . (3 marks)

Ans : The ratio of two numbers is 11 : 13

Let the common multiple of the given ratio be

First number $= 11x$

Second number $= 13x$

According to the given condition,

$11x + 13x = 264$

$24x = 264$

$$x = \frac{264}{24}$$

$$x = 11$$

∴ First number $= 11x = 11 \times 11 = 121$

Second number $= 13x = 13 \times 11 = 143$

Q 16) a, b, c are in continued proportion. If a =3 and c = 27, then find b. (3 marks)

Ans : a, b, c are in continued proportion.

∴ b^2 = ac

= (3) (27) …. (Substituting the given values)

= 81

∴ $b^2 = 81$ ∴ b = 9 … (taking square root of both the sides)

b = 9

Q 17) Find the mean proportional of $\frac{a^3 + b^3}{ab^3}, \frac{a^3b}{a^3 + b^3}$. (3 marks)

Ans : let the mean proportional be 'm'

∴ $\frac{a^3 + b^3}{ab^3}$, m, $\frac{a^3b}{a^3 + b^3}$ are in continued proportion.

∴ $m^2 = \frac{a^3 + b^3}{ab^3} \times \frac{a^3b}{a^3 + b^3}$

∴ $m^2 = \frac{a^3 b}{ab^3}$

$\therefore\ m^2 = \frac{a^2}{b^2}$

$\therefore\ m = \frac{a}{b}$ …… (taking square root on both sides)

$\therefore$ The mean proportional is $\frac{a}{b}$

Q 18) If x is geometric mean of 16 and 9 then the find value of x. . (2 marks)

Ans : x is the geometric mean of 16 and 9

$\therefore\ x^2 = 16 \times 9$

$\therefore\ x^2 = 144$

$\therefore\ \boxed{x = 12}$

Q. 19) The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age? (2 M)

(A) 3:2 (B) 2:3 (C) :3 (D) 3:4

Ans : (B) $\left[\frac{Nitin's\ age}{Mohasin's\ age} = \frac{24\ years}{36\ years}\right]$

(B) 2:3

Q 20) Population of a village is 14, 400. The ratio of illiterate to literate is 3 : 5. Find the number of literate and illiterate persons in the village.

Ans : Population of a village is 14, 400

The ratio of illiterate to literate is 3 : 5

Let the common multiple of the given ratio be x.

$\therefore$ Illiterate person $= 3x$

and literate person $= 5x$

According to the given condition,

$3x + 5x = 14,400$

$8x = 14,400$

$x = \frac{14,400}{8}$

$\therefore \boxed{x = 1800}$

illiterate person $= 3x = 3 \times 1800 = \boxed{5400}$

and literate person $= 5x = 5 \times 1800 = \boxed{9000}$

Q. 21) The ratio of boys and girls in a certain class is 5 : 4. If 4 boys and 4 girls are added. The ratio would be 6 : 5. Find the number of boys and girls in the class.

Ans : The ratio of boys and girls in a certain class is 5 : 4

Let the common multiple of the given ratio be x

$\therefore$ number of boys $= 5x$

number of girls $= 4x$

According to the given condition,

$\frac{5x + 4}{4x + 4} = \frac{6}{5}$

$\therefore (5x + 4) \times 5 = 6 \times (4x + 4)$

$\therefore 25x + 20 = 24x + 24$

$\therefore 25x - 24x = 24 - 20$

$\therefore x = 4$

$\therefore$ number of boys $= 5 \times 4 = 20$

and number of girls $= 4 \times 4 = 16$

Q 22) The measures of angles of a triangle are in the ratio 4 : 5 : 6. Determine the measures of each angle of the triangle.

Ans : Ratio of measures of angles of triangle is 4 : 5 : 6

Sum of angle of triangle is 180^0

Let the common multiple of the given ratio be x.

$4x + 5x + 6x = 180^0$

$15x = 180^0$

$x = \frac{180}{15}$

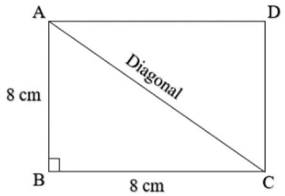
$\boxed{x = 12}$

$\therefore$ Measures of angles of triangle

$4x = 4 \times 12 = 48^0, 5x = 5 \times 12 = 60^0, 6x = 6 \times 12 = 72^0$

Find the following ratios.

Q 23) The ratio of diagonal of a square to its side, if the length of side is 8 cm.



□ ABCD is square

In right – angle Δ ABC,

$\therefore AC^2 = AB^2 + BC^2$ …… [Theorem of Pythagoras]

$\therefore AC^2 = 8^2 + 8^2$

$\therefore AC^2 = 2 \times 8^2$

$\therefore$ diagonal $= 8\sqrt{2}$ cm

$\therefore$ Ratio $= \frac{\text{diagonal}}{\text{length of side}} = \frac{8\sqrt{2}}{8} = \frac{\sqrt{2}}{1} = \sqrt{2} : 1$

Q. 24) The length of sides of a rectangle are 4 cm and 2 : 5 cm. Find the ratio of its perimeter to area.

Ans : length = 4 cm, width = 2.5 cm

$$\frac{\text{Perimeter of rectangle}}{\text{Area of Rectangle}} = \frac{2\,(4 + 2.5)}{4 \times 2.5}$$

$$= \frac{13.0}{10.0}$$

$$= \frac{13}{10}$$

$$= 13 : 10$$

Q 25) If the product of two numbers is 4032 and their ratio is 7 : 9 then find the numbers. Complete the following activity.

Ans : Ratio of two number is □

Let the common multiple of the given ratio be X

First number $= 7x$

Second number $\square$

According to the given condition,

$7x \times \square = \square$

$\square = \square$

$x^2 = \square$

$\therefore\ x^2 = \square$

$\therefore x = \square$

First number $= 7x = 7 \times \square = \square$

Second number $= 9x = 9 \times \square = \square$

Ans : Ratio of two number is $7 : 9$

Let the common multiple of the given ratio be

First number $= 7x$

Second number $= 9x$

According to the given condition,

$7x \times 9x = 4032$

$63x^2 = 4032$

$x^2 = \frac{4032}{63}$

$\therefore\ x^2 = 64$

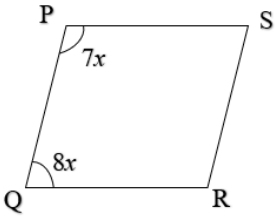
$\therefore x = \boxed{8}$

First number $= 7x = 7 \times \boxed{8} = \boxed{56}$

Second number $= 9x = 9 \times \boxed{8} = \boxed{72}$

Q. 26) □ PQRS is a parallelogram. The ratio of ∠ P and ∠ Q of this parallelogram is 7 : 8. Find the measure of ∠ Q

Ans :



Let the common multiple of given ratio be x.

$\therefore$ m∠p $= 7x^0$ and m∠Q $= 8x^0$

Now, m∠p + m∠Q $= 180^0$

……. [Adjacent angles of a parallelogram are supplementary.]

$\therefore 7x + 8x = 180^0$

$\therefore 15x = 180^0$

$\therefore x = \frac{180^0}{15}$

$\therefore x = 12^0$

$\therefore$ m∠Q $= 8x^0 = 8 \times 12^0 = 96^0$

$\therefore$ m∠Q $= 96^0$

Q 27) If a : b = 2 :1 and b : c = 3 : 1 then find the value of $\frac{a^2}{10bc}$

Ans : a : b = 2 :1 and b : c = 3 : 1

$\frac{a}{b} = \frac{2}{1}$ and $\frac{b}{c} = \frac{3}{1}$

$a = 2b$ and $b = 3c$ Putting these value in $a = 2b$

$\therefore\ a = 2\,(3c) \Rightarrow \boxed{a = 6c}$ and $\boxed{b = 3c}$

Now, $\frac{a^2}{10bc} = \frac{(6c)^2}{10 \times 3c \times c} = \frac{36c^2}{10 \times 3c^2} = \frac{12}{10}$

$\boxed{\therefore\ \frac{a^2}{10bc} = \frac{12}{10}}$

Q 28) If $\sqrt{0.03 \times 0.3 \times a} = 0.3 \times 0.03 \times \sqrt{b}$ then find the ratio $\frac{a}{b}$

Ans : $\sqrt{0.03 \times 0.3 \times a} = 0.3 \times 0.03 \times \sqrt{b}$

$\therefore\ \sqrt{\frac{3}{100} \times \frac{3}{10} \times a} = \frac{3}{10} \times \frac{3}{100} \times \sqrt{b}$

Squaring both sides,

$\therefore\ \frac{3}{100} \times \frac{3}{10} \times a = \frac{9}{100} \times \frac{9}{10000} \times b$

$\therefore\ \frac{a}{b} = \frac{9}{100} \times \frac{9}{10000} \times \frac{100}{3} \times \frac{10}{3}$

$\therefore\ \frac{a}{b} = \frac{9}{100} \times \frac{9}{100} \times \frac{10}{9}$

$\therefore\ \frac{a}{b} = \frac{9}{100} \times \frac{1}{10}$

$\therefore\ \frac{a}{b} = \frac{9}{1000}$

Q 29) $(x + 2) : (x + 12) = (x - 4) : (x + 3)$ then find the value of x

Ans : $\therefore \frac{x+2}{x+12} = \frac{x-4}{x+3}$

$\therefore (x + 2)(x + 3) = (x - 4)(x + 12)$

$x^2 + 3x + 2x + 6 = x^2 + 12x - 4x - 48$

$\therefore x^2 + 5x + 6 = x^2 + 8x - 48$

$\therefore x^2 - x^2 + 5x - 8x = -48 - 6$

$\therefore -3x = -54$

$\therefore x = \frac{54}{3}$

$\therefore \boxed{x = 18}$

Q 30) If $\frac{a}{b} = \frac{3}{2}$ then find $\frac{4a^2 + 3b^2}{4a^2 - 3b^2}$

Ans : $\frac{a}{b} = \frac{3}{2}$

$\frac{a^2}{b^2} = \frac{9}{4}$ …… (squaring both sides)

$\therefore \frac{a^2}{b^2} \times \frac{4}{3} = \frac{9}{4} \times \frac{4}{3}$ ….. (multiply both sides by $\frac{4}{3}$)

$\therefore \frac{4a^2}{3b^2} = \frac{36}{12}$

$\therefore \frac{4a^2 + 3b^2}{4a^2 - 3b^2} = \frac{36 + 12}{36 - 12}$ …… (using componendo - dividendo)

$\therefore \frac{4a^2 + 3b^2}{4a^2 - 3b^2} = \frac{48}{24} = \frac{2}{1} = 2 : 1$

Q 31) If $\frac{by+cz}{b^2 - c^2} = \frac{cz+ax}{c^2+ a^2} = \frac{ax+by}{a^2+ b^2}$ then prove that $\frac{x}{b} = \frac{y}{b} = \frac{z}{c}$ (3 marks)

Ans : $\frac{by+cz}{b^2 - c^2} = \frac{cz+ax}{c^2+ a^2} = \frac{ax+by}{a^2 - b^2}$

By the theorem on equal rations,

Each ratio $= \frac{by+cz-(cz+ax)+ax+by}{b^2+c^2-(c^2+a^2)+a^2+b^2}$

$= \frac{by+cz-cz-ax+ax+by}{b^2+ c^2 -c^2 + a^2+b^2}$

$= \frac{2by}{2b^2} = \frac{y}{b}$ .....(1)

Similarly, each ratio

$= \frac{by+cz+cz-ax-(ax+by)}{b^2+ c^2 -c^2 + a^2-(a^2+b^2)}$

$= \frac{by+cz+cz+ax-ax-by}{b^2+ c^2+c^2 + a^2-a^2-b^2}$

$= \frac{2cz}{2c^2} = \frac{z}{c}$ .....(2)

Also, each ratio

$= \frac{-(by+cz)+cz+ax+ax+by}{-(b^2+ c^2)+c^2 + a^2+a^2+b^2}$

$= \frac{-by-cz+cz+ax+ax+by}{-b^2+ c^2+c^2 + a^2+a^2+b^2}$

$= \frac{2ax}{2a^2} = \frac{x}{a}$ .....(3)

From (3), (1) and (2),

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

Q 32) If $\frac{x}{y} = \frac{13}{6}$ then the find the value of $\frac{2x^2 + 5y^2}{2x^2 - 5y^2}$ complete the following activity. (4 marks)

Ans : $\frac{x}{y} = \frac{13}{6}$

$\therefore \frac{x^2}{y^2} = \square$ …… (squaring both sides)

$\therefore \frac{x^2}{y^2} \times \square = \frac{169}{36} \times \frac{2}{5}$ …… [multiply by $\square$ both sides]

$\frac{2x^2}{5y^2} = \frac{338}{180}$

$\square$ …… [using componendo dividendo]

$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \frac{518}{158}$

$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \square$ …… [dividing by…..]

Ans : $\frac{x}{y} = \frac{13}{6}$

$\therefore \frac{x^2}{y^2} = \boxed{\frac{169}{36}}$ …… (squaring both sides)

$\therefore \frac{x^2}{y^2} \times \boxed{\frac{2}{5}} = \frac{169}{36} \times \frac{2}{5}$ …… [multiply by $\boxed{\frac{2}{5}}$ both sides]

$\frac{2x^2}{5y^2} = \frac{338}{180}$

$\boxed{\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \frac{338 + 180}{338 - 180}}$ …… [using componendo dividendo]

$$\frac{2x^2+5y^2}{2x^2-5y^2} = \frac{518}{158}$$

$$\frac{2x^2+5y^2}{2x^2-5y^2} = \boxed{\frac{259}{79}} \ldots\ldots \text{[dividing by 2]}$$

Q 33) If $\frac{x}{y+z-x} = \frac{y}{z+x-y} = \frac{z}{x+y-z}$ and $x + y + z \neq 0$ then show that each ratio is equal to 1. (4 marks)

Ans : Let $\frac{x}{y+z-x} = \frac{y}{z+x-y} = \frac{z}{x+y-z} = k$

by theorem on equal ratios,

each ratio $k = \frac{x+y+z}{y+z-x+z+x+y+x+y-z}$

$$k = \frac{x+y+z}{x+y+z}$$

$k = 1 \quad (\because x + y + z \neq 0)$

$\therefore$ **each ratio = 1**

Q 34) If $\frac{3a-5b}{5c+3b} = \frac{a+5c}{b-5a} = \frac{b-c}{a-c}$ then show that each ratio is $\frac{a}{b}$ (3 marks)

Ans : Let $\frac{3a-5b}{5c+3b} = \frac{a+5c}{b-5a} = \frac{b-c}{a-c} = k$

$\therefore$ By theorem on equal ratios,

$$k = \frac{(3a-5b) + (a+5c) + 5(b-c)}{(5c+3b) + (b-5a) + 5(a-c)}$$

$$k = \frac{3a-5b+a+5c+5b-5c}{5c+3b+b-5a+5a-5c}$$

$$k = \frac{3a + a}{3b + b}$$

$$k = \frac{4a}{4b}$$

$$\therefore k = \frac{a}{b}$$

$$\therefore \frac{3a - 5b}{5c + 3b} = \frac{a + 5c}{b - 5a} = \frac{b - c}{a - c} = \frac{a}{b}$$

$4m + n = 12$

$- 2n$. Find the values of the following expression.

Q 35) $\frac{n^2 + m^2}{n^2 - m^2}$ (3 marks)

Ans : $4m + n = 12m - 2n$

$\therefore 4m - 12m = -2n - n$

$\therefore -8m = -3n$

$\therefore \frac{m}{n} = \frac{3}{8}$ ..... (i)

$\therefore \frac{m^2}{n^2} = \frac{9}{64}$ ..... (squaring on both sides)

$\therefore \frac{m^2 + n^2}{m^2 - n^2} = \frac{9 + 64}{9 - 64}$ ..... (using componendo - dividendo)

$\therefore \frac{n^2 + m^2}{n^2 - m^2} = \frac{64 + 9}{64 - 9}$

$\therefore \boxed{\frac{n^2+m^2}{n^2-m^2} = \frac{73}{55}}$

4m + n = 12 – 2n. Find the values of the following expression.

Q 36) $\frac{3n+7m}{3n-7m}$ (3 marks)

Ans : 4m + n = 12m – 2n

$\therefore$ 4m – 12m = –2n – n

$\therefore$ – 8m = –3n

$\therefore \frac{m}{n} = \frac{3}{8}$

$\frac{n}{m} = \frac{8}{3}$ ……. (Invertendo)

$\frac{n}{m} \times \frac{3}{7} = \frac{8}{3} \times \frac{3}{7}$ ……. (multiplying on both sides by $\frac{3}{7}$)

$\frac{3n}{7m} = \frac{24}{21}$

$\therefore \frac{3n+7m}{3n-7m} = \frac{24+21}{24-21}$ …… (using componendo - dividendo)

$\therefore \frac{3n+7m}{3n-7m} = \frac{45}{3}$

$\therefore \boxed{\frac{3n+7m}{3n-7m} = \frac{15}{1}}$

Q 37) which number should be added to 11, 15 and 20 so that resultant numbers are in continued proportion ? (4 marks)

Ans : Let the number to be added to 11, 15 and 20 be x. so that resultant numbers are in continued proportion.

$\therefore (11 + x), (15 + x)$ and $(20 + x)$ are in continued proportion.

$$\frac{(11 + x)}{(15 + x)} = \frac{(15 + x)}{(20 + x)}$$

$$\therefore (15 + x)^2 = (11 + x)(20 + x)$$

$$\therefore 225 + 30x + x^2 = 220 + 11x + 20x + x^2$$

$$\therefore 225 + 30x + x^2 = 220 + 31x + x^2$$

$$\therefore 225 - 220 = x^2 - x^2 + 31x - 30x$$

$$\therefore \boxed{5 = x}$$

Q.38) Which number should be subtracted from each of 22, 32 and 47 in order that the differences would be in continued proportion. (4 marks)

Ans : Let the number to be subtracted to 22, 32 and 47 be x. so that resultant numbers are in continued proportion.

$\therefore (22 - x), (32 - x), (47 - x)$ are in continued proportion.

$$\frac{(22 - x)}{(32 - x)} = \frac{(32 - x)}{(47 - x)}$$

$$\therefore (32 - x)^2 = (22 - x)(47 - x)$$

$$\therefore 1024 - 64x + x^2 = 1034 - 22x - 47x + x^2$$

$$\therefore 1024 - 1034 = 64x - 69x$$

$\therefore\ -10 = -5x$

$\therefore x = \frac{10}{5}$

$\therefore \boxed{x = 2}$

Q 39) If $(x + y)^2$, m and $(x - y)^2$ are in continued proportion, then determine m.

Ans : $(x + y)^2$, m , $(x - y)^2$ are in continued proportion.

$\therefore m^2 = (x + y)^2 \times (x - y)^2$

$\therefore m^2 = (x^2 + 2xy + y^2)(x^2 - 2xy + y^2)$

$\therefore m^2 = x^4 - 2x^3y + x^2y^2 + 2x^3y - 4x^2y^2 + 2xy^3 + y^2x^2$
$- 2xy^3 + y^4$

$\therefore m^2 = x^4 + x^2y^2 + y^2x^2 - 4x^2y^2 + y^4$

$\therefore m^2 = x^4 - 2x^2y^2 + y^4$

$\therefore m^2 = (x^2 - y^2)^2$ ..... [$(a - b)^2 = a^2 - 2ab + b^2$]

$\therefore m = x^2 - y^2$ ..... (taking square root on both sides.)

Q 40) $\frac{a^2 + b^2}{a + c} = a$ complete the following activity. (3 marks)

$a =$ ☐ and $b =$ ☐ Putting these values.

Left Hand side = $\square$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \square$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \square$$

Right Hand side = a

$$= \square$$

$\therefore$ L. H. S = R. H. S

$\therefore$ $\square$

Ans : a = $\boxed{ck^2}$ and b = $\boxed{ck}$ Putting these values.

Left Hand side = $\boxed{\frac{a^2 + b^2}{a + c}}$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \boxed{\frac{c^2 k^4 + c^2 k^2}{c (k^2 + 1)}}$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \boxed{ck^2}$$

Right Hand side $= a$

$$= \boxed{ck^2}$$

$\therefore$ L. H. S = R. H. S

$$\therefore \boxed{\frac{a^2 + b^2}{a + c}} = a$$

➢ Solve the following equations.

Q 41) $\frac{\sqrt{7x+1} + \sqrt{3x+1}}{\sqrt{7x+1} - \sqrt{3x+1}} = \frac{5}{1}$

Ans : $\frac{\sqrt{7x+1} + \sqrt{3x+1}}{\sqrt{7x+1} - \sqrt{3x+1}} = \frac{5}{1}$

$$\therefore \frac{(\sqrt{7x+1} + \sqrt{3x+1}) + (\sqrt{7x+1} - \sqrt{3x+1})}{(\sqrt{7x+1} + \sqrt{3x+1}) - (\sqrt{7x+1} - \sqrt{3x+1})} = \frac{5+1}{5-1}$$

…… (Using componendo - dividendo)

$$\therefore \frac{\sqrt{7x+1} + \sqrt{3x+1} + \sqrt{7x+1} - \sqrt{3x+1}}{\sqrt{7x+1} + \sqrt{3x+1} - \sqrt{7x+1} + \sqrt{3x+1}} = \frac{6}{4}$$

$$\therefore \frac{2\sqrt{7x+1}}{2\sqrt{3x+1}} = \frac{6}{4}$$

$$\therefore \frac{\sqrt{7x+1}}{\sqrt{3x+1}} = \frac{3}{2}$$

$$\therefore \frac{7x+1}{3x+1} = \frac{9}{4}$$

$\therefore 4(7x + 1) = 9(3x + 1)$

$\therefore 28x + 4 = 27x + 9$

$\therefore 28x - 27x = 9 - 4$

$\therefore \boxed{x = 5}$

$\therefore x = 5$

Q 42) $\frac{(5x+2)^2 + (5x-2)^2}{(5x+2)^2 - (5x-2^2)} = \frac{13}{12}$

Ans : $\frac{(5x+2)^2 + (5x-2)^2}{(5x+2)^2 - (5x-2)^2} = \frac{13}{12}$

$$\frac{[(5x+2)^2 + (5x-2)^2] + [(5x+2)^2 - (5x-2)^2]}{[(5x+2)^2 - (5x-2)^2] + [(5x+2)^2 - (5x-2)^2]} = \frac{13+12}{13-12}$$

…… (Using componendo – dividendo)

$$\therefore \frac{(5x+2)^2 + (5x-2)^2 + (5x+2)^2 - (5x-2)^2}{(5x+2)^2 + (5x-2)^2 - (5x+2)^2 + (5x-2)^2} = \frac{25}{1}$$

$$\therefore \frac{2(5x+2)^2}{2(5x-2)^2} = \frac{25}{1}$$

$$\therefore \frac{(5x+2)^2}{(5x-2)^2} = \frac{25}{1}$$

$\therefore \frac{5x+2}{5x-2} = \frac{5}{1}$ …… (taking square root on both sides)

$\therefore 1(5x + 2) = 5(5x - 2)$

$\therefore 5x + 2 = 25x - 10$

$\therefore 5x - 25x = -10 - 2$

$\therefore -20x = -12$

$\therefore x = \frac{12}{20}$

$\therefore \boxed{x = \frac{3}{5}}$

$\therefore x = \frac{3}{5}$

Q 43) $\frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{x+7}{x+9}$

Ans : Let, $\frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{x+7}{x+9} = k$

$\therefore k = \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{3x\,(x+7)}{3x\,(x+9)}$

…… (by theorem on equal ratios)

$\therefore k = \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{3x^2 + 21x}{3x^2 + 27x}$

By theorem on equal ratios we get,

each ratio $k = \frac{3x^2 + 21x + 4 - 3x^2 - 21x}{3x^2 + 27x + 5 - 3x^2 - 27x}$

$k = \frac{4}{5}$

$\therefore \frac{x+7}{x+9} = \frac{4}{5}$ ……. [(i)]

$\therefore 5(x+7) = 4(x+9)$

$\therefore 5x + 35 = 4x + 36$

$\therefore 5x - 4x = 36 - 35$

$\therefore \boxed{x = 1}$

Q 44) $\frac{12x^2+18x+21}{24x^2-30x+9} = \frac{4x+6}{8x-10}$ Complete the following activity.

Let, $\frac{12x^2+18x+21}{24x^2-30x+9} = \frac{4x+6}{8x-10} = \square$

$\therefore \frac{12x^2+18x+21}{24x^2-30x+9} = \square$ …. (by theorem on equal ratios)

$\therefore \frac{12x^2+18x+21}{24x^2-30x+9} = \frac{12x^2+18x}{24x^2-30x}$

by theorem on equal ratios we get,

$\therefore \square = \square$

$\therefore \square = \square \; k = \frac{21}{9}$

$\therefore \square = \frac{7}{3}$ ……. (dividing by 3)

$\therefore \square$

$\therefore 12x + 18 = 56x - 70$

$\therefore$ $\square$

$\therefore$ $88 = 44x$

$\therefore$ $x = \frac{88}{44}$

$\therefore$ $x = \square$

Ans : Let, $\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{4x + 6}{8x - 10} = \boxed{k}$

$\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \boxed{\frac{3x\,(4x + 6)}{3x\,(8x - 10)}}$

……. (by theorem on equal ratios)

$\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{12x^2 + 18x}{24x^2 - 30x}$

by theorem on equal ratios we get,

$\therefore$ each ratio k = $\frac{12x^2 + 18x + 21 - (12x^2 - 18x)}{24x^2 - 30x + 9 - (24x^2 + 30x)}$

$\therefore$ $k = \frac{21}{9} = \frac{7}{3}$ …… (dividing by 3)

$\boxed{\frac{4x + 6}{8x - 10}} = \frac{7}{3}$

$\boxed{3\,(4x + 6) = 7\,(8x - 10)}$

$12x + 18 = 56x - 70$

$$\boxed{18 + 70 = 56x - 12x}$$

$$88 = 44x$$

$$\therefore x = \frac{88}{44}$$

$$\therefore \boxed{x = 2}$$

Q 45) If $\frac{x}{x + 2y + 2} = \frac{y}{y + 2z + x} = \frac{z}{z + 2x + y}$ and $x + y \neq 0$ then show that each ratio is equal to $\frac{1}{4}$

Ans : Let, $\frac{x}{x + 2y + z} = \frac{y}{y + 2z + x} = \frac{z}{z + 2x + y} = k$

by theorem on equal ratios,

$$\therefore k = \frac{x + y + z}{x + 2y + z + y + 2z + x + z + 2x + y}$$

$$\therefore k = \frac{x + y + z}{x + x + 2x + 2y + y + y + z + 2z + z}$$

$$\therefore k = \frac{x + y + z}{4x + 4y + 4z}$$

$$\therefore k = \frac{x + y + z}{4(x + y + z)}$$

$$\therefore k = \frac{1}{4}$$

$$\therefore \boxed{\frac{x}{x + 2y + z} = \frac{y}{y + 2x + x} = \frac{z}{z + 2x + y} = \frac{1}{4}}$$

Q 46) If $\frac{a}{b} = \frac{c}{d}$ then show that $\frac{a}{b} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}$

Ans : a, b, c, d are in continued proportion.

$\therefore$ Let $\frac{a}{b} = \frac{c}{d} = k$

$\therefore$ $\boxed{a = bk}$ and $\boxed{c = dk}$

Left Hand side $\frac{a}{b} = \frac{bk}{b}$

$\therefore$ $\frac{a}{b} = k$

Right Hand side $= \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}$

$= \frac{\sqrt{(bk)^2 + (dk)^2}}{\sqrt{b^2 + d^2}}$

$= \sqrt{\frac{b^2 k^2 + d^2 k^2}{b^2 + d^2}}$

$= \sqrt{\frac{k^2 (b^2 + d^2)}{b^2 + d^2}}$

$= \sqrt{k^2}$

$\therefore$ Right hand side $= k$

$\therefore$ $\sqrt{\frac{a^2 + c^2}{b^2 + d^2}} = k$

$\therefore$ L. H. S. = R. H. S

$$\therefore \boxed{\frac{a}{b} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}}$$

Q 47) Five numbers are in continued proportion. The first term is 4 and last term is 64. Find those numbers.

Ans : Let the numbers in continued Proportion be

a, ak, ak^2, ak^3, ak^4

Here, $a = 4$ and $ak^4 = 64$

$\therefore 4 \times k^4 = 64$

$\therefore k^4 = \frac{64}{4}$

$\therefore k^4 = 16$

$\therefore k = 2$ ..... ($\because 2^4 = 16$)

$\therefore a = 4$

$ak = 4 \times 2 = 8$, $ak^2 = 4 \times (2)^2 = 4 \times 4 = 16$

$ak^3 = 4 \times 2^3 = 4 \times 8 = 32$

$ak^4 = 64$

$\therefore$ the numbers are $4, 8, 16, 32, 64$

Q 48) If a, b, c and d are in proportion, then show that

$$\frac{2a - 3c}{2b - 3d} = \frac{5a + 4c}{5b + 4d}$$

Ans : $\frac{a}{b} = \frac{c}{d} = k$

Putting these value $a = bk$ and $c = dk$

Left Hand side $= \frac{2a - 3c}{2b - 3d}$

$$= \frac{2(bk) - 3(dk)}{2b - 3d} = \frac{2bk - 3dk}{2b - 3d}$$

$$= \frac{k(2b - 3d)}{(2b - 3d)}$$

$$= k$$

Right hand side $= \frac{5a + 4c}{5b + 4d}$

$$= \frac{5(bk) + 4(dk)}{5b + 4d} = \frac{5bk + 4dk}{5b + 4d}$$

$$= \frac{k(5b + 4d)}{5b + 4d}$$

$$= k$$

$\therefore$ L. H. S = R. H. S

$\therefore \frac{2a - 3c}{2b - 3d} = \frac{5a + 4c}{5b + 4d}$

Q 49) $\frac{a}{b} = \frac{b}{c}$ and $a, b, c > 0$ then show that $\frac{a^2+b^2}{b^2} = \frac{(a+c)^2}{b^2+c^2}$

Ans : $\frac{a}{b} = \frac{b}{c} = k$

$a = bk$ and $\boxed{b = ck}$

$\therefore a = (ck)k$

$\therefore \boxed{a = ck^2}$

$a = ck^2$ and $b = ck$

Putting these values $a = ck^2$ and $b = ck$

Left hand side $= \frac{a^2 + b^2}{b^2} = \frac{(ck^2)^2 + (ck)^2}{(ck)^2}$

$= \frac{c^2 k^4 + c^2 k^2}{c^2 k^2}$

$= \frac{c^2 k^2 (k^2 + 1)}{c^2 k^2}$

$= k^2 + 1$

Right hand side $= \frac{(a+c)^2}{b^2 + c^2} = \frac{a^2 + 2ac + c^2}{b^2 + c^2}$

$= \frac{(ck^2)^2 + 2(ck^2)c + c^2}{(ck)^2 + c^2}$

$= \frac{c^2 k^4 + 2c^2 k^2 + c^2}{c^2 k^2 + c^2}$

$$= \frac{c^2\ (k^4 + 2k^2 + 1)}{c^2\ (k^2 + 1)}$$

$$= \frac{(k^2 + 1)\,(k^2 + 1)}{(k^2 + 1)}$$

$$= k^2 + 1$$

$\therefore$ L. H. S = R. H. S

$$\therefore \frac{a^2 + b^2}{b^2} = \frac{(a + c)^2}{b^2 + c^2}$$

50) $\frac{a^2 + b^2}{a + c} = a$ Complete the following activity.

a = ☐ and b = ☐ putting these values.

Left Hand side = ☐

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \square$$

$$= \frac{c^2k^2(k^2 + 1)}{c\,(k^2 + 1)}$$

$$= \square$$

Right hand side = a

$$= \square$$

$\therefore$ L.H.S = R.H.S

$\therefore$ ☐

ck^2 ck

Ans : a = b = putting these values

Left Hand side = $\boxed{\frac{a^2 + b^2}{a + c}}$

$= \frac{(ck^2)^2+(ck)^2}{ck^2+c}$

$= \boxed{\frac{c^2k^2(k^2+1)}{c\,(k^2+1)}}$

$= \frac{c^2k^2(k^2+1)}{c\,(k^2+1)}$

$= \boxed{ck^2}$

Right Hand side = a

$= \boxed{ck^2}$

∴ L.H.S = R.H.S

∴ $\boxed{\frac{a^2 + b^2}{a + c} = a}$

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