

4. Ratio and proportion

Extra Questions

- From the following pairs of numbers find the reduced form of ratio of first number to second number. (1 mark)

Q. 1) 84, 96

$$\text{Ans : Ratio} = \frac{84}{96} = \frac{12 \times 7}{12 \times 8} = \frac{7}{8} = 7:8$$

Q. 2) 54, 72

$$\text{Ans : Ratio} = \frac{54}{72} = \frac{18 \times 3}{18 \times 4} = \frac{3}{4} = 3:4$$

- Express the following percentages as ratios the reduced from. (2 mark)

Q.3) 225 %

$$\begin{aligned} \text{Ans : } & \frac{225}{100} \\ &= \frac{45 \times 5}{20 \times 5} \\ &= \frac{45}{20} = \frac{5 \times 9}{5 \times 4} \\ &= \frac{9}{4} \\ &= 9:4 \end{aligned}$$

Q. 4) 25 : 100

$$\text{Ans : } \frac{25}{100}$$

$$= \frac{5 \times 5}{20 \times 5}$$

$$= \frac{5}{20}$$

$$= \frac{1}{4}$$

$$= 1:4$$

Q. 5) 0. 32%

$$\text{Ans : } 0. 32\%$$

$$= \frac{0. 32}{100}$$

$$= \frac{0.32 \times 100}{100 \times 100} \dots\dots\dots \text{(Multiply numerator and denominator by 100)}$$

$$= \frac{32}{10000}$$

$$= \frac{8 \times \cancel{4}}{25 \times 25 \times 4 \times \cancel{4}}$$

$$= \frac{\cancel{8}^2}{25 \times 25} \text{ (}\because \text{divide 8 by 4)}$$

$$= \frac{2}{25 \times 25}$$

$$= \frac{2}{625}$$

$$= 2 : 625$$

➤ Find the reduced form of the ratio of the first quantity to second quantity. (2 Mark)

Q. 6) 15 Rs, 13 Rs, 60 Paise

$$\begin{aligned} \text{Ans : Ratio of 15 Rs, 13Rs, 60 Paise} &= \frac{\text{Rs 15}}{\text{Rs 13, 60 Paise}} \\ &= \frac{15 \times 100}{13 \times 100 + 60} \\ &= \frac{1500}{1360} \\ &= \frac{20 \times 75}{20 \times 68} \\ &= \frac{75}{68} \\ &= 75 : 68 \end{aligned}$$

Here first and second quantity are different. For multiplication both the quantities should be same. So let's convert rupees in to paise.

$$\therefore 15 \text{ Rs} = 15 \times 100 = 1500$$

$$\text{and } 13 \text{ Rs} = 13 \times 100 = 1300$$

Q. 7) 4.2 kilogram, 1700 gram

Ans : Ratio of 4.2 kilogram, 1700 gram = $\frac{4.2 \times 1000 \text{ gm}}{1700 \text{ gm}}$

$$[\because 1\text{kg} = 1000\text{gm}] = \frac{4200 \text{ gm}}{1700 \text{ gm}}$$

$$= \frac{50 \times 84 \text{ gm}}{50 \times 34 \text{ gm}}$$

$$= \frac{84}{34} = \frac{42}{17}$$

$$= 42 : 17$$

Q 8) 9 Minutes , 40 seconds; 3 minutes ,8 seconds

Ans : Ratio of 9 minutes 40 ;seconds, 3 minutes

$$8 \text{ seconds} = \frac{9 \text{ minutes } 40 \text{ seconds}}{3 \text{ minutes } 8 \text{ seconds}}$$

$\because 1 \text{ minute} = 60 \text{ seconds}$

$$= \frac{(9 \times 60) + 40 \text{ seconds}}{(3 \times 60) + 8 \text{ seconds}}$$

$$= \frac{540 + 40}{180 + 8}$$

$$= \frac{580}{188}$$

$$= \frac{2 \times 290}{2 \times 94}$$

$$= \frac{290}{94}$$

$$= 290 : 94$$

➤ convert the following ratios into percentage.

Q 9) 14 : 20

Ans : Let $14 : 20 = x \%$

$$\therefore \frac{14}{20} = \frac{x}{100}$$

$$\therefore x = \frac{14}{20} \times 100$$

$$\therefore x = 14 \times 5$$

$$\therefore x = 70 \%$$

$$\therefore 14 : 20 = 70 \%$$

Q. 10) $\frac{7}{8}$

Ans : Let $\frac{7}{8} = x \%$

$$\therefore \frac{7}{8} = \frac{x}{100}$$

$$\therefore x = \frac{7}{8} \times 100$$

$$\therefore x = 7 \times 12.5$$

$$\therefore x = 87.5 \%$$

➤ Compare the following pairs of ratios.

Q. 11) $\frac{9}{13}, \frac{11}{15}$

Ans : $9 \times 15, 13 \times 11$

$$135 < 143$$

$$\therefore \frac{9}{13} < \frac{11}{15}$$

Q 12) $\frac{3\sqrt{3}}{2\sqrt{2}}, \frac{2\sqrt{2}}{3\sqrt{3}}$

Ans : $3\sqrt{3} \times 3\sqrt{3}; 2\sqrt{2} \times 2\sqrt{2}$

$$9 \times (\sqrt{3} \times \sqrt{3}); 2 \times (\sqrt{2} \times \sqrt{2})$$

$$9 \times 3 \qquad \qquad \qquad ; 2 \times 2$$

$$27 \qquad \qquad \qquad ; 4$$

$$27 > 4$$

$$\therefore \frac{3\sqrt{3}}{2\sqrt{2}} > \frac{2\sqrt{2}}{3\sqrt{3}}$$

Q 13) $\frac{\sqrt{12}}{\sqrt{9}}, \frac{\sqrt{24}}{\sqrt{18}}$

$$\sqrt{12} \times \sqrt{18}; \sqrt{9} \times \sqrt{24}$$

$$\sqrt{216}; \sqrt{216}$$

$$\therefore \sqrt{216} = \sqrt{216}$$

$$\therefore \frac{\sqrt{12}}{\sqrt{9}} = \frac{\sqrt{24}}{\sqrt{18}}$$

Q 14) The ratio of two numbers is 13 : 6 and their difference is 70. Find those numbers. (3 marks)

Ans : The ratio of two numbers is 13 : 6

Let the common multiple of the given ratio be x .

First number = $13x$

Second number = $6x$

According to the given condition, $13x - 6x = 70$

$$7x = 70$$

$$x = \frac{70}{7}$$

$$x = 10$$

First number = $13 \times x = 13 \times 10 = 130$

Second number = $6x = 6 \times 10 = 60$

Q . 15) The ratio of two numbers is 11 : 13 and their sum is 264. Find these numbers. . (3 marks)

Ans : The ratio of two numbers is 11 : 13

Let the common multiple of the given ratio be

First number = $11x$

Second number = $13x$

According to the given condition,

$$11x + 13x = 264$$

$$24x = 264$$

$$x = \frac{264}{24}$$

$$x = 11$$

$$\therefore \text{First number} = 11x = 11 \times 11 = 121$$

$$\text{Second number} = 13x = 13 \times 11 = 143$$

Q 16) If $a : b = 7 : 8$ and $b : c = 12 : 7$ then find $a : c$ in the simplest form. . (3 marks)

Ans : $a : b = 7 : 8$ and $b : c = 12 : 7$

$$\frac{a}{b} = \frac{7}{8} \text{ and } \frac{b}{c} = \frac{12}{7}$$

Now,

$$\begin{aligned} \frac{a}{b} \times \frac{b}{c} &= \frac{7}{8} \times \frac{12}{7} \\ &= \frac{12}{8} \end{aligned}$$

$$\Rightarrow \frac{3}{2} \dots\dots [\text{dividing by 4}]$$

Q 17) Find the mean proportional of $\frac{a^3 + b^3}{ab^3}, \frac{a^3b}{a^3 + b^3}$. (3 marks)

Ans : let the mean proportional be 'm'

$\therefore \frac{a^3 + b^3}{ab^3}, m, \frac{a^3b}{a^3 + b^3}$ are in continued proportion.

$$\therefore m^2 = \frac{a^3 + b^3}{ab^3} \times \frac{a^3b}{a^3 + b^3}$$

$$\therefore m^2 = \frac{a^3 b}{ab^3}$$

$$\therefore m^2 = \frac{a^2}{b^2}$$

$$\therefore m = \frac{a}{b} \dots\dots \text{(taking square root on both sides)}$$

$$\therefore \text{The mean proportional is } \frac{a}{b}$$

Q 18) If x is geometric mean of 16 and 9 then the find value of x . . (2 marks)

Ans : x is the geometric mean of 16 and 9

$$\therefore x^2 = 16 \times 9$$

$$\therefore x^2 = 144$$

$$\therefore \boxed{x = 12}$$

Q. 19) The ratio of the present ages of mother and son is 12 : 5. The mother's age at the time of birth of the son was 21 years. Find their present ages. (3 Mark)

Ans : The ratio of the present ages of mother and son is 12 : 5

Let the common multiple of the given ratio be x .

$$\therefore \text{Present age of mother} = 12x$$

$$\text{and present age of son} = 5x$$

According to the given condition,

$$12x - 5x = 21$$

$$\therefore 7x = 21$$

$$\therefore x = \frac{21}{7}$$

$$\therefore x = 3$$

$\therefore$ Present age of mother = $12x = 12 \times 3 = 36$ years.

and present age of son = $5x = 5 \times 3 = 15$ years.

Q 20) Population of a village is 14, 400. The ratio of illiterate to literate is 3 : 5. Find the number of literate and illiterate persons in the village.

Ans : Population of a village is 14, 400

The ratio of illiterate to literate is 3 : 5

Let the common multiple of the given ratio be x .

$\therefore$ Illiterate person = $3x$

and literate person = $5x$

According to the given condition,

$$3x + 5x = 14,400$$

$$8x = 14,400$$

$$x = \frac{14,400}{8}$$

$$\therefore \boxed{x = 1800}$$

$$\text{illiterate person} = 3x = 3 \times 1800 = \boxed{5400}$$

$$\text{and literate person} = 5x = 5 \times 1800 = \boxed{9000}$$

Q. 21) The ratio of boys and girls in a certain class is 5 : 4. If 4 boys and 4 girls are added. The ratio would be 6 : 5. Find the number of boys and girls in the class.

Ans : The ratio of boys and girls in a certain class is 5 : 4

Let the common multiple of the given ratio be x

$$\therefore \text{number of boys} = 5x$$

$$\text{number of girls} = 4x$$

According to the given condition,

$$\frac{5x + 4}{4x + 4} = \frac{6}{5}$$

$$\therefore (5x + 4) \times 5 = 6 \times (4x + 4)$$

$$\therefore 25x + 20 = 24x + 24$$

$$\therefore 25x - 24x = 24 - 20$$

$$\therefore x = 4$$

$$\therefore \text{number of boys} = 5 \times 4 = 20$$

$$\text{and number of girls} = 4 \times 4 = 16$$

Q 22) The measures of angles of a triangle are in the ratio 4 : 5 : 6. Determine the measures of each angle of the triangle.

Ans : Ratio of measures of angles of triangle is 4 : 5 : 6

Sum of angle of triangle is 180°

Let the common multiple of the given ratio be x .

$$4x + 5x + 6x = 180^\circ$$

$$15x = 180^0$$

$$x = \frac{180}{15}$$

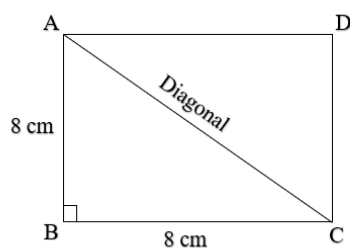
$$\boxed{x = 12}$$

∴ Measures of angles of triangle

$$4x = 4 \times 12 = 48^0, 5x = 5 \times 12 = 60^0, 6x = 6 \times 12 = 72^0$$

Find the following ratios.

Q 23) The ratio of diagonal of a square to its side, if the length of side is 8 cm.



□ ABCD is square

In right – angle Δ ABC,

$$\therefore AC^2 = AB^2 + BC^2 \dots\dots [\text{Theorem of Pythagoras}]$$

$$\therefore AC^2 = 8^2 + 8^2$$

$$\therefore AC^2 = 2 \times 8^2$$

$$\therefore \text{diagonal} = 8\sqrt{2} \text{ cm}$$

$$\therefore \text{Ratio} = \frac{\text{diagonal}}{\text{length of side}} = \frac{8\sqrt{2}}{8} = \frac{\sqrt{2}}{1} = \sqrt{2} : 1$$

Q. 24) The length of sides of a rectangle are 4 cm and 2 : 5 cm.
Find the ratio of its perimeter to area.

Ans : length = 4 cm, width = 2.5 cm

$$\begin{aligned}\frac{\text{Perimeter of rectangle}}{\text{Area of Rectangle}} &= \frac{2(4 + 2.5)}{4 \times 2.5} \\ &= \frac{13.0}{10.0} \\ &= \frac{13}{10} \\ &= 13 : 10\end{aligned}$$

Q 25) If the product of two numbers is 4032 and their ratio is 7 : 9 then find the numbers. Complete the following activity.

Ans : Ratio of two number is

Let the common multiple of the given ratio be X

First number = $7x$

Second number

According to the given condition,

$$7x \times \text{} = \text{}$$

$$\text{} = \text{}$$

$$x^2 = \underline{\underline{\text{>}}}$$

$$\therefore x^2 = \text{}$$

$$\therefore x = \text{}$$

$$\text{First number} = 7x = 7 \times \square = \square$$

$$\text{Second number} = 9x = 9 \times \square = \square$$

Ans : Ratio of two number is $\boxed{7 : 9}$

Let the common multiple of the given ratio be

$$\text{First number} = 7x$$

$$\text{Second number} = \boxed{9x}$$

According to the given condition,

$$7x \times \boxed{9x} = \boxed{4032}$$

$$\boxed{63x^2} = \boxed{4032}$$

$$x^2 = \frac{\boxed{4032}}{63}$$

$$\therefore x^2 = \boxed{64}$$

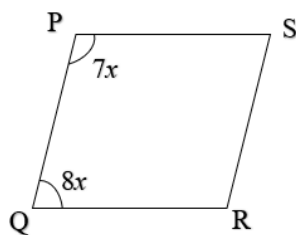
$$\therefore x = \boxed{8}$$

$$\text{First number} = 7x = 7 \times \boxed{8} = \boxed{56}$$

$$\text{Second number} = 9x = 9 \times \boxed{8} = \boxed{72}$$

Q. 26) $\square$ PQRS is a parallelogram. The ratio of $\angle P$ and $\angle Q$ of this parallelogram is 7 : 8. Find the measure of $\angle Q$

Ans :



Let the common multiple of given ratio be x .

$$\therefore m \angle p = 7x^0 \text{ and } m \angle Q = 8x^0$$

$$\text{Now, } m \angle p + m \angle Q = 180^0$$

..... [Adjacent angles of a parallelogram are supplementary.]

$$\therefore 7x + 8x = 180^0$$

$$\therefore 15x = 180^0$$

$$\therefore x = \frac{180^0}{15}$$

$$\therefore x = 12^0$$

$$\therefore m \angle Q = 8x^0 = 8 \times 12^0 = 96^0$$

$$\therefore m \angle Q = 96^0$$

Q 27) If $a : b = 2 : 1$ and $b : c = 3 : 1$ then find the value of $\frac{a^2}{10bc}$

Ans : $a : b = 2 : 1$ and $b : c = 3 : 1$

$$\frac{a}{b} = \frac{2}{1} \text{ and } \frac{b}{c} = \frac{3}{1}$$

$a = 2b$ and $b = 3c$ Putting these value in $a = 2b$

$$\therefore a = 2(3c) \Rightarrow \boxed{a = 6c} \text{ and } \boxed{b = 3c}$$

$$\text{Now, } \frac{a^2}{10bc} = \frac{(6c)^2}{10 \times 3c \times c} = \frac{36c^2}{10 \times 3c^2} = \frac{12}{10}$$

$$\boxed{\therefore \frac{a^2}{10bc} = \frac{12}{10}}$$

Q 28) If $\sqrt{0.03 \times 0.3 \times a} = 0.3 \times 0.03 \times \sqrt{b}$ then find the ratio $\frac{a}{b}$

Ans : $\sqrt{0.03 \times 0.3 \times a} = 0.3 \times 0.03 \times \sqrt{b}$

$$\therefore \sqrt{\frac{3}{100} \times \frac{3}{10} \times a} = \frac{3}{10} \times \frac{3}{100} \times \sqrt{b}$$

Squaring both sides,

$$\therefore \frac{3}{100} \times \frac{3}{10} \times a = \frac{9}{100} \times \frac{9}{10000} \times b$$

$$\therefore \frac{a}{b} = \frac{9}{100} \times \frac{9}{10000} \times \frac{100}{3} \times \frac{10}{3}$$

$$\therefore \frac{a}{b} = \frac{9}{100} \times \frac{9}{100} \times \frac{10}{9}$$

$$\therefore \frac{a}{b} = \frac{9}{100} \times \frac{1}{10}$$

$$\therefore \frac{a}{b} = \frac{9}{1000}$$

Q 29) $(x + 2) : (x + 12) = (x - 4) : (x + 3)$ then find the value of x

Ans : $\therefore \frac{x+2}{x+12} = \frac{x-4}{x+3}$

$$\therefore (x + 2)(x + 3) = (x - 4)(x + 12)$$

$$x^2 + 3x + 2x + 6 = x^2 + 12x - 4x - 48$$

$$\therefore x^2 + 5x + 6 = x^2 + 8x - 48$$

$$\therefore x^2 - x^2 + 5x - 8x = -48 - 6$$

$$\therefore -3x = -54$$

$$\therefore x = \frac{54}{3}$$

$$\therefore \boxed{x = 18}$$

Q 30) If $\frac{a}{b} = \frac{3}{2}$ then find $\frac{4a^2 + 3b^2}{4a^2 - 3b^2}$

$$\text{Ans : } \frac{a}{b} = \frac{3}{2}$$

$$\frac{a^2}{b^2} = \frac{9}{4} \dots\dots (\text{squaring both sides})$$

$$\therefore \frac{a^2}{b^2} \times \frac{4}{3} = \frac{9}{4} \times \frac{4}{3} \dots\dots (\text{multiply both sides by } \frac{4}{3})$$

$$\therefore \frac{4a^2}{3b^2} = \frac{36}{12}$$

$$\therefore \frac{4a^2 + 3b^2}{4a^2 - 3b^2} = \frac{36 + 12}{36 - 12} \dots\dots (\text{using componendo - dividendo})$$

$$\therefore \frac{4a^2 + 3b^2}{4a^2 - 3b^2} = \frac{48}{24} = \frac{2}{1} = 2 : 1$$

Q 31) If $\frac{a^2 + b^2}{a^2 - b^2} = \frac{25}{7}$ then find $\frac{a}{b}$ (3 marks)

$$\text{Ans : } \frac{a^2 + b^2}{a^2 - b^2} = \frac{25}{7}$$

$$\text{Using componendo - dividendo, } \frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - (a^2 - b^2)} = \frac{25 + 7}{25 - 7}$$

$$\frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2} = \frac{25 + 7}{25 - 7}$$

$$\frac{2a^2}{2b^2} = \frac{32}{18}$$

$$\frac{a^2}{b^2} = \frac{16}{9} \dots\dots (\because \text{dividing by 2})$$

$$\frac{a}{b} = \frac{4}{3} \dots\dots\dots (\text{Taking square root on both sides})$$

Q 32) If $\frac{x}{y} = \frac{13}{6}$ then find the value of $\frac{2x^2 + 5y^2}{2x^2 - 5y^2}$ complete the following activity. (4 marks)

$$\text{Ans : } \frac{x}{y} = \frac{13}{6}$$

$$\therefore \frac{x^2}{y^2} = \boxed{} \dots\dots (\text{squaring both sides})$$

$$\therefore \frac{x^2}{y^2} \times \boxed{} = \frac{169}{36} \times \frac{2}{5} \dots\dots [\text{multiply by } \boxed{} \text{ both sides}]$$

$$\frac{2x^2}{5y^2} = \frac{338}{180}$$

$$\boxed{} \dots\dots [\text{using componendo dividendo}]$$

$$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \frac{518}{158}$$

$$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \boxed{} \dots\dots [\text{dividing by } \dots\dots]$$

$$\text{Ans : } \frac{x}{y} = \frac{13}{6}$$

$$\therefore \frac{x^2}{y^2} = \boxed{\frac{169}{36}} \dots\dots (\text{squaring both sides})$$

$$\therefore \frac{x^2}{y^2} \times \boxed{\frac{2}{5}} = \frac{169}{36} \times \frac{2}{5} \dots\dots [\text{multiply by } \boxed{\frac{2}{5}} \text{ both sides}]$$

$$\frac{2x^2}{5y^2} = \frac{338}{180}$$

$$\boxed{\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \frac{338 + 180}{338 - 180}} \dots\dots [\text{ using componendo dividendo}]$$

$$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \frac{518}{158}$$

$$\frac{2x^2 + 5y^2}{2x^2 - 5y^2} = \boxed{\frac{259}{79}} \dots\dots [\text{ dividing by 2 }]$$

Q 33) If $\frac{x}{y+z-x} = \frac{y}{z+x-y} = \frac{z}{x+y-z}$ and $x + y + z \neq 0$ then show that each ratio is equal to 1. (4 marks)

Ans : Let $\frac{x}{y+z-x} = \frac{y}{z+x-y} = \frac{z}{x+y-z} = k$

by theorem on equal ratios,

$$\text{each ratio } k = \frac{x+y+z}{y+z-x+z+x+y+x+y-z}$$

$$k = \frac{x+y+z}{x+y+z}$$

$$k = 1 \quad (\because x + y + z \neq 0)$$

$$\therefore \boxed{\text{each ratio} = 1}$$

Q 34) If $\frac{3a-5b}{5c+3b} = \frac{a+5c}{b-5a} = \frac{b-c}{a-c}$ then show that each ratio is $\frac{a}{b}$ (3 marks)

Ans : Let $\frac{3a-5b}{5c+3b} = \frac{a+5c}{b-5a} = \frac{b-c}{a-c} = k$

$\therefore$ By theorem on equal ratios,

$$k = \frac{(3a - 5b) + (a + 5c) + 5(b - c)}{(5c + 3b) + (b - 5a) + 5(a - c)}$$

$$k = \frac{3a - 5b + a + 5c + 5b - 5c}{5c + 3b + b - 5a + 5a - 5c}$$

$$k = \frac{3a + a}{3b + b}$$

$$k = \frac{4a}{4b}$$

$$\therefore k = \frac{a}{b}$$

$$\therefore \frac{3a - 5b}{5c + 3b} = \frac{a + 5c}{b - 5a} = \frac{b - c}{a - c} = \frac{a}{b}$$

$$4m + n = 12$$

– 2n. Find the values of the following expression.

$$\text{Q 35) } \frac{n^2 + m^2}{n^2 - m^2} \text{ (3 marks)}$$

$$\text{Ans : } 4m + n = 12m - 2n$$

$$\therefore 4m - 12m = -2n - n$$

$$\therefore -8m = -3n$$

$$\therefore \frac{m}{n} = \frac{3}{8} \dots\dots (i)$$

$$\therefore \frac{m^2}{n^2} = \frac{9}{64} \dots\dots (\text{squaring on both sides})$$

$$\therefore \frac{m^2 + n^2}{m^2 - n^2} = \frac{9 + 64}{9 - 64} \dots\dots \text{(using componendo - dividendo)}$$

$$\therefore \frac{n^2 + m^2}{n^2 - m^2} = \frac{64 + 9}{64 - 9}$$

$$\therefore \boxed{\frac{n^2 + m^2}{n^2 - m^2} = \frac{73}{55}}$$

$4m + n = 12 - 2n$. Find the values of the following expression.

Q 36) $\frac{3n + 7m}{3n - 7m}$ (3 marks)

Ans : $4m + n = 12m - 2n$

$$\therefore 4m - 12m = -2n - n$$

$$\therefore -8m = -3n$$

$$\therefore \frac{m}{n} = \frac{3}{8}$$

$$\frac{n}{m} = \frac{8}{3} \dots\dots \text{(Invertendo)}$$

$$\frac{n}{m} \times \frac{3}{7} = \frac{8}{3} \times \frac{3}{7} \dots\dots \text{(multiplying on both sides by } \frac{3}{7} \text{)}$$

$$\frac{3n}{7m} = \frac{24}{21}$$

$$\therefore \frac{3n + 7m}{3n - 7m} = \frac{24 + 21}{24 - 21} \dots\dots \text{(using componendo - dividendo)}$$

$$\therefore \frac{3n + 7m}{3n - 7m} = \frac{45}{3}$$

$$\therefore \boxed{\frac{3n + 7m}{3n - 7m} = \frac{15}{1}}$$

Q 37) which number should be added to 11, 15 and 20 so that resultant numbers are in continued proportion ? (4 marks)

Ans : Let the number to be added to 11, 15 and 20 be x . so that resultant numbers are in continued proportion.

$\therefore (11 + x), (15 + x)$ and $(20 + x)$ are in continued proportion.

$$\frac{(11 + x)}{(15 + x)} = \frac{(15 + x)}{(20 + x)}$$

$$\therefore (15 + x)^2 = (11 + x)(20 + x)$$

$$\therefore 225 + 30x + x^2 = 220 + 11x + 20x + x^2$$

$$\therefore 225 + 30x + x^2 = 220 + 31x + x^2$$

$$\therefore 225 - 220 = x^2 - x^2 + 31x - 30x$$

$$\therefore \boxed{5 = x}$$

Q.38) Which number should be subtracted from each of 22, 32 and 47 in order that the differences would be in continued proportion. (4 marks)

Ans : Let the number to be subtracted to 22, 32 and 47 be x . so that resultant numbers are in continued proportion.

$\therefore (22 - x), (32 - x), (47 - x)$ are in continued proportion.

$$\frac{(22 - x)}{(32 - x)} = \frac{(32 - x)}{(47 - x)}$$

$$\therefore (32 - x)^2 = (22 - x)(47 - x)$$

$$\therefore 1024 - 64x + x^2 = 1034 - 22x - 47x + x^2$$

$$\therefore 1024 - 1034 = 64x - 69x$$

$$\therefore -10 = -5x$$

$$\therefore x = \frac{10}{5}$$

$$\therefore \boxed{x = 2}$$

Q 39) If $(x + y)^2$, m and $(x - y)^2$ are in continued proportion, then determine m .

Ans : $(x + y)^2$, m , $(x - y)^2$ are in continued proportion.

$$\therefore m^2 = (x + y)^2 \times (x - y)^2$$

$$\therefore m^2 = (x^2 + 2xy + y^2)(x^2 - 2xy + y^2)$$

$$\therefore m^2 = x^4 - 2x^3y + x^2y^2 + 2x^3y - 4x^2y^2 + 2xy^3 + y^2x^2 - 2xy^3 + y^4$$

$$\therefore m^2 = x^4 + x^2y^2 + y^2x^2 - 4x^2y^2 + y^4$$

$$\therefore m^2 = x^4 - 2x^2y^2 + y^4$$

$$\therefore m^2 = (x^2 - y^2)^2 \dots [(a - b)^2 = a^2 - 2ab + b^2]$$

$$\therefore m = x^2 - y^2 \dots \dots \dots \text{(taking square root on both sides.)}$$

Q 40) $\frac{a^2 + b^2}{a + c} = a$ complete the following activity. (3 marks)

$a = \square$ and $b = \square$ Putting these values.

Left Hand side = $\square$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \square$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \square$$

Right Hand side = a

$$= \square$$

$\therefore \text{L.H.S} = \text{R.H.S}$

$\therefore \square$

Ans : $a = \square$ and $b = \square$ Putting these values.

Left Hand side = $\frac{a^2 + b^2}{a + c}$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$\begin{aligned}
 &= \boxed{\frac{c^2 k^4 + c^2 k^2}{c (k^2 + 1)}} \\
 &= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)} \\
 &= \boxed{ck^2}
 \end{aligned}$$

Right Hand side = a

$$= \boxed{ck^2}$$

$\therefore$ L.H.S = R.H.S

$$\therefore \boxed{\frac{a^2 + b^2}{a + c}} = a$$

➤ Solve the following equations.

$$\text{Q 41)} \frac{\sqrt{7x+1} + \sqrt{3x+1}}{\sqrt{7x+1} - \sqrt{3x+1}} = \frac{5}{1}$$

$$\text{Ans : } \frac{\sqrt{7x+1} + \sqrt{3x+1}}{\sqrt{7x+1} - \sqrt{3x+1}} = \frac{5}{1}$$

$$\therefore \frac{(\sqrt{7x+1} + \sqrt{3x+1}) + (\sqrt{7x+1} - \sqrt{3x+1})}{(\sqrt{7x+1} + \sqrt{3x+1}) - (\sqrt{7x+1} - \sqrt{3x+1})} = \frac{5+1}{5-1}$$

..... (Using componendo - dividendo)

$$\therefore \frac{\sqrt{7x+1} + \sqrt{3x+1} + \sqrt{7x+1} - \sqrt{3x+1}}{\sqrt{7x+1} + \sqrt{3x+1} - \sqrt{7x+1} + \sqrt{3x+1}} = \frac{6}{4}$$

$$\therefore \frac{2\sqrt{7x+1}}{2\sqrt{3x+1}} = \frac{6}{4}$$

$$\therefore \frac{\sqrt{7x+1}}{\sqrt{3x+1}} = \frac{3}{2}$$

$$\therefore \frac{7x+1}{3x+1} = \frac{9}{4}$$

$$\therefore 4(7x+1) = 9(3x+1)$$

$$\therefore 28x+4 = 27x+9$$

$$\therefore 28x-27x = 9-4$$

$$\therefore \boxed{x=5}$$

$$\therefore x=5$$

$$\text{Q 42)} \frac{(5x+2)^2 + (5x-2)^2}{(5x+2)^2 - (5x-2)^2} = \frac{13}{12}$$

$$\text{Ans : } \frac{(5x+2)^2 + (5x-2)^2}{(5x+2)^2 - (5x-2)^2} = \frac{13}{12}$$

$$\frac{[(5x+2)^2 + (5x-2)^2] + [(5x+2)^2 - (5x-2)^2]}{[(5x+2)^2 - (5x-2)^2] + [(5x+2)^2 - (5x-2)^2]} = \frac{13+12}{13-12}$$

..... (Using componendo – dividendo)

$$\therefore \frac{(5x+2)^2 + (5x-2)^2 + (5x+2)^2 - (5x-2)^2}{(5x+2)^2 + (5x-2)^2 - (5x+2)^2 + (5x-2)^2} = \frac{25}{1}$$

$$\therefore \frac{2(5x+2)^2}{2(5x-2)^2} = \frac{25}{1}$$

$$\therefore \frac{(5x+2)^2}{(5x-2)^2} = \frac{25}{1}$$

$$\therefore \frac{5x+2}{5x-2} = \frac{5}{1} \dots\dots \text{(taking square root on both sides)}$$

$$\therefore 1(5x+2) = 5(5x-2)$$

$$\therefore 5x+2 = 25x-10$$

$$\therefore 5x-25x = -10-2$$

$$\therefore -20x = -12$$

$$\therefore x = \frac{12}{20}$$

$$\therefore \boxed{x = \frac{3}{5}}$$

$$\therefore x = \frac{3}{5}$$

$$\text{Q 43) } \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{x+7}{x+9}$$

$$\text{Ans : Let, } \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{x+7}{x+9} = k$$

$$\therefore k = \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{3x(x+7)}{3x(x+9)}$$

\dots\dots \text{(by theorem on equal ratios)}

$$\therefore k = \frac{3x^2 + 21x + 4}{3x^2 + 27x + 5} = \frac{3x^2 + 21x}{3x^2 + 27x}$$

By theorem on equal ratios we get,

$$\text{each ratio } k = \frac{3x^2 + 21x + 4 - 3x^2 - 21x}{3x^2 + 27x + 5 - 3x^2 - 27x}$$

$$k = \frac{4}{5}$$

$$\therefore \frac{x+7}{x+9} = \frac{4}{5} \dots\dots [(i)]$$

$$\therefore 5(x+7) = 4(x+9)$$

$$\therefore 5x + 35 = 4x + 36$$

$$\therefore 5x - 4x = 36 - 35$$

$$\therefore \boxed{x = 1}$$

Q 44) $\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{4x + 6}{8x - 10}$ Complete the following activity.

$$\text{Let, } \frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{4x + 6}{8x - 10} = \boxed{}$$

$$\therefore \frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \boxed{} \dots\dots (\text{by theorem on equal ratios})$$

$$\therefore \frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{12x^2 + 18x}{24x^2 - 30x}$$

by theorem on equal ratios we get,

$$\therefore \boxed{} = \boxed{}$$

$$\therefore \square = \square k = \frac{21}{9}$$

$$\therefore \square = \frac{7}{3} \dots\dots\dots (\text{dividing by 3})$$

$$\therefore \square$$

$$\therefore 12x + 18 = 56x - 70$$

$$\therefore \square$$

$$\therefore 88 = 44x$$

$$\therefore x = \frac{88}{44}$$

$$\therefore x = \square$$

$$\text{Ans : Let, } \frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{4x + 6}{8x - 10} = \boxed{k}$$

$$\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{3x(4x + 6)}{3x(8x - 10)}$$

..... (by theorem on equal ratios)

$$\frac{12x^2 + 18x + 21}{24x^2 - 30x + 9} = \frac{12x^2 + 18x}{24x^2 - 30x}$$

by theorem on equal ratios we get,

$$\therefore \text{each ratio } k = \frac{12x^2 + 18x + 21 - (12x^2 - 18x)}{24x^2 - 30x + 9 - (24x^2 - 30x)}$$

$$\therefore k = \frac{21}{9} = \frac{7}{3} \dots\dots (\text{dividing by } 3)$$

$$\boxed{\frac{4x + 6}{8x - 10} = \frac{7}{3}}$$

$$\boxed{3(4x + 6) = 7(8x - 10)}$$

$$12x + 18 = 56x - 70$$

$$\boxed{18 + 70 = 56x - 12x}$$

$$88 = 44x$$

$$\therefore x = \frac{88}{44}$$

$$\therefore \boxed{x = 2}$$

Q 45) If $\frac{x}{x + 2y + z} = \frac{y}{y + 2z + x} = \frac{z}{z + 2x + y}$ and $x + y \neq 0$ then show that each ratio is equal to $\frac{1}{4}$

$$\text{Ans : Let, } \frac{x}{x + 2y + z} = \frac{y}{y + 2z + x} = \frac{z}{z + 2x + y} = k$$

by theorem on equal ratios,

$$\therefore k = \frac{x + y + z}{x + 2y + z + y + 2z + x + z + 2x + y}$$

$$\therefore k = \frac{x + y + z}{x + x + 2x + 2y + y + y + z + 2z + z}$$

$$\therefore k = \frac{x + y + z}{4x + 4y + 4z}$$

$$\therefore k = \frac{x + y + z}{4(x + y + z)}$$

$$\therefore k = \frac{1}{4}$$

$$\therefore \boxed{\frac{x}{x + 2y + z} = \frac{y}{y + 2x + z} = \frac{z}{z + 2x + y} = \frac{1}{4}}$$

Q 46) If $\frac{a}{b} = \frac{c}{d}$ then show that $\frac{a}{b} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}$

Ans : a, b, c, d are in continued proportion.

$$\therefore \text{Let } \frac{a}{b} = \frac{c}{d} = k$$

$$\therefore \boxed{a = bk} \text{ and } \boxed{c = dk}$$

$$\text{Left Hand side } \frac{a}{b} = \frac{bk}{b}$$

$$\therefore \frac{a}{b} = k$$

$$\text{Right Hand side} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}$$

$$= \frac{\sqrt{(bk)^2 + (dk)^2}}{\sqrt{b^2 + d^2}}$$

$$= \sqrt{\frac{b^2 k^2 + d^2 k^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{k^2 (b^2 + d^2)}{b^2 + d^2}}$$

$$= \sqrt{k^2}$$

∴ Right hand side = k

$$\therefore \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} = k$$

∴ L. H. S. = R. H. S

$$\therefore \boxed{\frac{a}{b} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}}}$$

Q 47) Five numbers are in continued proportion. The first term is 4 and last term is 64. Find those numbers.

Ans : Let the numbers in continued Proportion be

$$a, ak, ak^2, ak^3, ak^4$$

Here, $a = 4$ and $ak^4 = 64$

$$\therefore 4 \times k^4 = 64$$

$$\therefore k^4 = \frac{64}{4}$$

$$\therefore k^4 = 16$$

$$\therefore k = 2 \dots\dots (\because 2^4 = 16)$$

$$\therefore a = 4$$

$$ak = 4 \times 2 = 8, ak^2 = 4 \times (2)^2 = 4 \times 4 = 16$$

$$ak^3 = 4 \times 2^3 = 4 \times 8 = 32$$

$$ak^4 = 64$$

$\therefore$ the numbers are 4, 8, 16, 32, 64

Q 48) If a, b, c and d are in proportion, then show that

$$\frac{2a - 3c}{2b - 3d} = \frac{5a + 4c}{5b + 4d}$$

$$\text{Ans : } \frac{a}{b} = \frac{c}{d} = k$$

Putting these value $a = bk$ and $c = dk$

$$\begin{aligned} \text{Left Hand side} &= \frac{2a - 3c}{2b - 3d} \\ &= \frac{2(bk) - 3(dk)}{2b - 3d} = \frac{2bk - 3dk}{2b - 3d} \\ &= \frac{k(2b - 3d)}{(2b - 3d)} \\ &= k \end{aligned}$$

$$\begin{aligned} \text{Right hand side} &= \frac{5a + 4c}{5b + 4d} \\ &= \frac{5(bk) + 4(dk)}{5b + 4d} = \frac{5bk + 4dk}{5b + 4d} \end{aligned}$$

$$= \frac{k(5b + 4d)}{5b + 4d}$$

$$= k$$

$$\therefore \text{L. H. S} = \text{R. H. S}$$

$$\therefore \frac{2a - 3c}{2b - 3d} = \frac{5a + 4c}{5b + 4d}$$

Q 49) $\frac{a}{b} = \frac{b}{c}$ and $a, b, c > 0$ then show that $\frac{a^2 + b^2}{b^2} = \frac{(a + c)^2}{b^2 + c^2}$

Ans : $\frac{a}{b} = \frac{b}{c} = k$

$a = bk$ and $b = ck$

$\therefore a = (ck)k$

$\therefore a = ck^2$

$a = ck^2$ and $b = ck$

Putting these values $a = ck^2$ and $b = ck$

Left hand side = $\frac{a^2 + b^2}{b^2} = \frac{(ck^2)^2 + (ck)^2}{(ck)^2}$

$$= \frac{c^2 k^4 + c^2 k^2}{c^2 k^2}$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c^2 k^2}$$

$$= k^2 + 1$$

$$\text{Right hand side} = \frac{(a+c)^2}{b^2+c^2} = \frac{a^2+2ac+c^2}{b^2+c^2}$$

$$= \frac{(ck^2)^2 + 2(ck^2)c + c^2}{(ck)^2 + c^2}$$

$$= \frac{c^2 k^4 + 2c^2 k^2 + c^2}{c^2 k^2 + c^2}$$

$$= \frac{c^2 (k^4 + 2k^2 + 1)}{c^2 (k^2 + 1)}$$

$$= \frac{(k^2 + 1)(k^2 + 1)}{(k^2 + 1)}$$

$$= k^2 + 1$$

$$\therefore \text{L. H. S} = \text{R. H. S}$$

$$\therefore \frac{a^2+b^2}{b^2} = \frac{(a+c)^2}{b^2+c^2}$$

50) $\frac{a^2+b^2}{a+c} = a$ Complete the following activity.

$a = \boxed{}$ and $b = \boxed{}$ putting these values.

Left Hand side = $\boxed{}$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \boxed{}$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \boxed{}$$

Right hand side = a

$$= \boxed{}$$

$\therefore \text{L.H.S} = \text{R.H.S}$

$$\therefore \boxed{}$$

Ans : a = $\boxed{ck^2}$ b = $\boxed{ck}$ ing these values

Left Hand side =

$$\boxed{\frac{a^2 + b^2}{a + c}}$$

$$= \frac{(ck^2)^2 + (ck)^2}{ck^2 + c}$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \frac{c^2 k^2 (k^2 + 1)}{c (k^2 + 1)}$$

$$= \boxed{ck^2}$$

Right Hand side = a

$$= \boxed{ck^2}$$

$\therefore \text{L.H.S} = \text{R.H.S}$

$$\therefore \boxed{\frac{a^2 + b^2}{a + c} = a}$$
