10. Division of Polynomials

1. Divide. Write the quotient and the remainder.

(1) $(3x^4 + 2x^3 + 15x^2 + 4x - 4) \div (3x + 2)$

Solution:

$$
\begin{array}{r|l}
3x+2 & x^3 + 5x - 2 \\
\hline
 & 3x^4 + 2x^3 + 15x^2 + 4x - 4 \\
 & \underline{-3x^4 \pm 2x^3} \\
 & 0 + 0 + 15x^2 + 4x - 4 \\
 & \underline{\quad - 15x^2 \pm 10x} \\
 & -6x - 4 \\
 & \underline{\quad \mp 6x \mp 4} \\
 & 0
\end{array}
$$

∴ **Quotient : $x^3 + 5x - 2$**

Remainder : 0

(2) $(x^4 + 2x^3 - 3x^2 + 2x - 4) \div (x^2 + 1)$

Solution :

$$
\begin{array}{r|l}
x^2+1 & x^2 + 2x - 4 \\
\hline
 & x^4 + 2x^3 - 3x^2 + 2x - 4 \\
 & \underline{-x^4 \quad \pm x^2} \\
 & + 2x^3 - 4x^2 + 2x - 4 \\
 & \underline{-2x^3 \quad \pm 2x} \\
 & -4x^2 + 0 - 4 \\
 & \underline{\mp 4x^2 + 0 \mp 4} \\
 & 0 + 0 + 0
\end{array}
$$

∴ **Quotient : $x^2 + 2x - 4$**

Remainder : 0

(3) $(x^4 + 2x^3 - 5x^2 - 13x + 9) \div (x^2 - 3)$

Solution:

$$
\begin{array}{r|l}
 & x^2 + 2x - 2 \\
\hline
x^2 - 3 & x^4 + 2x^3 - 5x^2 - 13x + 9 \\
 & \underline{-x^4 \qquad \mp 3x^2} \\
 & 2x^3 - 2x^2 - 13x + 9 \\
 & \underline{-2x^3 \qquad \mp 6x} \\
 & -2x^2 - 7x + 9 \\
 & \underline{\mp 2x^2 \qquad \pm 6} \\
 & -7x + 3
\end{array}
$$

$\therefore$ **Quotient : $x^2 + 2x - 2$**
Remainder : $-7x + 3$

(4) $(6x^4 - 5x^3 + 3x^2 + 5x - 9) \div (x^2 - 1)$

Solution:

$$
\begin{array}{r|l}
 & 6x^2 - 5x + 9 \\
\hline
x^2 - 1 & 6x^4 - 5x^3 + 3x^2 + 5x - 9 \\
 & \underline{-6x^4 \qquad \mp 6x^2} \\
 & -5x^3 + 9x^2 + 5x - 9 \\
 & \underline{\mp 5x^3 \qquad \pm 5x} \\
 & +9x^2 - 9 \\
 & \underline{-9x^2 \qquad \pm 9} \\
 & 0 \qquad 0
\end{array}
$$

$\therefore$ **Quotient : $6x^2 - 5x + 9$**
Remainder : 0

(5) $(2x^4 - 6x^3 - 5x^2 + 19x - 12) \div (x - 3)$

Solution:

$$
\begin{array}{r|l}
 & 2x^3 - 5x + 4 \\
\hline
x - 3 & 2x^4 - 6x^3 - 5x^2 + 19x - 12 \\
 & -2x^4 \mp 6x^3 \\
\hline
 & 0 \quad -5x^2 + 19x - 12 \\
 & \quad \mp 5x^2 \pm 15x \\
\hline
 & \quad +4x - 12 \\
 & \quad -4x \pm 12 \\
\hline
 & \quad 0 \quad 0
\end{array}
$$

∴ **Quotient : $2x^3 - 5x + 4$**

Remainder : 0

(6) $(2a^2 - 4a^3 + 5a^4 + 8 + 4a) \div (a^2 - 1)$

Solution:

First write the polynomial $(2a^2 - 4a^3 + 5a^4 + 8 + 4a)$ in the descending order of indices and then divide.

∴ **$5a^4 - 4a^3 + 2a^2 + 4a + 8$**

$$
\begin{array}{r|l}
 & 5a^4 - 4a + 7 \\
\hline
a^2 - 1 & 5a^4 - 4a^3 + 2a^2 + 4a + 8 \\
 & -5a^4 \quad \mp 5a^2 \\
\hline
 & \quad -4a^3 + 7a^2 + 4a + 8 \\
 & \quad \mp 4a^3 \quad \pm 4a \\
\hline
 & \quad +7a^2 \quad +8 \\
 & \quad -7a^2 \quad \pm 7 \\
\hline
 & \quad +15
\end{array}
$$

∴ Quotient: $5a^2 - 4a + 7$

Remainder : 15

(7) $(8m^4 - 14m^3 + 3m^2 - 16m + 8) \div (4m - 1)$

Solution:

$$
\begin{array}{r|l}
 & 2m^3 - 3m^2 - 4 \\
\hline
4m - 1 & 8m^4 - 14m^3 + 3m^2 - 16m + 8 \\
 & -8m^4 \mp 2m^3 \\
\hline
 & -12m^3 + 3m^2 - 16m + 8 \\
 & \mp 12m^3 \pm 3m^2 \\
\hline
 & -16m + 8 \\
 & \mp 16m \pm 4 \\
\hline
 & +4
\end{array}
$$

∴ Quotient : $2m^3 - 3m^2 - 4$

Remainder : 4

(8) $(24x^4 - 25x^3 + 6x^2 - 40x + 18) \div (8x - 3)$

Solution:

$$
\begin{array}{r|l}
 & 3x^3 - 2x^2 - 5 \\
\hline
8x - 3 & 24x^4 - 25x^3 + 6x^2 - 40x + 18 \\
 & -24x^4 \mp 9x^3 \\
\hline
 & 0 \quad -16x^3 + 6x^2 - 40x + 18 \\
 & \mp 16x^3 \pm 6x^2 \\
\hline
 & -40x + 18 \\
 & \mp 40x \pm 15 \\
\hline
 & +3
\end{array}
$$

∴ **Quotient :** $3x^3 - 2x^2 - 5$

Remainder : 3

(9) $(3x^4 - x^3 - 6x^2 + 20x + 4) \div (3x - 1)$

Solution:

$$
\begin{array}{r|l}
3x - 1 & x^3 - 2x + 6 \\
\hline
3x^4 - x^3 - 6x^2 + 20x + 4 & \\
-3x^4 \mp x^3 & \\
\hline
0 \quad -6x^3 + 20x + 4 & \\
\mp 6x^3 \pm 2x & \\
\hline
18x + 4 & \\
-18x \mp 6 & \\
\hline
+10 &
\end{array}
$$

∴ **Quotient :** $x^3 - 2x + 6$

Remainder : 10

(10) $(6p^4 - 8p^3 + p^2 + 14p - 20) \div (2p^2 - 3)$

Solution:

$$
\begin{array}{r|l}
2p^2 - 3 & 3p^3 - 4p + 5 \\
\hline
6p^4 - 8p^3 + p^2 + 14p - 20 & \\
-6p^4 \quad \mp 9p^3 & \\
\hline
-8p^3 + 10p^2 + 14p - 20 & \\
\mp 8p^3 \quad \pm 12p & \\
\hline
+10p^2 + 2p - 20 & \\
\pm 10p^2 \quad \mp 15 & \\
\hline
+2p - 5 &
\end{array}
$$

$\therefore$ **Quotient : $3p^3 - 4p + 5$**

Remainder : $2p - 5$

2. Divide. Write the quotient and the remainder

(1) $(2x^3 - 3x^2 + 4x + 11) \div (x^2 - 3)$

Solution:

$$
\begin{array}{r|l}
 & 2x - 3 \\
\hline
x^2 - 3 & 2x^3 - 3x^2 + 4x + 11 \\
 & \underline{-\,2x^3 \qquad \mp 6x} \\
\hline
 & -3x^2 + 10x + 11 \\
 & \underline{\mp 3x^2 \qquad \pm 9} \\
\hline
 & +\,10x + 2
\end{array}
$$

$\therefore$ **Quotient : $2x - 3$**

Remainder : $10x + 2$

(2) $(2x^3 + 7x^2 + 6x + 1) \div (x + 1)$

Solution:

$$
\begin{array}{r|l}
 & 2x^2 + 5x + 1 \\
\hline
x + 1 & 2x^3 + 7x^2 + 6x + 1 \\
 & \underline{-\,2x^3 \pm 2x^2} \\
\hline
 & 5x^2 + 6x + 1 \\
 & \underline{-\,5x^2 \pm 5x} \\
\hline
 & x + 1 \\
 & \underline{-\,x \pm 1} \\
\hline
 & 0
\end{array}
$$

$\therefore$ **Quotient : :** $2x^2 + 5x + 1$

Remainder : 0

(3) $(2x^3 + x^2 + 4x - 10) \div (2x - 3)$

Solution:

$$
\begin{array}{r}
x^2 + 2x + 5 \\
\hline
2x - 3 \;\big)\; 2x^3 + x^2 + 4x - 10 \\
-2x^3 \mp 3x^2 \\
\hline
4x^2 + 4x - 10 \\
-4x^2 \mp 6x \\
\hline
+10x - 10 \\
\pm 10x \mp 15 \\
\hline
5
\end{array}
$$

$\therefore$ **Quotient :** $x^2 + 2x + 5$

Remainder : 5

(4) $(a^3 - 2a^2 - a + 1) \div (a - 2)$

Solution:

$$
\begin{array}{r}
a^2 - 1 \\
\hline
a - 2 \;\big)\; a^3 - 2a^2 - a + 1 \\
-a^3 \mp 2a^2 \\
\hline
-a + 1 \\
\mp a \pm 2 \\
\hline
-1
\end{array}
$$

∴ **Quotient :** $a^2 - 1$

Remainder : -1

(5) $(2n^3 + n^2 + n + 9) \div (2n - 1)$

Solution:

$$
\begin{array}{r|l}
 & n^2 + n + 1 \\
\hline
2n - 1 & 2n^3 + n^2 + n + 9 \\
 & -2n^3 \mp n^2 \\
\hline
 & \quad 2n^2 + n + 9 \\
 & \quad -2n^2 \mp n \\
\hline
 & \qquad + 2n + 9 \\
 & \qquad \pm 2n \mp 1 \\
\hline
 & \qquad\qquad + 10
\end{array}
$$

∴ **Quotient :** $n^2 + n + 1$

Remainder : 10

(6) $(b^3 - 5b^2 + 4b + 13) \div (b + 2)$

Solution:

$$
\begin{array}{r|l}
 & b^2 - 7b + 18 \\
\hline
b + 2 & b^3 - 5b^2 + 4b + 13 \\
 & -b^3 \pm 2b^2 \\
\hline
 & \quad -7b^2 + 4b + 13 \\
 & \quad \mp 7b^2 \mp 14b \\
\hline
 & \qquad + 18b + 13 \\
 & \qquad \pm 18b \pm 36 \\
\hline
 & \qquad\qquad - 23
\end{array}
$$

∴ Quotient : $b^2 - 7b + 18$

Remainder : -23

(7) $(3y^3 - y^2 - 23y - 12) \div (3y + 2)$

Solution:

$$
\begin{array}{r|l}
 & y^2 - y - 6 \\
\hline
3y + 2 & 3y^3 - y^2 - 23y - 12 \\
 & -3y^3 \pm 2y \\
\hline
 & -3y^2 - 23y - 12 \\
 & \mp 3y^2 \mp 2y \\
\hline
 & -21y - 12 \\
 & \mp 18y \mp 12 \\
\hline
 & -3y + 0
\end{array}
$$

∴ Quotient : $y^2 - y - 6$

Remainder : $-3y$

(8) $(m^3 - 2m^2 + 7m - 10) \div (m - 4)$

Solution:

$$
\begin{array}{r|l}
 & m^2 + 2m + 15 \\
\hline
m - 4 & m^3 - 2m^2 + 7m - 10 \\
 & -m^3 \mp 4m^2 \\
\hline
 & 2m^2 + 7m - 10 \\
 & -2m^2 \mp 8m \\
\hline
 & +15m - 10 \\
 & \pm 15m \mp 60 \\
\hline
 & +50
\end{array}
$$

∴ **Quotient : $m^2 + 2m + 15$**

Remainder : 50

(9) $(4x^3 - 20x^2 + 8x + 25) \div (2x - 4)$

Solution:

$$
\begin{array}{r}
2x^2 - 6x - 8 \\
2x - 4 \overline{)\,4x^3 - 20x^2 + 8x + 25} \\
\underline{-\,4x^3 \mp 8x^2} \\
0 \quad -12x^2 + 8x + 25 \\
\underline{\mp\,12x^2 \pm 24x} \\
-16x + 25 \\
\underline{\mp\,16x \pm 32} \\
0 - 7
\end{array}
$$

∴ **Quotient : $2x^2 - 6x - 8$**

Remainder : -7

(10) $(3x^2 + x^3 - 6x + 18) \div (x - 3)$

Solution:

First write the polynomial $3x^2 + x^3 - 6x + 18$ in the descending order of indices and then divide.

∴ $x^3 + 3x^2 - 6x + 18$

$$
\begin{array}{r|l}
 & x^2 - 6 \\
\hline
x - 3 & x^3 + 3x^2 - 6x + 18 \\
 & -x^3 \mp 3x^2 \\
\hline
 & 6x^2 - 6x + 18 \\
 & \mp 6x \pm 18 \\
\hline
 & 6x^2 + 0 + 0
\end{array}
$$

∴ Quotient : $x^2 - 6$

Remainder : $6x^2$

3. Divide. Write the quotient and the remainder.

(1) $(x^2 + 4x - 8) \div (x - 1)$

Solution:

$$
\begin{array}{r|l}
 & x + 5 \\
\hline
x - 1 & x^2 + 4x - 8 \\
 & -x^2 \mp x \\
\hline
 & 5x - 8 \\
 & \pm 5x \mp 5 \\
\hline
 & -3
\end{array}
$$

∴ Quotient : $x + 5$

Remainder : -3

(2) $\left(x^2 + 5x - 6\right) \div (x + 2)$

Solution:

$$
\begin{array}{r|l}
 & x + 3 \\
\hline
x + 2 & x^2 + 5x - 6 \\
 & -x^2 \pm 2x \\
\hline
 & 3x - 6 \\
 & -3x \pm 6 \\
\hline
 & -12
\end{array}
$$

∴ **Quotient :** $x + 3$

Remainder : -12

(3) $\left(5x^2 - 11x - 17\right) \div (x - 3)$

Solution:

$$
\begin{array}{r|l}
 & 5x + 4 \\
\hline
x - 3 & 5x^2 - 11x - 17 \\
 & -5x^2 \mp 15x \\
\hline
 & +4x - 17 \\
 & \pm 4x \mp 12 \\
\hline
 & 0 - 5
\end{array}
$$

∴ **Quotient :** $5x + 4$

Remainder : -5

(4) $\left(3m^2 + 24m + 40\right) \div (3m + 9)$

Solution:

$$
\begin{array}{r}
m + 4 \\
3m + 9 \overline{)\ 3m^2 + 24m + 40} \\
\underline{-\ 3m^2 \pm 9m} \\
15m + 40 \\
\underline{\mp\ 12m \pm 36} \\
3m + 4
\end{array}
$$

∴ **Quotient :** $m + 4$

Remainder : $3m + 4$

(5) $\left(12x^2 + 5x - 30\right) \div (3x + 5)$

Solution:

$$
\begin{array}{r}
4x - 5 \\
3x + 5 \overline{)\ 12x^2 + 5x - 30} \\
\underline{-\ 12x^2 \pm 20x} \\
-\ 15x - 30 \\
\underline{\mp\ 15x \mp 25} \\
-\ 5
\end{array}
$$

∴ **Quotient :** $4x - 5$

Remainder : -5

(6) $\left(x^3 - 4x^2 - x\right) \div x$

Solution:

$$
\begin{array}{r}
x^2 - 4x - 1 \\
\hline
x \;\big)\; x^3 - 4x^2 - x \\
\underline{-\,x^3} \\
-\,4x^2 - x \\
\underline{\mp\,4x^2} \\
-\,x \\
\underline{\mp\,x} \\
0
\end{array}
$$

∴ **Quotient :** $x^2 - 4x - 1$

Remainder : 0

(7) $\left(4x^4 - 6x^3 + 2x^2 - 6x + 10\right) \div 2x$

Solution:

$$
\begin{array}{r}
2x^3 - 3x^2 + x - 3 \\
\hline
2x \;\big)\; 4x^4 - 6x^3 + 2x^2 - 6x + 10 \\
\underline{-\,4x^4} \\
-\,6x^3 + 2x^2 - 6x + 10 \\
\underline{\mp\,6x^3} \\
+\,2x^2 - 6x + 10 \\
\underline{\pm\,2x^2} \\
-\,6x + 10 \\
\underline{\pm\,6x} \\
+\,10
\end{array}
$$

∴ Quotient : $2x^3 - 3x^2 + x - 3$

Remainder : 10

(8) $\left(x^4 - x^3 + x^2 + x - 3\right) \div x^2$

Solution:

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2 & x^4 - x^3 + x^2 + x - 3 \\
 & -x^4 \\
\hline
 & 0 - x^3 + x^2 + x - 3 \\
 & \mp x^3 \\
\hline
 & +x^2 + x - 3 \\
 & -x^2 \\
\hline
 & +x - 3
\end{array}
$$

∴ Quotient : $x^2 - x + 1$

Remainder : $x - 3$

(9) $\left(x^5 - x^4 - x^3 - x^2 - x\right) \div x^3$

z **Solution:**

$$
\begin{array}{r|l}
 & x^2 - x - 1 \\
\hline
x^3 & x^5 - x^4 - x^3 - x^2 - x \\
 & -x^5 \\
\hline
 & -x^4 - x^3 - x^2 - x \\
 & \mp x^4 \\
\hline
 & -x^3 - x^2 - x \\
 & \mp x^3 \\
\hline
 & -x^2 - x
\end{array}
$$

∴ Quotient : $x^2 - x - 1$

Remainder : $-x^2 - x$

(10) $(2x^3 + 4x - 1) \div 2x$

Solution:

$$
\begin{array}{r|l}
 & x^2 + 2 \\
\hline
2x & 2x^3 + 4x - 1 \\
 & -2x^3 \\
\hline
 & +4x - 1 \\
 & \pm 4x \\
\hline
 & -1
\end{array}
$$

∴ Quotient : $x^2 + 2$

Remainder : -1

4. Complete the following activity.

(1) $\left(8x^2 + 2x - 7\right) \div 2x$

$$
\begin{array}{r|l}
 & \square + \square \\
\hline
2x & 8x^2 + 2x - 7 \\
 & -\square \\
\hline
 & +\square - 7 \\
 & -\square \\
\hline
 & \square
\end{array}
$$

∴ **Quotient :** $\square$

Remainder : $\square$

Solution:

$$
\begin{array}{r|l}
 & \boxed{4x} + \boxed{1} \\
\hline
2x & 8x^2 + 2x - 7 \\
 & -\boxed{8x^2} \\
\hline
 & +\boxed{2x} - 7 \\
 & -\boxed{2x} \\
\hline
 & \boxed{-7}
\end{array}
$$

∴ **Quotient :** $\boxed{4x + 1}$

Remainder : $\boxed{-7}$

(2) $\left(25x^3 - 8x^2\right) \div 5x$

$$\begin{array}{r|l} \square - \square & \\ \hline \square & 25x^3 - 8x^2 \\ - \square & \\ \hline - \square & \\ - \square & \\ \hline 0 & \end{array}$$

∴ **Quotient :** $\square$

Remainder : $\square$

Solution:

$$\begin{array}{r|l} 5x^2 - \frac{8}{5}x & \\ \hline 5x & 25x^3 - 8x^2 \\ - 25x^3 & \\ \hline - 8x^2 & \\ \mp 8x^2 & \\ \hline 0 & \end{array}$$

∴ **Quotient :** $5x^2 - \frac{8}{5}x$

Remainder : 0

(3) $(x^3 + 7x - 6) \div (x + 3)$

Write the polynomial $(x^3 + 7x - 6)$ **in descending order and divide.**

$\therefore x^3 + \square + \square - 6$

$$
\begin{array}{r}
\square + 3x - \square \\
x + 3 \overline{)\ x^3 + \square + \square - 6} \\
-\square + \square \\
\hline
\square + \square\, x - 6 \\
- \\
\square + \square \\
\hline
+ \square \quad - 6 \\
- \square \quad - \square \\
\hline
+ \square
\end{array}
$$

$\therefore$ **Quotient :** $\square$

Remainder : $\square$

Solution :

Write the polynomial $(x^3 + 7x - 6)$ **in descending order and divide.**

$\therefore x^3 + \boxed{0x^2} + \boxed{7x} - 6$

$$
\begin{array}{r|l}
 & \boxed{x^2} + 3x - \boxed{2} \\
\hline
x + 3 & x^3 + \boxed{0x^2} + \boxed{7x} - 6 \\
 & -\boxed{x^3} + \boxed{3x^2} \\
\hline
 & +\boxed{3x^2} + \boxed{7}x - 6 \\
 & \pm\boxed{3x^2} \pm \boxed{9x} \\
\hline
 & -\boxed{2x} - 6 \\
 & \mp\boxed{2x} \mp \boxed{6} \\
\hline
 & +\boxed{0}
\end{array}
$$

$\therefore$ **Quotient :** $\boxed{x^2 + 3x - 2}$

Remainder : $\boxed{0}$

5. Identify which of the following polynomials.

(1) $\frac{5}{3}x^2 + 6x + 7$

Ans : $\frac{5}{3}x^2 + 6x + 7$ **is a polynomial.**

(2) $\sqrt{2}.\sqrt{y} + y^{\frac{1}{2}} + 3$

Ans : $\sqrt{2}.\sqrt{y} + y^{\frac{1}{2}} + 3$ **is not a polynomial because the index of** $y^{\frac{1}{2}}$ **is** $\frac{1}{2}$ **which is fraction and not a whole number.**

(3) $\sqrt{5}\,.\,y^2 + 4$

Ans : $\sqrt{5}\,.\,y^2 + 4$ **is a polynomial.**

(4) $7x^2 + \frac{1}{x} + 5$

Ans : $7x^2 + \frac{1}{x} + 5$ **i.e.** $7x^2 + x^{-1} + 5$ **this is not a polynomial because the index of** x^{-1} **is** -1 **which is negative.**

(5) $a^2 + 7a + 8$

Ans : $a^2 + 7a + 8$ **is a polynomial.**

6. Write the degree of the following polynomial.

(1) $3x^2 + 7x + 2$

Ans : The degree of the polynomial $3x^2 + 7x + 2$ **is 2.**

(2) -11

Ans : The degree of the polynomial -11 **is 0.**

(3) $8 + 6y^2 + y + y^3$

Ans : The degree of the polynomial $8 + 6y^2 + y + y^3$ **is 0.**

(4) $5x + 2$

Ans : The degree of the polynomial $5x + 2$ **is 1.**

(5) $(m^2 + 2m + 1) - (m^2 + 2m + 1)$

Ans : $(m^2 + 2m + 1) - (m^2 + 2m + 1) = 0$

Therefore the polynomial $(m^2 + 2m + 1) - (m^2 + 2m + 1)$ has no degree.

7. Write the indices of the variables in descending order of the following polynomial.

(1) $x^4 + x^2 + x^5 + 3$

Ans : $x^5 + x^4 + 0x^3 + x^2 + 0x + 3$

(2) $1 + x^2 + x^3 - 2x$

Ans : $x^3 + x^2 - 2x + 1$

(3) $m^2 - 5m + m^3$

Ans : $m^3 + m^2 - 5m$

(4) $1 - 8a + 2a^4 + 3a^3$

Ans : $2a^4 + 3a^3 - 8a + 1$

(5) $2x + 4x^4 - 3x^2$

Ans : $4x^4 - 3x^2 + 2x$

8. Match the following pairs.

'A' Group	'B' Group
(1) $5x^3$	(*a*) Binomial polynomial
(2) $x^2 + 2x + 1$	(b) Monomial polynomial
(3) $x^2 - 3$	(c) Trinomial polynomial

Ans :

'A' Group	'B' Group
(1) $5x^3$	(b) Monomial polynomial
(2) $x^2 + 2x + 1$	(c) Trinomial polynomial
(3) $x^2 - 3$	(*a*) Binomial polynomial

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