2. Real Numbers

Extra Questions

- Classify the decimal form of the given rational numbers into terminating and non-terminating recurring type.(1mark)

1) $\frac{4}{5}$

Ans : Since, 5 is the only prime factor in the denominator

∴ the decimal form of rational number $\frac{4}{5}$ will be terminating type.

or method (II)

$$
\begin{array}{r|l}
 & 0.8 \\
\hline
5 & 40 \\
 & -\,0 \\
\hline
 & 40 \\
 & -\,40 \\
\hline
 & 00
\end{array}
$$

∴ $\frac{4}{5}$ is terminating type

2) $\frac{1}{2}$

Ans : Since, 2 is the only prime factor in the denominator.

∴ the decimal form of rational number $\frac{1}{2}$ will be terminating type.

or method (II)

$$
\begin{array}{r|l}
2 & 1.0 \\
 & -\ 0 \\
\hline
 & 10 \\
 & -\ 10 \\
\hline
 & 00
\end{array}
$$

$\therefore \frac{1}{2}$ is terminating type

3) $\frac{16}{11}$

Ans : Since, the denominator is other than prime factors 2 or 5

$\therefore$ the decimal form of rational number $\frac{16}{11}$ will be non-terminating type recurring type.

or method (II)

$$
\begin{array}{r|l}
11 & 16.0000 \ldots \\
 & -\ 11 \\
\hline
 & 50 \\
 & -\ 44 \\
\hline
 & 60 \\
 & -\ 55 \\
\hline
 & 50 \\
 & -\ 44 \\
\hline
 & 60 \\
 & -\ 55 \\
\hline
 & 05
\end{array}
$$

$\therefore \frac{16}{11} = 1.454545 \ldots.$

$\therefore \frac{16}{11} = 1.\overline{45}$

$\therefore \frac{16}{11}$ is non-terminating recurring type

4) $\frac{2}{3}$

Ans : Since, the denominator is other than prime factors 2 or 5.

$\therefore$ the decimal form of the rational number $\frac{2}{3}$ will be non-terminating recurring type.

or method (II)

```
      1.666
    ---------
3 ) 2.000
    - 0
    -----
      20
     -18
    -----
      20
     - 18
    -----
       20
      -18
    -----
       02
    -----
```

$\therefore \frac{2}{3} = 0.666 \ldots = 0.\dot{6}$

$\therefore \frac{2}{3}$ = is non-terminating recurring type.

Write the following rational numbers in decimal form. (3M)

5) $3\frac{1}{8}$

Ans : Given, $3\frac{1}{8} = \frac{3\times8+1}{8} = \frac{24+1}{8} = \frac{25}{8}$

$$
\begin{array}{r|l}
8 & 3.125 \\
\hline
 & 25.000 \\
 & -24 \\
\hline
 & 10 \\
 & -8 \\
\hline
 & 20 \\
 & -16 \\
\hline
 & 40 \\
 & -40 \\
\hline
 & 00
\end{array}
$$

$\therefore\ 3\frac{1}{8} = \frac{25}{8} = 3.125$

6) $3\frac{4}{5}$ (CBSE 2015)

Ans : Given, $3\frac{4}{5} = \frac{3\times5+4}{5} = \frac{15+4}{5} = \frac{19}{5}$

$$
\begin{array}{r|l}
5 & 3.8 \\
\hline
 & 19.0 \\
 & -15 \\
\hline
 & 40 \\
 & -40 \\
\hline
 & 00
\end{array}
$$

$\therefore\ 3\frac{4}{5} = \frac{19}{5} = 3.8$

7) $\frac{135}{1125}$

Ans :

$$
\begin{array}{r|l}
1125 & 0.12 \\
\hline
 & 135.00 \\
 & -\ 15 \\
\hline
 & 1350 \\
 & -1125 \\
\hline
 & 2250 \\
 & -\ 2250 \\
\hline
 & 0000
\end{array}
$$

$\therefore \frac{135}{1125} = 0.12$

8) $\frac{17}{99}$

Ans :

$$
\begin{array}{r|l}
99 & 0.171717\ldots \\
\hline
 & 17.0000 \\
 & -\ 0 \\
\hline
 & 170 \\
 & -99 \\
\hline
 & 710 \\
 & -\ 693 \\
\hline
 & 170 \\
 & -\ 99 \\
\hline
 & 710 \\
 & -\ 693 \\
\hline
 & 17
\end{array}
$$

$\therefore \frac{17}{99} = 0.1717\ldots..$

$\therefore \frac{17}{99} = 0.\overline{17}$

➢ Write the following national numbers in $\frac{p}{q}$ form (3 mark)

9) $0.\dot{2}$ (CBSE 2016)

Ans : Let x = $0.\dot{2}$ = 0.222…… (i)

Since, one number i.e. 2 is repeating after decimal point.

Thus, multiplying both sides by 10

10x = 2.222 …… (ii)

Subtracting (ii) from (i)

10x – x = 2. $\dot{2}$ – 0. $\dot{2}$

9x = 2

∴ x = $\frac{2}{9}$

∴ $0.\dot{2}$ = $\frac{2}{9}$

10) $17.\overline{82}$

Ans : let x = 17. $\overline{82}$ = 17.8282 …… (i)

Multiplying both sides by 100,

100x = $17.\overline{82}$ ……..(ii)

Subtracting (ii) from (i)

100x – x = 1782 . $\overline{82}$ – 17.82

∴ 99x = 1765

x = $\frac{1765}{99}$

∴ $17.\overline{82}$ = $\frac{1765}{99}$

11) $0.\overline{513}$

Ans : let x = $0.\overline{513}$ = 0.513513 ……. (i)

Multiplying both sides by 1000.

1000x = 513.513 …….. (ii)

Subtracting (ii) from (i),

∴ 1000x – x = 513.513 – 0.513

∴ 999x = 513

∴ x = $\frac{513}{999}$

∴ $0.\overline{513}$ = $\frac{513}{999}$

12) $0.\overline{3178}$ (CBSE2015)

Ans : let x = $0.\overline{3178}$ = 0.31783178 ……. (i)

Multiplying both sides by 10000,

1000x = 3178.3178 …….. (ii)

Subtracting (ii) from (i),

∴ 10000x – x = 3178.3178 – 0.3178

∴ 9999x = 3178

∴ x = $\frac{3178}{9999}$

∴ $0.\overline{3178}$ = $\frac{3178}{9999}$

13) Show that $5\sqrt{2}$ is an irrational number. (4mark)

Ans : let us assume that $5\sqrt{2}$ is a rational number. so, we can find co-prime integers 'a' and 'b' ($b \neq 0$) such that.

$5\sqrt{2} = \frac{a}{b}$

$\therefore 5b\sqrt{2} = a$

Squaring both sides,

$(5)^2 b^2 (\sqrt{2})^2 = a^2$

$25b^2 \times 2 = a^2$

$50b^2 = a^2$ ..... (i)

$\therefore b^2 = \frac{a^2}{50}$

Since 50 divides a^2, so 50 divides 'a' as well.

So, we write a = 50c, where c is an integer.

$\therefore a^2 = (50c)^2$ ..... [squaring both sides]

$\therefore 50b^2 = 50 \times 50c^2$ ..... [from (i)]

$\therefore b^2 = 50c^2$

$\therefore c^2 = \frac{b^2}{50}$

Since, 50 divides b^2, so 50 divides 'b'

$\therefore$ 50 divides both a and b

$\therefore$ a and b have at least 50 as a common factor but this contradicts the fact that a and b have no common factor other than 1.

∴ our assumption that $5\sqrt{2}$ is a rational number is wrong.

∴ $5\sqrt{2}$ is an irrational number.

14) Prove that $6 + \sqrt{7}$ is an irrational number. (4 mark)

Ans : Let $6+\sqrt{7}$ not be an irrational number.

It means $6+\sqrt{7}$ is a rational number.

If $6+\sqrt{7}$ is rational then let $6+\sqrt{7} = \frac{p}{q}$

∴ we get $\sqrt{7} = \frac{p}{q} - 6$

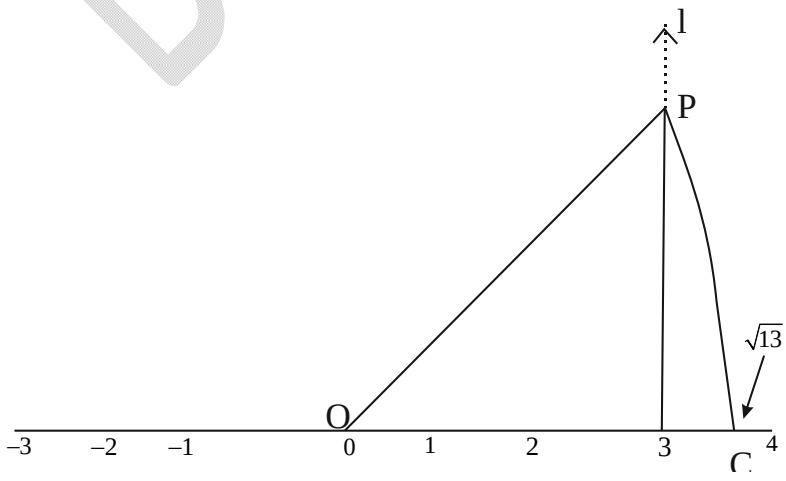
In this equation left side = $\sqrt{7}$ is an irrational number. and right side = $\frac{p}{q} - 6$ is rational number, which contradictory.

∴ $6+\sqrt{7}$ is rational number is not correct.

∴ $6+\sqrt{7}$ is an irrational number.

15) Represent the number $\sqrt{13}$ on a number line [CBSE2015] (4 mark)

Ans : To represent $\sqrt{13}$ on the number line.



Take a number line with o as zero. Mark point A on it such that OA = 3 units. Draw a line AP = 2 units passing through A and perpendicular to line *l*. Join OP.

Then OP = $\sqrt{13}$ units.

Draw an arc with centre O and radius OP, which intersects the number line at c.

Then, C represents $\sqrt{13}$ on the number line. In right angle triangle ΔOAP,

$$OP^2 = OA^2 + AP^2$$
$$OP^2 = 3^2 + 2^2$$
$$OP^2 = 9 + 4$$
$$OP^2 = 13$$
$$\therefore OP = \sqrt{13}$$
$$\therefore OC = \sqrt{13}$$

Hence verified.

16) Write any four rational numbers between -3 and -4 .

Ans : -3.1, -3.2, -3.3, ……. -3.9.

17) Write any three rational numbers between 6.2 and 6.3 (1 mark)

Ans : - 6.21, 6.22, 6.23, ……..

Write the following root of numbers may be rational or irrational verify it. (2 mark)

18) 3^6

Ans : - $3^6 = 729$

∴ 3 is the 6^{th} root of 729.

But if $x^6 = 4$ then x = $\sqrt[6]{3}$ which is an irrational number.

19) 4^4

Ans : - 4^4 = 256

∴ 4 is the 4^{th} root of 256 but if $x^4 = 4$ then x = $\sqrt[4]{4}$ which is an irrational number.

- State which of the following are surds. Justify (2 marks)

20) $\sqrt[3]{17}$

Ans : $\sqrt[3]{17}$ is surd because 17 is a positive rational number, 3 is a positive integer greater than 1 and $\sqrt[3]{17}$ is irrational.

21) $\sqrt[3]{125}$

Ans : $\sqrt[3]{125}$ is not surd because $\sqrt[3]{125}$ = 5 is not irrational number.

22) $\frac{1}{\sqrt{15}}$

Ans : $\frac{1}{\sqrt{15}}$ is surd because $\sqrt{15}$ is irrational number.

∴ $\frac{1}{\sqrt{15}}$ is irrational number.

- Classify the given pair of surds into like surds and unlike surds. (2 marks)

23) $\sqrt{75}, \sqrt{147}$

Ans : $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$

$\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$

$\therefore \sqrt{75}, \sqrt{147}$ are like surds.

24) $\sqrt{117}, 3\sqrt{2}$

Ans : $\sqrt{117} = \sqrt{9 \times 13} = 3\sqrt{13}$

And $3\sqrt{2}$

$\therefore \sqrt{117}, \ 3\sqrt{2}$ are unlike surds.

Express the following surds are pure surds. (2 marks)

25) $4\sqrt{5}$

Ans : $= \sqrt{4^2 \times 5}$

$= \sqrt{16 \times 5}$

$= \sqrt{80}$

$\therefore 4\sqrt{5} = \sqrt{80}$

26) $3\sqrt{6}$

Ans : $= \sqrt{3^2 \times 6}$

$= \sqrt{9 \times 6}$

$= \sqrt{54}$

$\therefore 3\sqrt{6} = \sqrt{54}$

➢ Simplify the following surds . (2 marks)

27) $\sqrt{882}$

Ans : $= \sqrt{2 \times 441}$

$= \sqrt{2 \times 3 \times 147}$

$= \sqrt{2 \times 3 \times 3 \times 49}$

$\therefore 4\sqrt{5} = \sqrt{2 \times 9 \times 49}$

$= 3 \times 7\sqrt{2}$

$= 21\sqrt{2}$

28) $\sqrt{98}$

Ans : $= \sqrt{98}$

$= \sqrt{2 \times 49}$

$= 7\sqrt{2}$

➢ Compare the following pair of surds. (2 marks)

29) $5\sqrt{6}, 6\sqrt{5}$

Ans : $5\sqrt{6} = \sqrt{5^2 \times 6} = \sqrt{25 \times 6} = \sqrt{150}$

$6\sqrt{5} = \sqrt{6^2 \times 5} = \sqrt{36 \times 5} = \sqrt{180}$

$\therefore \sqrt{150} < \sqrt{180}$

$\therefore 5\sqrt{6} < 6\sqrt{5}$

30) $3\sqrt{17}, 19\sqrt{2}$

Ans : $3\sqrt{17} = \sqrt{3^2 \times 17} = \sqrt{9 \times 17} = \sqrt{153}$

$19\sqrt{2} = \sqrt{19^2 \times 2} = \sqrt{361 \times 2} = \sqrt{722}$

$\therefore \sqrt{153} < \sqrt{722}$

$\therefore 3\sqrt{17} < 19\sqrt{2}$

31) $4\sqrt{7}, 5\sqrt{2}$

Ans : $4\sqrt{7} = \sqrt{4^2 \times 7} = \sqrt{16 \times 7} = \sqrt{112}$

$5\sqrt{2} = \sqrt{5^2 \times 2} = \sqrt{25 \times 2} = \sqrt{50}$

$\therefore \sqrt{112} > \sqrt{50}$

$\therefore 4\sqrt{7} > 5\sqrt{2}$

➢ Simplify. (3 marks)

32) $\frac{3}{5}\sqrt{7} + 2\sqrt{7}$

Ans : $\frac{3}{5}\sqrt{7} + 2\sqrt{7}$

$= \left(\frac{3}{5} + 2\right)\sqrt{7}$

$= \left(\frac{3+5\times2}{5}\right)\sqrt{7}$

$= \frac{13}{5}\sqrt{7}$

33) $\frac{1}{4}\sqrt{243} + \sqrt{\frac{27}{4}}$

Ans : $\frac{1}{4}\sqrt{3 \times 81} + \frac{\sqrt{3\times9}}{2}$

$= \frac{1}{4}\ 9\sqrt{3} + \frac{3\sqrt{3}}{2}$

$= \frac{9}{4}\sqrt{3} + \frac{3}{2}\sqrt{3}$

$= \left(\frac{9}{4} + \frac{3}{2}\right)\sqrt{3}$

$= \left(\frac{9}{4} + \frac{3\times2}{2\times2}\right)\sqrt{3}$

$= \left(\frac{9}{4} + \frac{6}{4}\right)\sqrt{3}$

$= \frac{15}{4}\sqrt{3}$

34) $\sqrt{7} - \frac{2}{5}\sqrt{7}$

Ans : $1\sqrt{7} - \frac{2}{5}\sqrt{7}$

$= \left(1 - \frac{2}{5}\right)\sqrt{7}$

$= \left(\frac{5-2}{5}\right)\sqrt{7}$

$= \frac{3}{5}\sqrt{7}$

35) $6\sqrt{32} - \sqrt{18} - \sqrt{2}$

Ans : $6\sqrt{16\times2} - \sqrt{9\times2} - \sqrt{2}$

$= 6\times 4\sqrt{2} - 3\sqrt{2} - \sqrt{2}$

$= 24\sqrt{2} - 3\sqrt{2} - \sqrt{2}$

$= (24 - 3 - 1)\sqrt{2}$

$= 20\sqrt{2}$

36) $\sqrt{24} \times \sqrt{27}$

Ans : $\sqrt{24} \times \sqrt{27}$

$= \sqrt{24 \times 27}$

$= \sqrt{648}$

$= \sqrt{2 \times 324}$

$= \sqrt{2 \times 3 \times 108}$

$= \sqrt{2 \times 3 \times 2 \times 54}$

$= \sqrt{12 \times 54}$

$= \sqrt{12 \times 3 \times 18}$

$= \sqrt{36 \times 18}$

$= 6 \times 3\sqrt{2}$

$= 18\sqrt{2}$

37) $4\sqrt{12} \times 7\sqrt{16}$

Ans : $4\sqrt{12} \times 7\sqrt{16}$

$= 4 \times 7\sqrt{12 \times 16}$

$= 28 \times 4\sqrt{12}$

$= 112\sqrt{12}$

$= 112\sqrt{4 \times 3}$

$= 112 \times 2\sqrt{3}$

$= 224\sqrt{3}$

38) $\sqrt{147} \div \sqrt{3}$

Ans : $\frac{\sqrt{147}}{\sqrt{3}}$

$= \frac{\sqrt{3 \times 49}}{\sqrt{3}}$

$= \frac{7\sqrt{3}}{\sqrt{3}}$

$= 7$

39) $4\sqrt{28} \div 3\sqrt{7}$

Ans : $\frac{4\sqrt{28}}{3\sqrt{7}}$

$= \frac{4\sqrt{4 \times 7}}{3\sqrt{7}}$

$= \frac{4 \times 2\sqrt{7}}{3\sqrt{7}}$

$= \frac{8\sqrt{7}}{3\sqrt{7}}$

$= \frac{8}{3}$

➢ Rationalize the denominator . (4 marks)

40) $\frac{-5}{2\sqrt{5}}$

Ans : $\frac{-5}{2\sqrt{5}}$

$= \frac{-5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$$= \frac{-5\sqrt{5}}{2\sqrt{5} \times \sqrt{5}}$$

$$= \frac{-5\sqrt{5}}{2(\sqrt{5})^2}$$

$$= \frac{-5\sqrt{5}}{2 \times 5}$$

$$= \frac{-5\sqrt{5}}{10}$$

$$= \frac{-\sqrt{5}}{2}$$

41) $\frac{5}{\sqrt{27}}$

Ans : $\frac{5}{\sqrt{3 \times 9}}$

$$= \frac{5}{3\sqrt{3}}$$

$$= \frac{5}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{3 \times \sqrt{3} \times \sqrt{3}}$$

$$= \frac{5\sqrt{3}}{3 \times (\sqrt{3})^2}$$

$$= \frac{5\sqrt{3}}{3 \times 3}$$

$$= \frac{5\sqrt{3}}{9}$$

42) Rationalize the denominator $\frac{8}{7-3\sqrt{5}}$ (CBSE 2017). (4 marks)

Ans : The conjugate pair of $7-3\sqrt{5}$ is $7+3\sqrt{5}$

$\therefore \frac{8}{7-3\sqrt{5}} = \frac{8}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ .....[multiplying the numerator and denominator by $7+3\sqrt{5}$]

$= \frac{8 \times (7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})}$ ..... [$\because$(a-b) (a+b) = $a^2 - b^2$]

$= \frac{8 \times (7+3\sqrt{5})}{(7)^2-(3\sqrt{5})^2}$

$= \frac{8\times (7+3\sqrt{5})}{49-9\times 5}$

$= \frac{8\ (7+3\sqrt{5})}{49-45}$

$= \frac{8\ (7+3\sqrt{5})}{4}$

$= 2\ (7 + 3\sqrt{5})$

43) $\frac{6}{2\sqrt{3}+\sqrt{6}}$

Ans : $\frac{6}{2\sqrt{3}+\sqrt{6}}$

The conjugate pair of $2\sqrt{3} + \sqrt{6}$ is $2\sqrt{3} - \sqrt{6}$

$= \frac{6}{2\sqrt{3}+\sqrt{6}} \times \frac{2\sqrt{3}-\sqrt{6}}{2\sqrt{3}-\sqrt{6}}$ ... [multiplying the numerator and denominator by $2\sqrt{3} - \sqrt{6}$]

$= \frac{6 \times (2\sqrt{3}-\sqrt{6})}{(2\sqrt{3}+\sqrt{6})(2\sqrt{3}-\sqrt{6})}$ ..... [$\because$(a-b) (a+b) = $a^2 - b^2$]

$= \frac{12\sqrt{3}-6\sqrt{6}}{4\times 3-6}$

$= \frac{12\sqrt{3}-6\sqrt{6}}{12-6}$

$= \frac{12\sqrt{3}-6\sqrt{6}}{6}$

$= \frac{12\sqrt{3}}{6} - \frac{6\sqrt{6}}{6}$

$= 2\sqrt{3} - \sqrt{6}$

➢ Find the value . (2 mark)

44) $|5 \times (-4) + 4 \times (-5)|$

Ans : $|-20 + (-20)|$

$= |-20 - 20|$

$= |-40|$

$= 40$

45) $|7(5 - 7) - 3 \times (4 - 5)|$

Ans : $|7(-2) - 3(-1)|$

$= |7(-2) + 3|$

$= |-14 + 3|$

$= |-11|$

$= 11$

46) $|-3| \times |7|$

Ans : $|-3| \times |7|$

$= |3| \times |7|$

$= |21|$

$= 21$

47) $\left|5 - \frac{1}{2}x\right| = \frac{1}{4}$.(3 mark)

Ans : $5 - \frac{1}{2}x = \frac{1}{4}$ or $5 - \frac{1}{2}x = \frac{-1}{4}$

$\frac{-x}{2} = \frac{1}{4} - 5$ or $\frac{-x}{2} = \frac{-1}{4} - 5$

$\frac{-x}{2} = \frac{1-20}{4}$ or $\frac{-x}{2} = \frac{-20-1}{4}$

$\frac{-x}{2} = \frac{-19}{4}$ or $\frac{-x}{2} = \frac{-21}{4}$

$x = \frac{19}{2}$ or $x = \frac{21}{2}$

48) $\left|x - \frac{1}{2}\right| = \frac{3}{2}$.(3 mark)

Ans : $x - \frac{1}{2} = \frac{3}{2}$ or $x - \frac{1}{2} = \frac{-3}{2}$

$\frac{2x-1}{2} = \frac{3}{2}$ or $\frac{2x-1}{2} = \frac{-3}{2}$

$2x - 1 = 3$ or $2x - 1 = -3$

$2x = 3 + 1$ or $2x = -3 + 1$

$2x = 4$ or $2x = -2$

$x = \frac{4}{2}$ or $x = \frac{-2}{2}$

$x = 2$ or $x = -1$

49) $|4x - 2| = 10$. (3 mark)

Ans : $4x - 2 = 10$ or $4x - 2 = -10$

$4x = 10 + 2$ or $4x = -10 + 2$

$4x = 12$ or $4x = -8$

$x = \frac{12}{4}$ or $x = \frac{-8}{4}$

$x = 3$ or $x = -2$

50) show that $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}} = 0$ (CBSE2017). (4 mark)

Ans : $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2+\sqrt{5}}{(2)^2-(\sqrt{5})^2}$

$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2+\sqrt{5}}{4-5}$

$= \frac{2-\sqrt{3}}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2+\sqrt{5}}{(-1)}$

$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2-\sqrt{5}$

$= 0$

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