8. Quadrilateral : Constructions and Types

1. The measures of angles of a quadrilateral is in the proportion of 3 : 4 : 5 : 6 then find the measure of its each angle. Write with reason, what type of a quadrilateral it is.

Solution : The measures of angles of a quadrilateral is in the proportion of 3 : 4 : 5 : 6 .

∴ Suppose, □ PQRS is a quadrilateral.

∴ m ∠ P : m ∠ Q : m ∠ R : m ∠ S = 3 : 4 : 5 : 6

Let the measures of ∠ P , ∠ Q, ∠ R and ∠ S be $3x$, $4x$, $5x$ and $6x$ respectively.

Sum of the measures of all interior angles of a quadrilateral is 360^0.

$$\therefore 3x + 4x + 5x + 6x = 360^0$$

$$\therefore 18x = 360^0$$

$$\therefore x = \frac{360}{18}$$

$$\therefore x = 20$$

$$\therefore m\angle P = 3x = 3 \times 20 = 60^0$$

$m\angle Q = 4x = 4 \times 20 = 80^0$

$m\angle R = 5x = 5 \times 20 = 100^0$

$m\angle S = 6x = 6 \times 20 = 120^0$

∴ Sum of the measures of two angles = $m\angle P + m\angle S = 180^0$

$= 60^0 + 120^0 = 180^0$

Sum of the measures of adjacent angles of a quadrilateral is 180^0.

∴ $\angle P$ and $\angle S$ are the adjacent angles.

∴ $m\angle Q + m\angle R = 180^0$

$80^0 + 100^0 = 180^0$

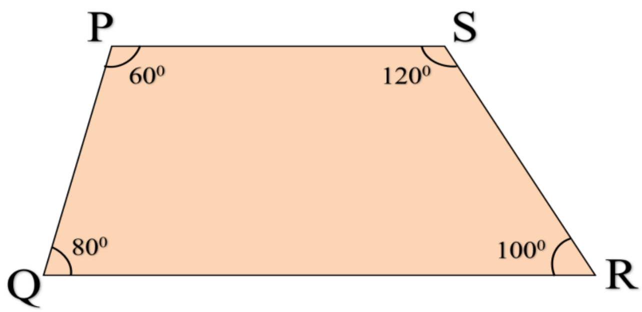
Sum of the measures of adjacent angles of a quadrilateral is 180^0.

∴ $\angle Q$ and $\angle R$ are the adjacent angles.

Sum of the measures of two interior angles of a quadrilateral is 180^0.

If the same side interior angles are supplementary then the line are parallel.

∴ Seg PS ∥ Seg QR



$$m\angle P + m\angle R = 60^0 + 80^0 = 140^0 \neq 180^0$$

∴ side PQ is not parallel to side SR

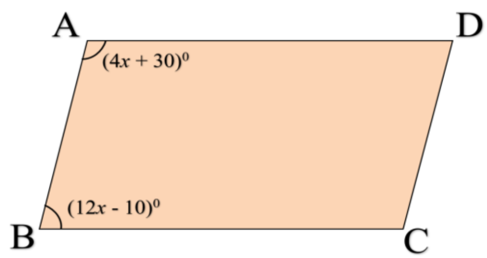
∴ Only one pair of opposite sides of □ PQRS is parallel.

∴ □ PQRS is trapezium.

∴ □ PQRS is trapezium.

2. Measure of adjacent angles of a parallelogram are $(4x + 30)^0$ and $(12x + 10)^0$ respectively. Find the measures of its each angle.

Solution :



In parallelogram ABCD,

$\angle$ A and $\angle$ B are adjacent angles.

Here, m $\angle$ A = $(4x + 30)^0$, m $\angle$ B = $(12x + 10)^0$

...... (Given)

Adjacent angles of a parallelogram are supplementary.

$$\therefore (4x + 30) + (12x - 10) = 180$$

$$\therefore 16x + 20 = 180$$

$$\therefore 16x = 180 - 20$$

$$\therefore 16x = 160$$

$$\therefore x = \frac{160}{16}$$

$$\therefore x = 10$$

$\therefore$ m∠A = $(4x + 30)^0 = 4 \times 10 + 30 = 40 + 30 = 70^0$

m∠B = $(12x - 10)^0 = 12 \times 10 - 10 = 120 - 10 = 110^0$

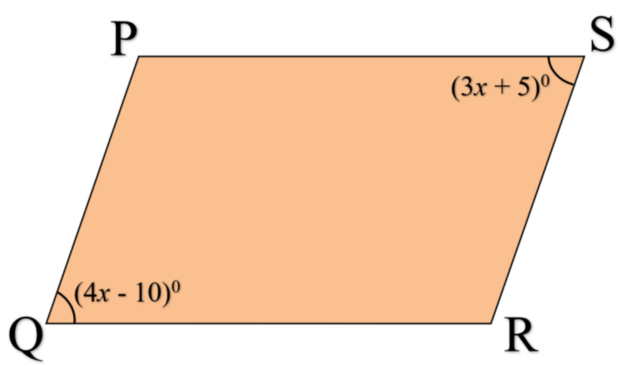
The opposite angles of parallelogram are congruent.

$\therefore$ m∠A = m∠C = 70^0 and m∠B = m∠D= 110^0

$\therefore$ **The measure of each angle of a parallelogram are $70^0, 110^0, 70^0$ and 110^0**

3. Measures of opposite angles of a parallelogram are $(4x + 30)^0$ and $(3x + 5)^0$. Find the measure of its each angle.

Solution :



In parallelogram PQRS,

∠PQR and ∠PSR are the opposite angles of parallelogram.

Here, m∠PQR = $(4x + 30)^0$, m∠PSR = $(3x + 5)^0$

Measures of the opposite angles of the parallelogram are same.

m ∠ PQR = m ∠ PSR

$\therefore 4x - 10 = 3x + 5$

$\therefore 4x - 3x = 10 + 5$

$\therefore x = 15$

$\therefore$ **m ∠ PQR =** $(4x + 30)^0 = 4 \times 15 - 10$

$= 60 - 10 = 50^0$

m ∠ PSR = $(3x + 5)^0 = 3 \times 15 + 5 = 45 + 5$

$= 50^0$

Sum of the measures of all interior angles of a quadrilateral is 360^0**.**

The measures of remaining two opposite angles

= 360 − m ∠ PQR − m ∠ PSR

= $360 - 50 - 50$

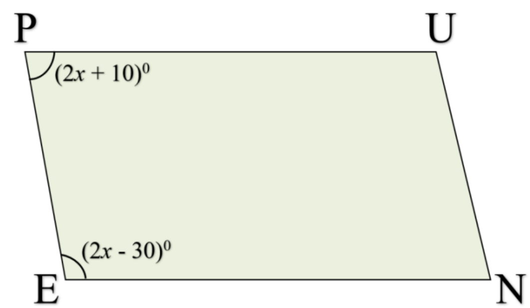
= $360 - 100$

$= 260^0$

∴ Measure of one angle = $\frac{260}{2} = 130^0$

∴ The measure of each angle of a parallelogram are $50^0, 130^0, 50^0, 130^0$.

4. In the figure alongside, ◻ PUNE is a parallelogram. If m ∠ P = $(2x + 10)^0$ and m ∠ E = $(2x - 30)^0$ then find the value of x and m ∠ N.



Solution :

In parallelogram PUNE,

Seg PU ∥ Seg EN

m ∠ P = $(2x + 10)^0$, m ∠ E = $(2x - 30)^0$ …… (Given)

Adjacent angles of a parallelogram are supplementary.

∴ m ∠ P + m ∠ E = 180^0

∴ $(2x + 10)^0 + (2x - 30)^0 = 180^0$

$$\therefore 4x - 20 = 180$$

$$\therefore 4x = 180 + 20$$

$$\therefore 4x = 200$$

$$\therefore x = \frac{200}{4}$$

$$\therefore x = 50$$

∠ P and ∠ N are opposite angles.

Measures of the opposite angles of the parallelogram are same.

$\therefore$ **m ∠ P = m ∠ N =** $(2x + 10)^0$

$= 2 \times 50 + 10 \qquad \ldots\ldots (\because x = 50)$

$= 100 + 10$

$= 110^0$

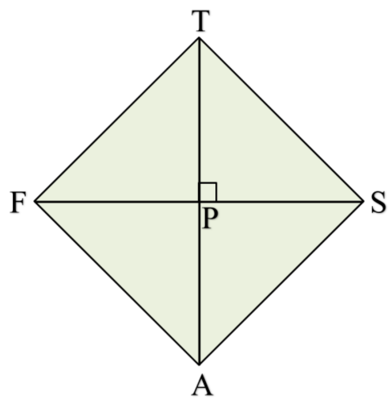
$\therefore x = 50$ **and m ∠ N =** 110^0

5. If the length of diagonals of a rhombus FAST are 16 cm and 30 cm then find the side and perimeter of the rhombus.

Solution : Diagonals of a rhombus FAST intersect at P.

$\therefore l\,(FS) = 16$ cm

$l\,(TA) = 30$ cm



Diagonals of a rhombus are perpendicular bisectors of each other.

∴ In Δ TPS, m ∠ TPS = 90^0

$l\,(PS) = \frac{1}{2}\,l\,(FS) = \frac{1}{2} \times 16 = 8$ cm

$l\,(TP) = \frac{1}{2}\,l\,(TA) = \frac{1}{2} \times 30 = 15$ cm

Δ TPS is a right angled triangle.

By Pythagoras theorem,

$l\,(TS)^2 = l(TP)^2 + l\,(PS)^2$

$= (15)^2 + (8)^2$

$= 225 + 64$

$= 289$

$\therefore l\,(TS)^2 = (17)^2$

$\therefore l\,(TS) = 17$ cm

∴ The length of the side of the rhombus FAST = 17 cm

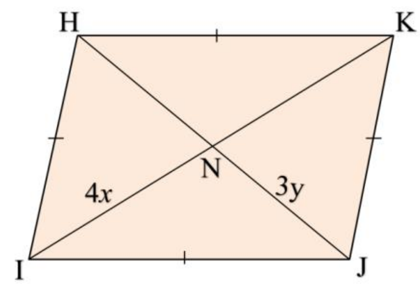
Perimeter of rhombus FAST = 4 × side

$= 4 \times 17$

$= 68$ cm

∴ The length of the side of rhombus FAST is 17 cm and the perimeter is 68 cm.

6. In □ HIJK, l (HI) = l (IJ) = l (JK) = l (KH)



(i) Identify the type of the quadrilateral

(ii) If l (IK) = 32 and l (IN) = $4x$, then find the value of x .

(iii) If l (HJ) = 24 and l (JN) = 3y, then find the value of x .

(iv) Find the perimeter of □ HIJK.

Solution :

(i) □ HIJK is rhombus.

(ii) l(IK) = 32 and l (IN) = $4x$

Diagonals of a rhombus are perpendicular bisectors of each other.

∴ In Δ HNI, m ∠ HNI = 90^0

$\therefore l\ (\text{IN}) = \frac{1}{2} \times l\ (\text{IK})$

$\therefore 4x = \frac{1}{2} \times 32$

$\therefore 4x = 16$

$\therefore x = \frac{16}{4}$

$\therefore x = 4$

∴ The value of x is 4.

(iii) $l\ (\text{HJ}) = 24$ and $l\ (\text{JN}) = 3\text{y}$

Diagonals of a rhombus are perpendicular bisectors of each other.

In Δ KNJ, m ∠ KNJ = 90^0

$\therefore l\ (\text{NJ}) = \frac{1}{2} \times l\ (\text{HJ})$

$\therefore 3\text{y} = \frac{1}{2} \times 24$

$\therefore 3\text{y} = 12$

$\therefore \text{y} = \frac{12}{3}$

$\therefore y = 4$

(iv) Δ INJ is the right angled triangle.

$l\,(\text{IN}) = 4x = 4 \times 4 = 16$ **….. From (ii)**

$l\,(\text{NJ}) = 3y = 3 \times 4 = 12$ **….. From (iii)**

By Pythagoras theorem,

$l\,(\text{IJ})^2 = l\,(\text{IN})^2 + l\,(\text{NJ})^2$

$= (16)^2 + (12)^2$

$= 256 + 144$

$= 400$

$l\,(\text{IJ})^2 = (20)^2$

$l\,(\text{IJ}) = 20$ **cm**

∴ The length of the side of rhombus is 20 cm.

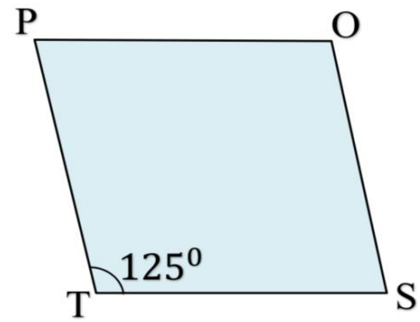
Perimeter of rhombus HIJK = 4 × side

= 4 × 20

= 80 cm

∴ The perimeter of rhombus HIJK is 80 cm.

7. Measure of one angle of a rhombus is 125^0, find the measures of remaining angles.



Solution :

Suppose, ◻ POST is a rhombus. Sum of the measures of four internal angles of a rhombus is 360^0 .

$\therefore m\angle P + m\angle O + m\angle S + m\angle T = 360^0$

Opposite angles of a rhombus are congruent.

$\therefore m\angle PTS = m\angle POS = 125^0$

$\therefore m\angle PTS + m\angle OST + m\angle POS + m\angle OPT = 360^0$

$\therefore 125^0 + m\angle OST + 125^0 + m\angle OPT = 360^0$

$\therefore m\angle OST + m\angle OPT + 250^0 = 360^0$

$\therefore m\angle OST + m\angle OPT + 360^0 - 250^0$

$\therefore m\angle OST + m\angle OPT = 110^0$ ….. (i)

But $m\angle OST = m\angle OPT$ …… (Opposite angles of a rhombus are congruent.)

$\therefore$ m $\angle$ OPT + m $\angle$ OPT = 110^0 ….. [from (i)]

$\therefore 2\ m \angle OPT = 110^0$

$\therefore m \angle OPT = \frac{110}{2}$

$\therefore m \angle OPT = 55^0$

$\therefore m \angle OPT = m \angle OST = 55^0$

$\therefore$ The measures of remaining angles of a rhombus are 55^0, 125^0 and 55^0.

8. O is the point of intersection of diagonals of rectangle MELT.

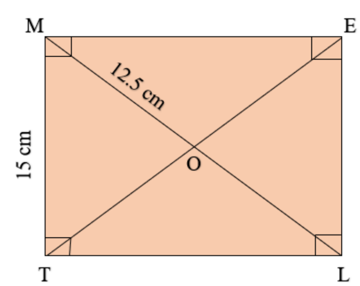
(i) If *l* (MT) = 15 cm the find *l* (EL)

(ii) If *l* (MO) = 12.5 then find *l* (ML)

(iii) Find the perimeter of rectangle MELT.

Solution :

(i) Opposite sides of a rectangle are congruent.



$\therefore$ ***l*** **(EL) =** ***l*** **(MT) = 15 cm**

(ii) Diagonals of a rectangle bisect each other.

∴ l (ML) = 2 × l (MO) = 2 × 12.5 = 25 cm

(iii) Δ MTL is the right angled triangle.

By Pythagoras theorem,

$l\ (\mathrm{ML})^2 = l\ (\mathrm{MT})^2 + l\ (\mathrm{TL})^2$

$(25)^2 = (15)^2 + l\ (\mathrm{TL})^2$

$\therefore l\ (\mathrm{TL})^2 = (25)^2 - (15)^2$

$= 625 - 225$

= 400

$\therefore l\ (\mathrm{TL})^2 = (20)^2$

$\therefore l\ (\mathrm{TL}) = 20$ cm

∴ The length is 20 cm and breadth is 15 cm of rectangle MELT.

Perimeter of the rectangle = 2 (length + breadth)

= 2 (20 + 15)

= 2 (35)

= 70 cm

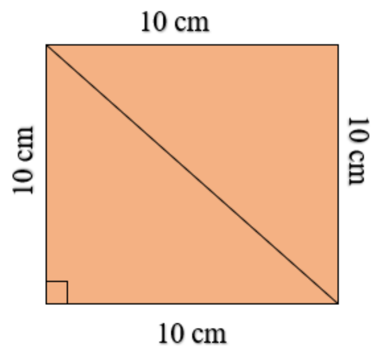
∴ The perimeter of rectangle MELT is 70 cm.

9. The length of a side of a square is 10 cm.

(i) Find the length of a diagonal.

(ii) Find its perimeter.

Solution :



(i) □ LEAF is a square of side 10 cm.

Seg LA is a diagonal.

Δ LFA is a right angled triangle.

∴ By Pythagoras theorem,

$$l\,(LA)^2 = l\,(LF)^2 + l\,(FA)^2$$

$$= (10)^2 + (10)^2$$

$$= 100 + 100$$

$$= 200$$

$$\therefore l\,(LA) = \sqrt{200}$$

$$= \sqrt{2 \times 100}$$

$$= 10\sqrt{2} \text{ cm}$$

∴ The length of a diagonal of a square is 10 $\sqrt{2}$ cm.

(ii) The length of a side of a square = 10 cm ….. (given)

∴ Perimeter of a square = 4 × side

= 4 × 10

= 40 cm

∴ The perimeter of a square is 40 cm.

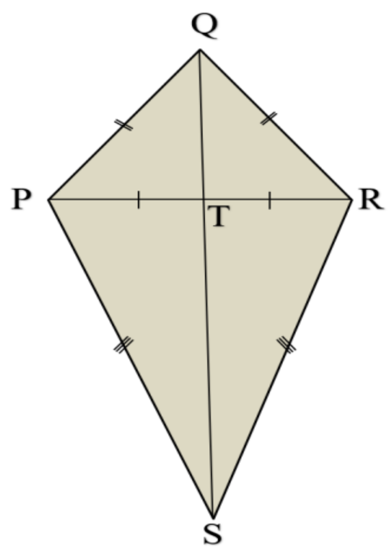
10. In the figure alongside,

l (TR) = 8 cm, l (QR) = 10 cm

l (RS) = 17 cm. Find

(i) l (QT) (ii) l (PQ) (iii) l (PS)

(iv) the length of the diagonals.



Solution : (i) □ PQRS is a kite.

In kite PQRS, m ∠ QTR = 90^0

l (TR) = 8 cm , l (QR) = 10 cm ….. (given)

Δ QTR is a right angled triangle.

∴ By Pythagoras theorem,

$l\,(QR)^2 = l\,(QT)^2 + l\,(TR)^2$

$\therefore (10)^2 = l\,(QT)^2 + (8)^2$

$\therefore 100 = l\,(QT)^2 + 64$

$\therefore l\,(QT)^2 = 100 - 64$

$\therefore l\,(QT)^2 = 36$

$\therefore l\,(QT)^2 = (6)^2$

$\therefore l\,(QT) = 6$ cm

(ii) □ PQRS is a kite. $l\,(QR) = 10$ cm

∴ Seg PQ ≅ Seg QR

$\therefore l\,(PQ) = l\,(QR) = 10$ cm

(iii) □ PQRS is a kite, *l* (RS) = 17 cm

Seg PS ≅ Seg RS

$\therefore l\,(PS) = l\,(RS) = 17$ cm

(iv) One diagonal of a kite is the perpendicular bisector of the other diagonal.

$\therefore PT = TR = 8$ cm

Diagonal PR = 2 PT = 2 × 8 = 16 cm

But, l (TR) = 8 cm ….. (given)

Δ STR is a right angled triangle.

By Pythagoras theorem,

$l\,(RS)^2 = l\,(TS)^2 + l\,(TR)^2$

$\therefore\ (17)^2 = l\,(TS)^2 + (8)^2$

$\therefore\ l\,(TS)^2 = (17)^2 - (8)^2$

$= 289 - 64$

$= 225$

$\therefore\ l\,(TS)^2 = (15)^2$

$\therefore\ (TS) = 15\ cm$

The length of the other diagonal = $l\,(QS) = l\,(QT) + l\,(TS)$

$= 6 + 15$

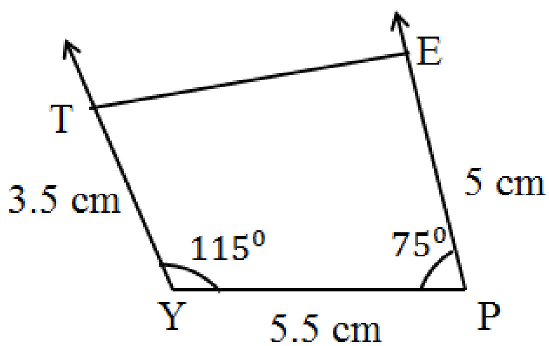
= 21 cm

$\therefore$ The length of one diagonal is 16 cm and other diagonal is 21 cm of a kite.

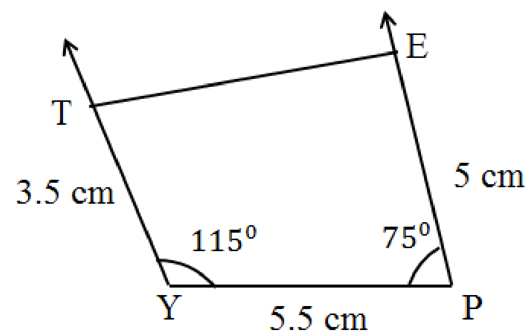
11. Construct □ TYPE, such that l (TY) $= 3.5\ cm$,
l (YP) $= 5.5$ cm, l (PE) $= 5$ cm , m ∠ P $= 75^0$,
m ∠ Y $= 115^0$

Ans :

Rough figure :



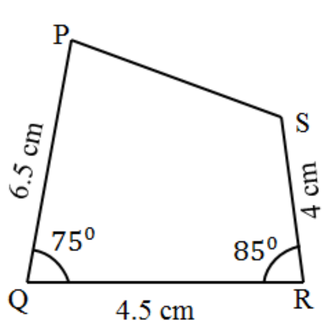
Right figure :



12. Construct □ PQRS, such that l (PQ) $= 6.5$ cm,
l (QR) $= 4.5$ cm , l (RS) $= 4$ cm, m ∠ Q $= 75^0$,
m ∠ R $= 85^0$.

Ans :

Rough figure :



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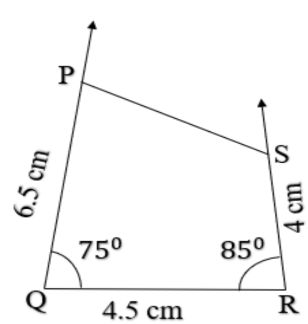
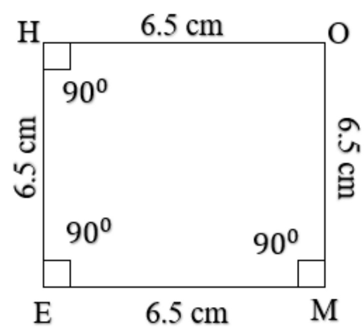


13. Construct ◻ HOME, such that l (HO) = l (OM) = 6.5 cm, m∠H = m∠O = m∠M = 90^0.

Ans :

Rough figure :



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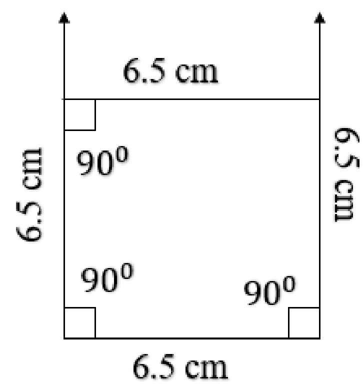
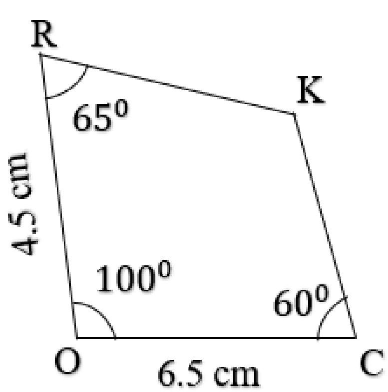


14. Construct ◻ ROCK, such that l (RO) = 4.5 cm, l (OC) = 6.5 cm, m∠R = 65^0, m∠O = 100^0, m∠C = 60^0.

Ans : Rough figure :



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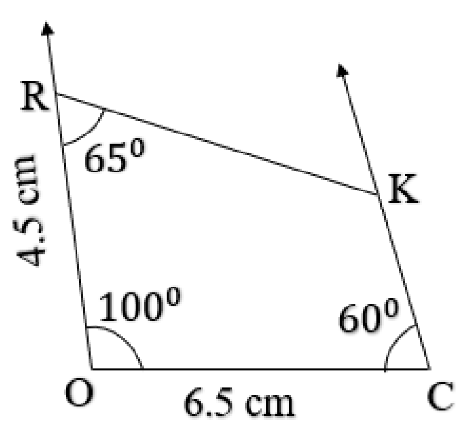
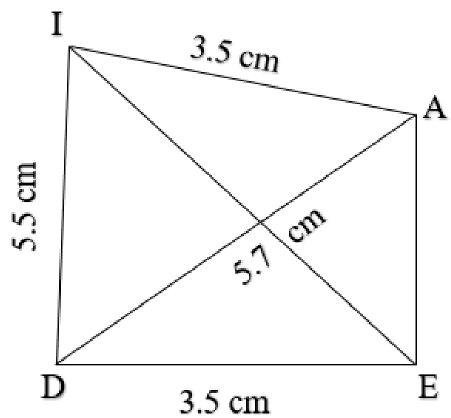


15. Construct □ IDEA, such that l (IA) = l (DE) = 3.5 cm, l (ID) = 5.5 cm, l (DA) = l (IE) = 5.7 cm.

Ans: Rough figure :



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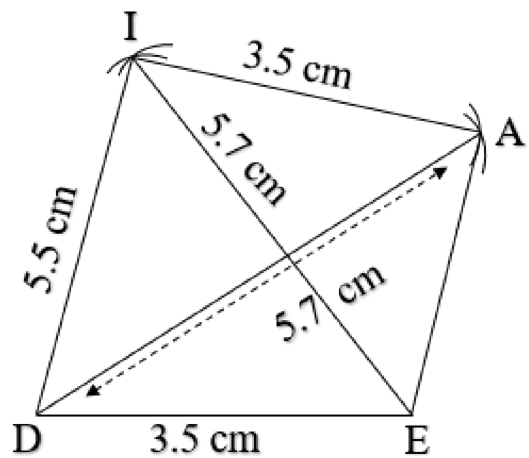
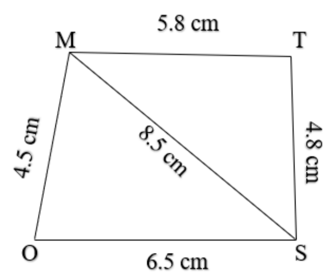


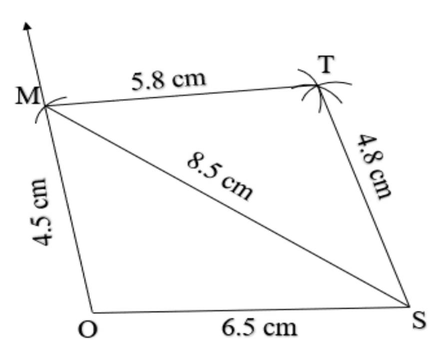
16. Construct □ MOST, such that l (MO) = 4.5 cm, l (OS) = 6.5 cm, l (ST) = 4.8 cm , l (MT) = 5.8 cm and hype l (MS) = 8.5 cm.

Ans:

Rough figure :



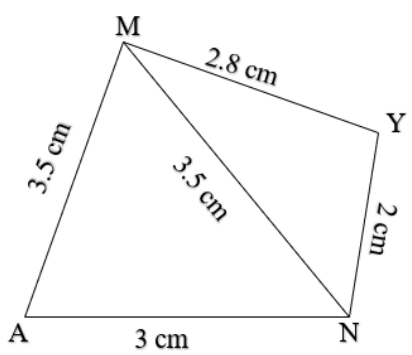
Right figure :



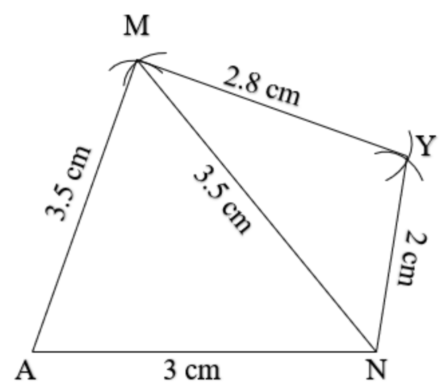
17. Construct □ MANY, such that l (MA) = 3.5 cm, l (AN) = 3 cm, l (NY) = 2 cm , l (MY) = 2.8 cm, l (MN) = 3.5 cm.

Ans:

Rough figure :



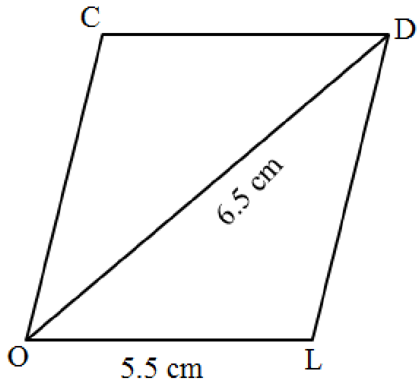
Right figure :



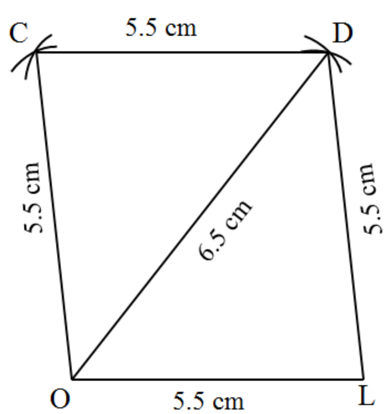
18. Construct □ COLD, such that the COLD is 5.5 cm, and one diagonal is 6.5 cm.

Ans:

Rough figure :



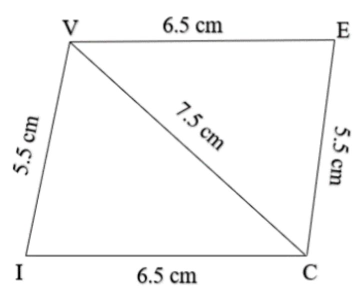
Right figure :



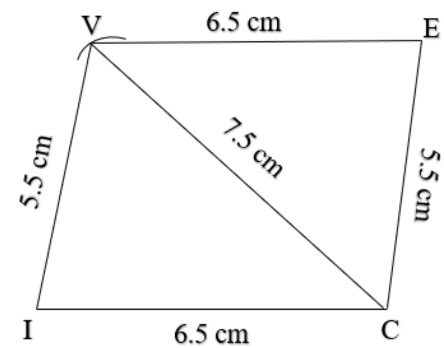
19. Construct a parallelogram VICE such that, l (VI) $= 5.5$ cm, l (IC) $= 6.5$ cm, l (VC) $= 7.5$ cm.

Ans:

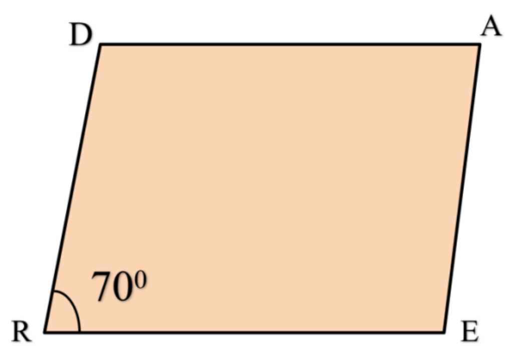
Rough figure :



Right figure :



20. In parallelogram READ, $\angle R = 70^0$. To find the remaining angles of parallelogram, complete the following activity.



Activity :

- **READ is a parallelogram.**

Adjacent angles of a parallelogram are supplementary.

$\therefore$ $m\angle R + m\angle E = \square$

But, $m\angle R = \square$ …… (Given)

$\therefore$ $\square + m\angle E = \square$

$\therefore$ $m\angle E = \square - \square = \square$

Opposite angles of parallelogram are congruent.

$\therefore$ $m\angle D = m\angle E = \square$ and $m\angle A = m\angle R = \square$

$\therefore$ $m\angle R = \square$, $m\angle E = \square$, $m\angle A = \square$ and $m\angle D = \square$

Solution : ◻ READ is a parallelogram.

Adjacent angles of a parallelogram are supplementary.

$\therefore$ $m\angle R + m\angle E = \boxed{180^0}$

But, $m\angle R = \boxed{70^0}$ …… (Given)

$\therefore$ $\boxed{70^0} + m\angle E = \boxed{180^0}$

$\therefore$ $m\angle E = \boxed{180^0} - \boxed{70^0} = \boxed{110^0}$

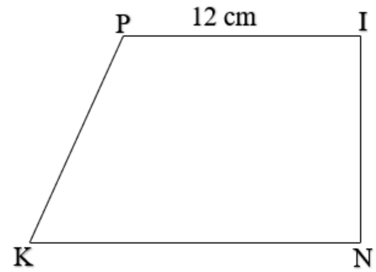
Opposite angles of parallelogram are congruent.

$\therefore$ **m ∠ D = m ∠ E =** $\boxed{110^0}$ **and m ∠ A = m ∠ R =** $\boxed{70^0}$

$\therefore$ **m ∠ R =** $\boxed{70^0}$ **, m ∠E =** $\boxed{110^0}$ **, m ∠ A =** $\boxed{70^0}$

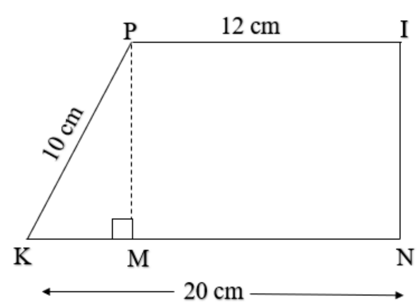
and m ∠ D = $\boxed{110^0}$

21. In the figure alongside, ◻ PINK is a trapezium. IN is the distance between two parallel lines of a trapezium. If *l* (KN) = 20 cm then find *l* (IN), Complete the following activity.



Activity :

Draw seg PM ⊥ side KN. Name the point of their intersection as M.



A rectangle PINM is formed. Opposite sides of a rectangle are congruent.

$\therefore$ ***l*** **(MN) =** ***l*** **(PI) =** $\boxed{}$ **cm**

Also ***l*** **(KN) =** ***l*** **(KM) +** ***l*** **(MN)**

But, l (KN) = 18 cm, l (MN) = $\square$

$\therefore$ 18 = l (KM) + $\square$

$\therefore$ l (KM) = 18 − $\square$

$\therefore$ l (KM) = $\square$ cm

Δ PMK is a right angled triangle.

By Pythagoras theorem,

$\therefore$ l (PK)^2 = $\square$ + l (PM)^2

But, l (PK) = $\square$ cm

$\therefore$ $\square$ = $\square$ + l (PM)^2

$\therefore$ l (PM)^2 = $\square$ − $\square$

= 100 − $\square$

= $\square$

$\therefore$ l (PM) = $\square$ cm

Opposite sides of a rectangle are congruent.

$\therefore$ l (IN) = l (PM) = $\square$ cm

∴ The distance between two parallel lines of a trapezium is ………. cm.

Solution :

Draw seg PM ⊥ side KN. Name the point of their intersection as M.

A rectangle PINM is formed.

Opposite sides of a rectangle are congruent.

$\therefore l\,(MN) = l\,(PI) = \boxed{12}$ **cm**

Also $l\,(KN) = l\,(KM) + l\,(MN)$

But, $l\,(KN) = 18$ **cm,** $l\,(MN) = \boxed{12}$ **cm**

$\therefore 18 = l\,(KM) + \boxed{12}$

$\therefore l\,(KM) = 18 - \boxed{12}$

$\therefore l\,(KM) = \boxed{6}$ **cm**

Δ PMK is a right angled triangle.

By Pythagoras theorem,

$\therefore l\,(PK)^2 = \boxed{l\,(KM)^2} + l\,(PM)^2$

But, l (PK) $= \boxed{10}$ cm

$\therefore \boxed{(10)^2} = \boxed{(6)^2} + l\,(\text{PM})^2$

$\therefore l\,(\text{PM})^2 = \boxed{(10)^2} - \boxed{(6)^2}$

$= 100 - \boxed{36}$

$= \boxed{64}$

$\therefore l\,(\text{PM}) = \boxed{8}$ cm

Opposite sides of a rectangle are congruent.

$\therefore l\,(\text{IN}) = l\,(\text{PM}) = \boxed{8}$ cm

∴ The distance between two parallel lines of a trapezium is 8 cm.

22. Write the following statement true or false.

1. Every square is a parallelogram.

Ans : True

Explanation : Opposite sides of a square are both congruent and parallel.

2. Every rhombus is a rectangle.

Ans : False, Each angle of rectangle is right angle. But it is not necessary that a rhombus also has each angle as right angle. So every rhombus is not a rectangle.

3 Diagonals of a parallelogram are congruent.

Ans : False, Diagonals of a parallelogram are not congruent because the opposite angles of a parallelogram are congruent.

4. Every square is a rhombus.

Ans : True

Explanation : All sides of a square and rhombus are congruent.

5. Every rectangle is a parallelogram.

Ans : True

Explanation : Opposite sides of a rectangle are congruent and parallel.

6. Every equilateral rectangle is a square.

Ans : True

Explanation : A square is a rectangle and also quadrilateral.

7. Every rectangle is a square.

Ans : False, Every rectangle is not a square because all sides of square must have same length and opposite sides of rectangle have same length.

8. Every square is a rectangle.

Ans : True

Explanation : Diagonals of rectangle and square are congruent and they are perpendicular to each other.

9. If the diagonals of a parallelogram are perpendicular to each other, it must be a rhombus

Ans : True

10. Diagonals of square are congruent and bisect each other.

Ans : True

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