# 7. Variation

**1. The grass in the field is enough for 30 cows for 120 days. In the same field if 50 cows are set for grazing how many days it will be enough for them.**

**Solution :**

**Suppose the number of cows be *x* and the number of days for grazing be y.**

**There are inverse variation between the number of cows and the number of days for grazing.**

$$\therefore x \propto \frac{1}{y}$$

$\therefore x \times y = k$ ….. (k is constant of variation)

When $x = 30$ cows then y = 120 days

$\therefore k = x \times y = 30 \times 120 = 3600$

$\therefore xy = 3600$…… Equation of variation

Now, $x = 50$ then $y = ?$

$\therefore xy = 3600$

$\therefore 50 \times y = 3600$

$$\therefore y = \frac{3600}{50}$$

$$\therefore y = 72$$

$\therefore$ 72 days grass will be enough for 50 cows.

**2. A car running at a speed covers a distance of 378 km using 9 litres of diesel. How much diesel will be filled it to cover a distance of 231 km.**

**Solution :**

**Let us assume the travelling distance of a car to be $x$ and filled diesel in a car to be y.**

**There is a direct variation in the travelling distance of a car and filled diesel in a car.**

$\therefore x \propto y$

$\therefore x = \mathrm{k}y$

When $= 378$ , $y = 9$ is given

$\therefore x = \mathrm{k}y$ ……. (k is constant of variation)

$\therefore 378 = \mathrm{k} \times 9$

$\therefore \mathrm{k} = \frac{378}{9}$

$\therefore \mathrm{k} = 42$ …….. (constant of variation)

Equation of variation is $x = 42y$

$\therefore$ If $x = 231$ then $y = ?$

$\therefore 231 = 42 \times y$

$\therefore y = \frac{231}{42}$

$\therefore y = 5.5$

**$\therefore$ 5.5 litres diesel will be fitted in a car to cover a distance of 231 km.**

**3. Mandar can typed 4500 english words in 25 minutes on computer. How many words will be typed in 13 second on computer?**

**Solution :**

**Suppose typing words to be $x$ and time taken to type be y. There is a direct variation in typing words and time.**

$\therefore x \propto y$

$\therefore x = ky$ …….(k is a constant of variation)

If $x = 4500$ words then

$y = 25$ minutes $= 25 \times 60 = 1500$ sec. is given.

$\therefore x = ky$

$\therefore 4500 = k \times 1500$

$\therefore k = \frac{4500}{1500}$

$\therefore k = 3$

$\therefore$ Equation of variation is $x = 3y$

$\therefore$ if $y = 13$ sec. then $x = ?$

$\therefore x = 3 \times 13$

$\therefore x = 39$

$\therefore$ **Mandar take 13 seconds to type 39 english words in 13 seconds.**

**4. 6 workers take 12 days to complete the work of a road. How many more workers should be employed, if the same work is to be completed in 9 days ?**

**Solution :**

**Suppose, the number of days be $x$ and number of workers be y.**

**There is inverse variation in days and workers.**

$$\therefore x \propto \frac{1}{y}$$

$$\therefore x \times y = k \ldots\ldots\ \text{(k is constant of variation)}$$

$$\text{If } x = 12 \text{ then } y = 6 \text{ is given.}$$

$$\therefore k = x \times y = 12 \times 6 = 72$$

$$\text{If } x = 9 \text{ then } y = ?$$

$$x \times y = 72$$

$$\therefore 9 \times y = 72$$

$$\therefore y = \frac{72}{9}$$

$$\therefore y = 8$$

**∴ 8 workers will be required.**

**5. Dhaval earns Rs. 4225 in 13 days. How many days he has to work to earn Rs. 17225**

**Solution :**

**Suppose earned money $= x$ and required days $=$ y**

**There is direct variation in earned money and required days.**

$$\therefore x \propto y \ldots\ldots. \text{ (k is constant of variation)}$$

**If $x = 4225$ then y$=$ 13 is given**

$$\therefore k = x \times y$$

$$\therefore 4225 = k \times 13$$

$$\therefore k = 325$$

$\therefore$ **Equation of variation is $x = 325$ y**

**If y $=$ 17225 then y $=$ ?**

$$\therefore 17225 = 325y$$

$$\therefore y = \frac{17225}{325}$$

$$\therefore y = 53$$

$\therefore$ **Dhaval works 53 days to earn Rs. 17225.**

**2. Solve the following questions :**

**1)** $x \propto \frac{1}{\sqrt{y}}$**, when** $x = 20$ **,** $y = 4$**.**

**Find (i) y when** $x = 8$

**(ii)** $x$ **when y = 6**

**Solution :**

$$x \propto \frac{1}{\sqrt{y}}$$

$$\therefore x \times \sqrt{y} = k \text{ ..... (I)}$$

If $x = 20$ then $y = 4$

$$\therefore 20 \times \sqrt{4} = k$$

$$\therefore 20 \times 2 = k$$

$$\therefore k = 40 \text{ ...... (k is constant)}$$

$$\therefore x \times \sqrt{y} = 40$$

(i) if $x = 8$ then $y = ?$

$$x \times \sqrt{y} = 40$$

$$\therefore 8 \times \sqrt{y} = 40$$

$$\therefore \sqrt{y} = \frac{40}{8}$$

$$\therefore \sqrt{y} = 5$$

Square on both the sides

$$\therefore \left(\sqrt{y}\right)^2 = (5)^2$$

$$\therefore y = 25$$

$$\therefore x = 8 \text{ then } y = 25$$

(ii) if $y = 16$ then $x = ?$

$$x \times \sqrt{y} = 40$$

$$x \times \sqrt{16} = 40$$

$\therefore x = 4 = 40$ ..($\because$ 4 is the square root of 16)

$$\therefore x = \frac{40}{4}$$

$$\therefore x = 10$$

$\therefore y = 16$ then $x = 10$

**2) $x \propto \sqrt{y}, y = 64$ then $x = 56$**

**Find (i) the value of y if**

**(ii) the value of if y = 36**

**Solution :**

$$x \propto \sqrt{y}$$

$$\therefore x = k\sqrt{y}$$

$\therefore k = \frac{x}{\sqrt{y}}$ ….. (constant of variation)

if $y = 64$ the $x = 56$ …… (given)

$$\therefore k = \frac{56}{\sqrt{64}}$$

$\therefore k = \frac{56}{8}$ ….. (Constant of variation)

(i) if $x = 126$ then $y = ?$

$$\because \frac{x}{\sqrt{y}} = 7$$

$$\therefore \frac{126}{\sqrt{y}} = 7$$

$$\therefore \sqrt{y} = \frac{126}{7}$$

$$\therefore \sqrt{y} = 18$$

**Square on both the sides**

$$\therefore \left(\sqrt{y}\right)^2 = (18)^2$$

$$\therefore y = 324$$

$$\therefore x = 126 \text{ then } y = 324$$

**(ii) if $y = 36$ then $x = ?$**

$$\because \frac{x}{\sqrt{y}} = 7$$

$$\therefore \frac{x}{\sqrt{36}} = 7$$

$$\therefore \frac{x}{6} = 7$$

$$\therefore x = 7 \times 6$$

$$\therefore x = 42$$

$$\therefore y = 36 \text{ then } x = 42$$

**3)** $a \propto \frac{1}{\sqrt[3]{b}}$, $a = 15$ then $b = 27$.

Find (i) the value of b if $a = 5$

(ii) the value of $a$ if $b = 512$

**Solution :** $a \propto \frac{1}{\sqrt[3]{b}}$

$\therefore a \times \sqrt[3]{b} = k$ …… (k is constant of variation)

if $a = 15$ then $b = 27$

$\therefore 15 \times \sqrt[3]{27} = k$

$\therefore 15 \times 3 = k$

$\therefore k = 45$

(i) if $a = 5$ then $b = ?$

$a \times \sqrt[3]{b} = 45$

$\therefore 5 \times \sqrt[3]{b} = 45$

$\therefore \sqrt[3]{b} = \frac{45}{5}$

$\therefore \sqrt[3]{b} = 9$

Square on both the sides

$\therefore \left(\sqrt[3]{b}\right)^3 = (9)^3$

$\therefore b = 729$

$\therefore a = 5$ then $b = 729$

(ii) if $b = 512$ then $a = ?$

$a \times \sqrt[3]{b} = 45$

$\therefore a \times \sqrt[3]{512} = 45$

6 is the cube root of 512

$\therefore a \times 6 = 45$

$\therefore a = \frac{45}{6}$

$\therefore$ b = 512 then $a = \frac{45}{6}$

**4) $x \propto$ y. $x$ = 0.15 then y = 0.03.**

**Find (i) the value of $x$ if y =0.06**

**(ii) the value of y if $x$ = 0.025**

**Solution :** $x \propto$ y

$\therefore x$ = ky …… (k is constant of variation)

If $x$ = 0.15 then y = 0.03 ….. (given)

$\therefore 0.15 = k \times 0.03$

$\therefore k = \frac{0.15}{0.03} = \frac{0.15 \times 100}{0.03 \times 100} = \frac{15}{3}$

$\therefore k = 5$

(i) if y = 0.06 then $x$ = ?

$\therefore x = 5 \times y$

$\therefore x = 5 \times 0.06$

$\therefore x = 0.3$

$\therefore$ y = 0.06 then $x$ = 0.3

(ii) if $x = 0.025$ then y = ?

$\therefore x = 5 \times y$

$\therefore 0.025 = 5 \times y$

$\therefore y = \frac{0.025}{5}$

$\therefore y = 0.005$

$\therefore x = 0.025$ then $y = 0.005$

**3. Find the constant of variation and the equation of variation in the following.**

**1) $x$ varies inversely as y, when $x$ =30 then y = 2.5**

**Solution : $x$ inversely varies as y**

$\therefore x \propto \frac{1}{y}$

$\therefore x \times y = k$ …….. (I)

If $x = 30$ then $y = 2.5$

$\therefore k = 30 \times 2.5 = 75$ ….. (constant of variation)

Put the value of k = 75 in $eq^n$. (I)

$xy = 75$ …… this is the equation of variation

$\therefore$ k = 75 is the constant of variation and $xy = 75$ is the constant of variation.

**2) m varies directly as n. when m = 19.5 , n = 6.5**

**Solution : m varies directly as n**

$\therefore m \propto n$

$\therefore m = kn$ …….. (I)

When m = 19.5 , n = 6.5 , is given

$\therefore 19.5 = k \times 6.5$

$\therefore k = \frac{19.5}{6.5}$

$\therefore$ **k = 3 ……. (constant of variation)**

**Put the value of k = 3 in eq$^n$. (I)**

$\therefore$ **m = 3n is the equation of variation**

$\therefore$ **k = 3 is the constant of variation and m = 3n is the equation of variation.**

**3) $x \propto \frac{1}{y}$ if $x = 0.04$ then y = 0.50**

**Solution : $x$ inversely varies as y**

$$\therefore\ x \propto \frac{1}{y}$$

$$\therefore\ x \times y = k \ \ldots\ldots\ldots\ (I)$$

**If $x = 0.04$ then y = 0.50**

$\therefore$ **k = 0.04 × 0.50 = 0.02….. (constant of variation)**

**Put the value of k = 0.02 in eq$^n$. (I) , we get**

$xy = 0.02$ **…….. (equation of variation)**

$\therefore$ **k = 0.02 is the constant of variation and $xy = 0.02$ is the equation of variation.**

**4) q varies directly as cube of p. When q = 108 then p = 3**

**Solution : q varies directly as cube of p.**

$$\therefore q \propto (p)^3$$

$$\therefore q = kp^3 \ \ldots\ldots\ldots\ (I)$$

**if q = 108 then p = 3**

$$\therefore 108 = k \times (3)^3$$

$\therefore 108 = k \times 27$

$\therefore k = \frac{108}{27}$

$\therefore k = 4$ ……. (constant of variation)

Put the value of $k = 4$ in $eq^n$. (I) , we get

$\therefore q = 4p^3$

$\therefore k = 4$ is the constant of variation and $q = 4p^3$ is the equation of variation.

**4. solve the following.**

**1) If $m \propto n$ and when $m = 15$ , $n = 12$. Find the value of n, when $m = 10$**

Solution : $m \propto n$

$\therefore m = kn$ ……. (I)

if $m = 15$ then $n = 12$

$\therefore 15 = k \times 12$

$\therefore k = \frac{15}{12}$

$\therefore k = \frac{5}{4}$ ……… (constant of variation)

Put the value of $k = \frac{5}{4}$ in $eq^n$. (I) , We get

$\therefore m = \frac{5}{4} \times n$ ……. ($eq^n$. of variation)

When $m = 10$ then $n = ?$

$\therefore 10 = \frac{5}{4} \times n$

$$\therefore n = \frac{10 \times 4}{5}$$

$$\therefore n = 8$$

$$\therefore m = 10 \text{ then } n = 8$$

**2) If $b \propto a$ when $b = 6$ then $a = 14$. Find the value of, when $b = 9$.**

Solution : $b \propto a$

$$\therefore b = ka \ldots\ldots. (I)$$

$$\text{If } b = 6 \text{ then } a = 14$$

$$\therefore 6 = k \times 14$$

$$\therefore k = \frac{6}{14}$$

$$\therefore k = \frac{3}{7}$$

$$\text{When } b = 9 \text{ then } a = ?$$

$$9 = k \times a$$

$$\therefore 9 = \frac{3}{7} \times a$$

$$\therefore a = \frac{9 \times 7}{3}$$

$$\therefore a = 21$$

$$\therefore b = 9 \text{ then } a = 21$$

**3) $x \propto \frac{1}{y}$ when $x = 22$ then $y = 5$. Find the value of $x$ when $y = 2.2$**

Solution : $x \propto \frac{1}{y}$

$\therefore xy = k$

When $x = 22$ then $y = 5$

$\therefore 22 \times 5 = k$

$\therefore k = 110$

When $y = 2.2$ then $x = ?$

$\therefore x \times 2.2 = 110$

$\therefore x = \frac{110}{2.2}$

$\therefore x = 50$

$\therefore y = 2.2$ then $x = 50$

**4) $x \propto y$. When $x = 176$ then $y = 8$. Find the value of $x$, if $y = 3.5$**

Solution : $x \propto y$

$\therefore x = ky$

When $x = 176$ then $y = 8$

$\therefore 176 = k \times 8$

$\therefore k = \frac{176}{8}$

$\therefore k = 22$

If $y = 3.5$ then $x = ?$

$\therefore x = 22 \times 3.5$

$\therefore x = 77$

$\therefore y = 3.5$ then $x = 77$

**5. If p varies directly as q, complete the following table.**

| | | | | |
|---|---|---|---|---|
| p | 4 | ……… | ……… | ……… |
| q | 180 | 225 | ……… | 450 |

**Solution : p varies directly as q.**

$\therefore p \propto q$

$\therefore p = kq$

if $p = 4$ then $q = 180$

$\therefore 4 = k \times 180$

$\therefore k = \frac{4}{180}$

$\therefore k = \frac{1}{45}$ …….. (constant of variation)

(i) if $q = 225$ then $p = ?$

$\therefore p = \frac{1}{45} q$ …… (equation of variation)

$\therefore p = \frac{1}{45} \times 225$

$\therefore p = 5$

$\therefore p = 225$ then $p = 5$

(ii) if $p = 8$ then $q = ?$

$\because p = \frac{1}{45} q$ ……. (equation of variation)

$\therefore 8 = \frac{1}{45} q$

$\therefore q = 8 \times 45$

$\therefore q = 360$

$\therefore p = 8$ then $q = 360$

(iii) if $q = 450$ then $p = ?$

$\because p = \frac{1}{45} q$ ……. (equation of variation)

$\therefore p = \frac{1}{45} \times 450$

$\therefore p = 10$

$\therefore q = 450$ then $p = 10$

| p | 4 | 5 | 8 | 10 |
|---|---|---|---|---|
| q | 180 | 225 | 360 | 450 |

**6. If m varies inversely as n then complete the following table.**

| | | | | | |
|---|---|---|---|---|---|
| m | 18 | 24 | ……… | ……… | 20 |
| n | 8 | ……… | 12 | 4 | ……… |

Solution : m varies inversely as n

$\therefore m \propto \frac{1}{n}$

$\therefore m \times n = k$

if $m = 18$ then $n = 8$

$\therefore k = 144$ ……. (constant of variation)

(i) if $m = 24$ then $n = ?$

$\because m \times n = 144$

$\therefore 24 \times n = 144$

$\therefore n = \frac{144}{24}$

$\therefore n = 6$

$\therefore m = 24$ then $n = 6$

(ii) When $n = 12$ then $m = ?$

$\because m \times n = 144$

$\therefore m \times 12 = 144$

$\therefore m = \frac{144}{12}$

$\therefore m = 12$

$\therefore n = 12$ then $m = 12$

(iii) $n = 4$ then $m = ?$

$\because m \times n = 144$

$\therefore m \times 4 = 144$

$\therefore m = \frac{144}{4}$

$\therefore m = 36$

$\therefore n = 4$ then $m = 36$

(iv) if $m = 20$ then $n = ?$

$\because m \times n = 144$

$\therefore 20 \times n = 144$

$\therefore n = \frac{144}{20}$

$\therefore n = 7.2$

$\therefore$ m = 20 then n = 7.2

**Table :**

| | | | | | |
|---|---|---|---|---|---|
| m | 18 | 24 | 12 | 36 | 20 |
| n | 8 | 6 | 12 | 4 | 7.2 |

**7. Which of the following examples are of direct variation and inverse variation ?**

**1)**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 20 | 10 | 65 | 45 | 5 |
| y | 8 | 4 | 26 | 18 | 2 |

**Ans : From the given table, the ratio of $x$ and y in each pair is $\frac{5}{2}$ . It is constant. So $x$ and y are in direct variation.**

**2)**

| | | | | |
|---|---|---|---|---|
| $a$ | 10 | 15 | 6 | 24 |
| b | 12 | 8 | 20 | 5 |

**Ans : From the given table. The product of a and b in each pair is 120. It is constant . So a and b are in inverse proportion.**

**3)**

| | | | | |
|---|---|---|---|---|
| Number of workers (**m**) | 5 | 8 | 10 | 13 |
| Granted amount (**n**) | 350 | 560 | 700 | 910 |

**Ans : From the given table, the ratio of number of workers (m) and the granted amount in each pair is $\frac{1}{70}$, which is constant.**

**Therefore there is direct proportion between workers and granted amount.**

**4)**

| Number of workers ($x$) | 16 | 28 | 7 | 11.2 |
|---|---|---|---|---|
| Number of days ($y$) | 14 | 8 | 32 | 20 |

**Ans : From the given table, the product of number of workers ($x$) and number of days in each pair is 224 , which is constant.**

**Therefore there is inverse proportion between workers and days.**

**8. Write the following statements using symbol of variation.**

**1) At constant temperature, the volume (v) of a fixed amount of gas is inversely proportional to its pressure (p).**

**Ans : $v \propto \frac{1}{p}$**

**2) The force (f) of an object is directly proportional to its mass (m).**

**Ans : $m \propto f$**

**3) The resistance (R) of a wire is directly proportional to its length (*l*).**

**Ans : $R \propto l$**

**4) The resistance (R) of a wire is directly proportional to its cross sectional area (A).**

**Ans : $R \propto \frac{1}{A}$**

**5) The amount of heat production in a conductor is directly proportional to the square of electric current flowing through it.**

**Ans : $H \propto I^2$**

**6) The circumference (c) of a circle is directly proportional to its diameter (d).**

**Ans : $c \propto d$**

**9. Which of the following are in direct and inverse proportion.**

**1) The distance travelled by a vehicle and the petrol filled in the tank of a vehicle.**

**Ans : The distance travelled by a vehicle is directly proportional to the amount of petrol used.**

**2) Fruits inward in market and their cost.**

**Ans : If fruits inward in market increases then their cost decreases . Therefore this is an example of inverse variation.**

**3) The water is used and its bill.**

**Ans : As the use of water increases, its bill also increases. Therefore, this is an example of direct variation.**

**4) The value of rupee in international market and exportation.**

**Ans : As the value of rupee increases, exportation of goods decreases.**
**Therefore, this is an example of inverse variation.**

**5) Swine flew and the people affected by swine flew.**

**Ans : As the swine flew spread, the people affected by swine flew increases.**

**10. Match the following pairs :**

| Group ' A' | Group 'B' |
|---|---|
| 1) $x \propto y$ | (a) Equation of inverse variation |
| 2) $xy = k$ | (b) Direct variation |
| 3) $x \propto \frac{1}{y}$ | (c) Inverse variation |
| 4) $x = ky$ | (d) Equation of direct variation |

**Ans :**

| ‘ A ’ Group | ‘ B ’ Group |
|---|---|
| 1) $x \propto y$ | (b) Direct variation |
| 2) $xy = k$ | (a) Equation of inverse variation |
| 3) $x \propto \frac{1}{y}$ | (c) Inverse variation |
| 4) $x = ky$ | (d) Equation of direct variation |

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