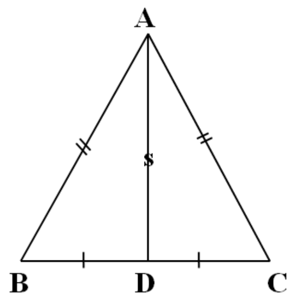
13. Congruence of triangles.

1. In the adjacent figure, seg AB ≅ seg AC.
The point D is the midpoint of side BC.
To show that ∠BAD ≅ ∠CAD



Solution : In Δ BAD and Δ CAD ,

seg AB ≅ seg AC…….. (Given)

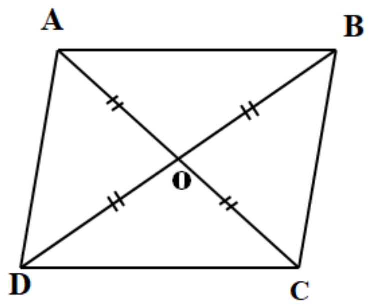
seg BD ≅ seg DC…….. (The point D is the midpoint of side BC.)

seg AD ≅ seg AD …….. (Common side)

∴ ΔBAD ≅ ΔCAD ……..(S – S - S test)

∴ ∠BAD ≅ ∠CAD…….. (corresponding angles of congruent triangles)

2. In the adjacent figure,
seg AC and seg BD intersect each other at point O. To show that ,
(i) seg AD ≅ seg BC
(ii) seg AB ≅ seg DC



Solution : (i) In ΔADO and ΔCBO,

AO = CO ……. (O is the point of intersection of seg AC)

DO ≅ BO ……. (O is the point of intersection of seg DB)

∠AOD ≅ ∠COB ……. (Pair of opposite angles)

∴ ΔADO ≅ ΔCBO…… (S – A – S test)

∴ seg AD ≅ seg BC …….(corresponding sides of congruent triangles.)

(ii) In ΔABO and ΔCDO

AO = CO…..(O is the point of intersection of seg AC)

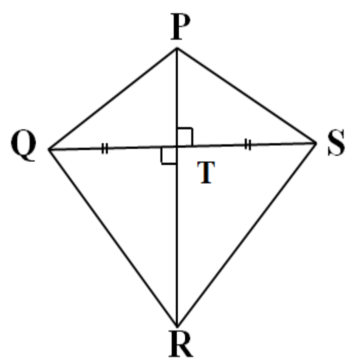
BO = DO…… (O is the point of intersection of seg BD)

∠AOB ≅ ∠COD ……. (Pair of opposite angles)

∴ ΔABO ≅ ΔCDO…… (S-A-S test)

∴ seg AB ≅ seg DC……. (corresponding sides of congruent triangles.)

3. In the figure alongside, seg QS ⊥ seg PR.
T is the midpoint of seg QS.
To show that
(i) seg PQ ≅ seg PS
(ii) seg QR ≅ seg RS



Solution : seg QS ⊥ seg PR

∴ ∠PTQ = PST = RTQ = RTS = Each angle is 90^0

(i) In ΔPTQ and ΔPST ,

QT = ST ……. (T is the midpoint of seg QS)

PT = PT ……. (Common)

$m\angle$PTQ = $m\angle$PTS …….. (Each 90^0)

∴ ΔPQT ≅ ΔPST …… (S – A- S test)

∴ seg PQ ≅ seg PS ……. (corresponding sides of congruent triangles)

(ii) In ΔRQT and ΔRST ,

QT = ST……. (T is the midpoint of Seg QS)

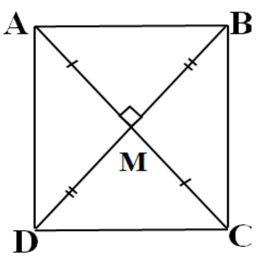
RT = RT ……. (Common)

$m\angle RTQ = m\angle RTS$……. (Each 90^0)

∴ ΔRQT ≅ ΔRST…….. (S – A – S test)

∴ seg QR ≅ seg RS ……. (corresponding sides of congruent triangles)

4. In the adjacent figure, seg AC ⊥ seg BD. seg AC and seg BD intersect each other at point M. To show that
(i) AB ≅ BC (ii) AB ≅ AD



Solution : (i) In ΔABM and ΔCBM,

AM ≅ CM …….(M is the point of intersection of Seg AC)

BM ≅ BM…… (Common)

$m\angle BMA = m\angle BMC$…… (each 90^0)

∴ ΔABM ≅ ΔCBM …….. (S – A – S test)

∴ AB ≅ BC …….. (corresponding sides of congruent triangles)

(ii) In ΔABM and ΔADM,

BM $\cong$ DM ……(M is the point of intersection of seg BD)

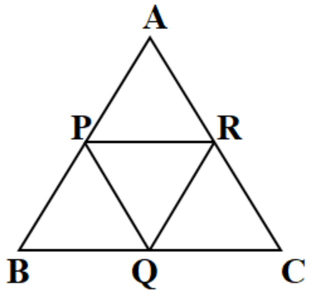
AM $\cong$ AM …… (Common)

$m\angle$AMB $= m\angle$AMD…… (each 90^0)

ΔABM $\cong$ ΔADM …….. (S – A – S test)

$\therefore$ AB $\cong$ AD…….. (corresponding sides of congruent triangles)

5. In the figure alongside, ΔABC is a rhombus the point P , Q and R are the midpoint of seg AB , seg BC and seg AC respectively.
Then show that $\Delta QCR \cong \Delta PBQ \cong \Delta APR$



Solution : ΔABC is a rhombus.

$\therefore$ Suppose AB = BC = CA = a

Also $m\angle A = m\angle B = m\angle C$

P , Q , R are the midpoint of side AB , side BC , side AC.

$\therefore$ BQ = CQ = $\frac{1}{2}$ BC = $\frac{1}{2}$ a

AR = CR = $\frac{1}{2}$ AC = $\frac{1}{2}$ a

AP = BP = $\frac{1}{2}$ AB = $\frac{1}{2}$ a

In ΔAPR and ΔBQP ,

seg AP ≅ seg BQ , seg AR ≅ seg BP …..(each $\frac{1}{2}$ a)

∠A ≅ ∠B

∴ ΔAPR ≅ ΔBQP……. (S – A – S test) ….. (1)

∴ seg PR ≅ seg QP

Also ΔAPR ≅ ΔCQR …….(2)

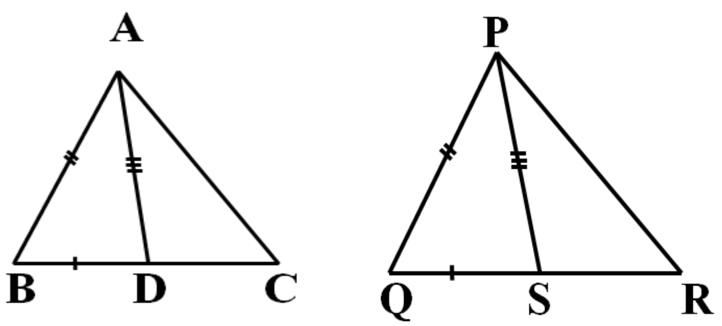
∴ seg PR ≅ seg RQ

∴ ΔAPR ≅ ΔBQP ≅ ΔCRQ ….. [from (1) and (2)] …. (3)

∴ seg PR ≅ seg QP ≅ seg RQ….. (4)

∴ ΔQCR ≅ ΔPBQ ≅ ΔAPR ….. [from (3) and (4)]

6. In the adjacent figure, seg AD and seg PS are the medians of ΔABC and ΔPQR respectively. If seg AB ≅ seg PQ, seg BC ≅ seg QR, and seg AD ≅ seg PS then show that ΔABC ≅ ΔPQR



Solution : In ΔABC , seg AD is a median.

∴ Point D is the midpoint of side BC.

∴ BD = $\frac{1}{2}$BC

Also in ΔPQR, seg PS is a median.

Point S is the midpoint of seg PS

$\therefore$ QS = $\frac{1}{2}$ QR

But BC = QR ……. (Given)

$\therefore$ BD = QS

seg AB ≅ seg PQ and seg AD ≅ seg PS ……. (Given)

$\therefore$ ΔABD ≅ ΔPQS ……. (S – S – S test)

$\therefore$ ∠ABD ≅ ∠PQS ……. (corresponding angles of congruent triangles)

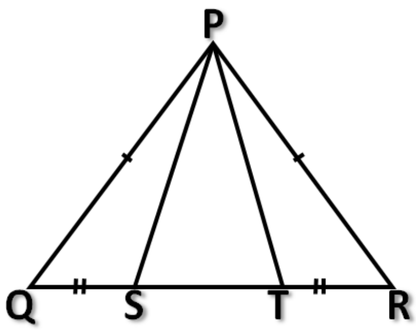
Also ∠ABC ≅ ∠PQR

seg AB ≅ seg PQ…… (Given)

And seg BC ≅ seg QR ……. (Given)

$\therefore$ ΔABC ≅ ΔPQR …… (S – A – S test)

7. In the figure alongside, seg PQ ≅ seg PR seg QS ≅ seg RT. Q – S –T and R – T – S then show that (i) ΔPQS ≅ ΔPRT (ii) ΔPQT ≅ ΔPRS



Solution : (i) In ΔPQS and ΔPRT,

In the correspondence PQS ↔ PRT

seg PQ ≅ seg PR …… (Given)

seg QS ≅ seg RT …… (Given)

∠PQS ≅ ∠PRT

$\therefore$ ΔPQS ≅ ΔPRT …….. (S – A – S test)

(ii) QS + ST = QT ……. (Q – S –T)

∴ QS = QT – ST

And ST + TR = SR ……. (S – T – R)

∴ TR = SR – ST

But , QS = RT ……. (Given)

∴ QT – ST = SR – ST

∴ QT = SR …… (i)

In ΔPQT and ΔPRS ,

In the correspondence PQT↔ PRS

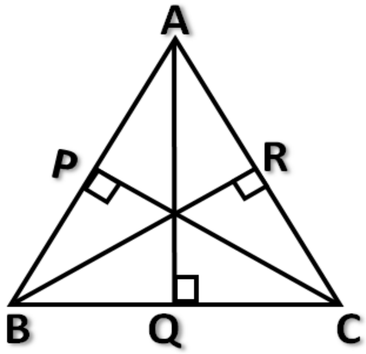
seg PQ ≅ seg PR ……. (Given)

seg QT ≅ seg RS ……. (from (i))

∠PQT ≅ ∠PRS

∴ ΔPQT ≅ ΔPRS …….. (S – A – S test)

8. In ΔABC , seg AQ ⊥ seg BC ,
seg BR ⊥ seg AC, seg CP ⊥ seg AB
and AQ = BR = CP
then show that AB = BC = AC



Solution : In ΔPBC and ΔRCB ,

PC = RB ……. (Given)

m∠BPC = m∠CRB ……. (Each 90^0)

seg BC ≅ seg CB ….. (common)

∴ ΔPBC ≅ ΔRCB ……(Hypotenuse – side test)

∴ ∠PBC = ∠RCB …… (corresponding angles of congruent triangles)

∠ABC = ∠ACB

∴ AB = AC ……. (Sides of opposite equal angles)

Now, In ΔABQ and ΔBAR

AQ = BR …….. (given)

m∠AQB = m∠ARB ……. (Each 90^0)

seg AB ≅ seg BA ….. (common)

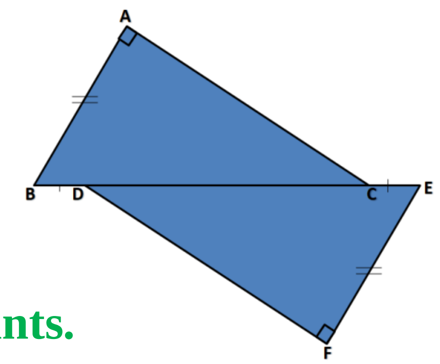
ΔABQ ≅ ΔBAR …… (Hypotenuse side theorem)

∴ ∠ABQ ≅ ∠BAR …… (corresponding angles of congruent triangles)

∴ AC = BC ……. (2)

∴ AB = BC = AC ……. [from (1) and (2)]

9. In the figure alongside, seg AB ⊥ seg AC , seg EF ⊥ seg DF. AB – EF and BD = CF then to show that AC = FD



Solution : B , D ,C and E are the collinear points.

∴ B – D – C

∴ BD + DC = BC

And D – C – E

∴ DC + CE = DE

BD = CE ……. (given)

∴ BD + DC = DC + CE

∴ BC + DE

And AB = EF ……. (given)

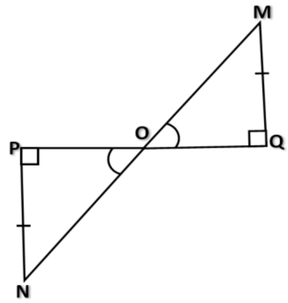
∠BAC ≅ ∠EFD ……(each right angle)

ΔABC ≅ ΔFED ……. (Hypotenuse side theorem)

∴ seg AC ≅ seg FD ….. (corresponding sides of congruent triangles)

∴ AC = FD

10. In the adjacent figure, seg QM ≅ seg NP, seg QM and seg NP are perpendicular to seg PQ. Then to show that the point O is the midpoint of seg MN .



Solution : In ΔMQO and ΔNPO

Seg QM ≅ Seg PN ……. (given)

∠MQO ≅ ∠NPO …….. (each right angle)

∠MOQ ≅ ∠NOP …… (Pair of opposite angles)

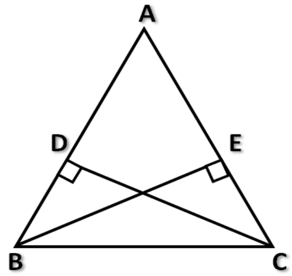
∴ ΔMQO ≅ ΔNPO……. (S –A – S test)

∴ seg OQ ≅ seg OP ……. (corresponding sides of congruent triangles)

∴ OQ = OP and OM = ON

∴ The point O is the midpoint of seg PQ. Also the midpoint of seg MN.

11. In the figure alongside, seg CD ≅ seg BF. $\angle CDB = \angle CEB = 90^0$ then to show that seg AC ≅ seg AB.



Solution : In ΔDBC and ΔECB

seg CD ≅ seg BE …… (given)

∠CDB ≅ ∠CEB ……. (each 90^0)

hypt BC ≅ hypt CB ……. (common side)

∴ ΔDBC ≅ ΔECB……. (hypotenuse side theorem)

∠DBC ≅ ∠ECB ……. (corresponding angles of congruent triangles)

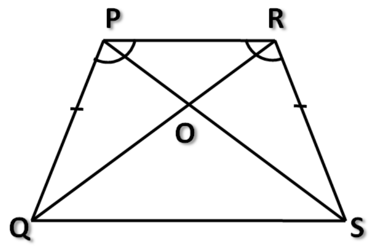
∴ ∠ABC ≅ ∠BAC

In ΔABC,

Side AC is the opposite side of ∠ABC and side AB is opposite side of ∠ACB.

∴ seg AC ≅ seg AB.

12. In the adjacent figure, m∠QPR = m∠SRP seg PQ ≅ seg RS, then to show that ΔPQR ≅ ΔRSP. Also write the remaining congruent parts of triangles.



Solution : In ΔPQR and ΔRSP

seg PQ ≅ seg RS …….. (given)

seg PR ≅ seg RP …….. (common side)

And ∠QPR ≅ ∠SRP ……. (given)

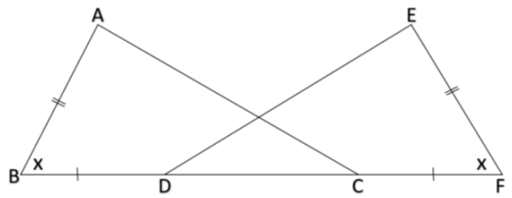
∴ ΔPQR ≅ ΔRSP…….. (S – A – S)

The remaining congruent parts of triangles :

seg QR ≅ seg SP, ∠PQR ≅ ∠RSP and ∠PRQ ≅ ∠RPS.

13. In the figure alongside,

seg AB ≅ seg EF,
seg BD ≅ seg CF,
∠ABC ≅ ∠EFD. State ΔABC and ΔAFD are congruent. Give reason.



Solution : seg BD ≅ seg CF i.e . BD = CF

∠ABC ≅ ∠EFD ……. (given)

and seg AB ≅ seg EF ……. (given)

∴ ΔABC ≅ ΔEFD …….. (S – A – S test)

seg AC ≅ seg ED ……. (corresponding sides of congruent triangles)

∠ACB ≅ ∠EDF …….. (corresponding angles of congruent triangles)

seg BC ≅ seg FD ……. (corresponding angles of congruent triangles)

Reason : If two sides and the included angle of a triangle are congruent with two corresponding sides and the included angle of the other triangle then the two triangles are congruent with each other by S – A – S test.

14. In the adjacent figure,

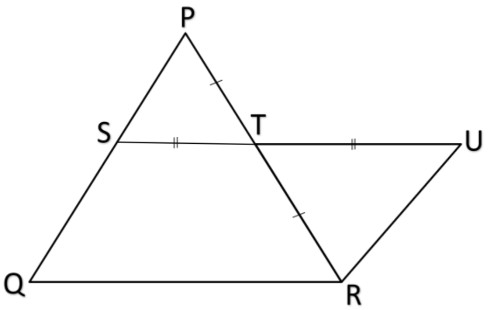
seg ST ≅ seg TU,

Seg PT ≅ seg TR , Also

P – T – R and S – T – U then

to show that ΔPST ≅ ΔRUT.

Write the remaining congruent parts of triangle.



Solution : In ΔPST and ≅ ΔRUT

seg ST ≅ seg TU ……. (i)

seg PT ≅ seg TR ……..(ii)

P –T – R that means the three points P ,T, R are the points on seg PR.

and S – T – U that means the three points S , T , U are the points on seg SU.

i. e. seg PR and seg SU intersect at point T.

∴ ∠PTS ≅ ∠RTU …… (Opposite angles)….. (iii)

From (i) , (ii) , and (iii) ,

ΔPST ≅ ΔRUT ….. (S – A – S test)

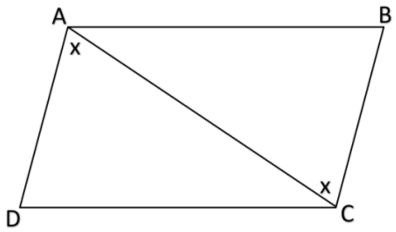
The remaining congruent parts of triangle :

seg PS ≅ seg RU , ∠SPT ≅ ∠URT and

∠PST ≅ ∠RUT

15. In the adjacent figure,

∠DAC ≅ ∠ACB then



(i) Which additional information is needed to show that ΔADC and ΔCAB will be congruent by S – A – S test ?

(ii) Which additional information should be provided to get ΔADC and ΔCAB are congruent by S – A – A test.

Solution : (i) In ΔADC and ΔCBA ,

∠DAC ≅ ∠BCA ……. (given)

The two sides seg AC and seg AD are included by ∠DAC. Also the two sides seg CA and seg CB are Included by ∠BCA.

∴ seg AC ≅ seg CA ….. (Common side)

∴ The additional necessary information is seg AD ≅ seg BC to show that ΔADC and ΔCBA are congruent by S – A – S test.

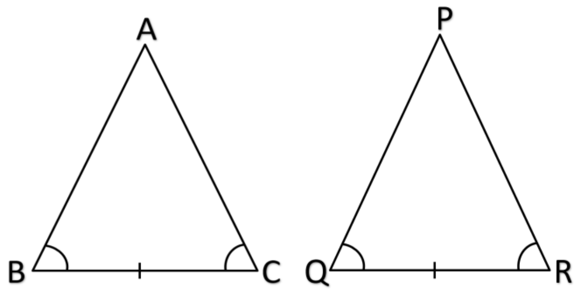
(ii) In ΔADC and ΔCBA,

∠DAC ≅ ∠BCA ……. (given)

seg AC ≅ seg CA ……. (common side)

In ΔADC , ∠ADC is the angle opposite to the side AC and in ΔCBA, ∠CBA is the angle opposite to the side CA should be congruent to show that ΔADC and ΔCBA are congruent by S – A – A test.

16. State one to one correspondence between the vertices of the triangles in which the two triangles are congruent by A – S – A test in the adjacent figure.



Solution : (i) In ΔABC and ΔPQR

∠B ≅ ∠Q, seg BC ≅ seg QC and ∠C ≅ ∠R

Therefore , ΔABC ≅ ΔPQR by A – S – A test in the correspondence of vertices ABC ↔ PQR

∴ ABC ↔ PQR

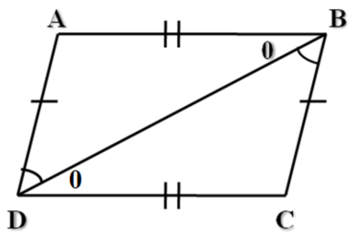
(ii) In ΔABC and ΔPQR

∠B ≅ ∠R, seg BC ≅ seg RQ and ∠C ≅ ∠Q

Therefore, ΔABC ≅ ΔPRQ by A – S – A test in the correspondence ABC ↔ PRQ of vertices.

∴ ABC ↔ PRQ

17. In the figure alongside, mention the tests by which triangles are congruent.



Solution : (i) In ΔADB and ΔCBD,

seg AD ≅ seg BC, seg DB ≅ seg BD

and seg AB ≅ seg CD

∴ ΔADB ≅ ΔCBD …… (S – S – S test)

(ii) In ΔADB and ΔCBD,

∠ADB ≅ ∠CBD …… (given)

seg DB ≅ seg BD ……. (common side)

∠ABD ≅ ∠CDB ……. (given)

∴ ΔADB ≅ ΔCBD …… (A – S – A test)

(iii) In ΔADB and ΔCBD,

seg AD ≅ seg BC …… (given)

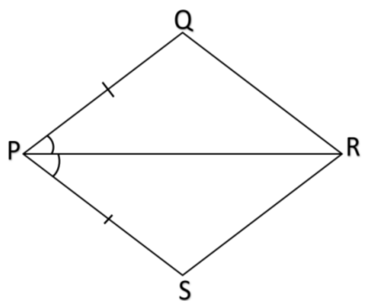
∠ADB ≅ ∠CBD ...... (given)

seg DB ≅ seg BD ……. (given)

∴ ΔADB ≅ ΔCBD ……. (S – A – S test)

18. In the adjacent figure,

seg PQ ≅ seg PS and seg PR is the bisector of ∠QPS then show ΔPRQ ≅ ΔPRS. Also state seg RQ and seg RS are congruent.



Solution : In ΔPRQ and ΔPRS,

seg PQ ≅ seg PS …… (given)

∠QPR = ∠SPR ……. (seg PR is the bisector of ∠QPS)

seg PR ≅ seg PR …….. (common side)

ΔPRQ ≅ ΔPRS ……. (S – A – S test)

∴ seg RQ ≅ seg RS ……. (corresponding sides of congruent triangles)

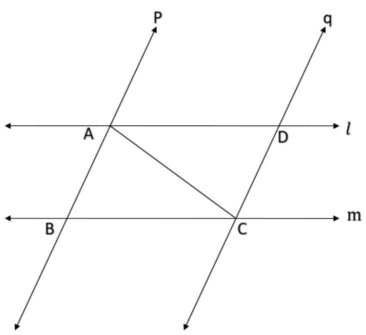
∴ seg PQ and seg RS are congruent.

19. In the figure alongside ,

line p ∥ line q and line *l* ∥ line m.

Line p, line q intersects line *l* and line m. To show that

ΔABC ≅ ΔCDA.



Solution : In ΔABC and Δ CDA,

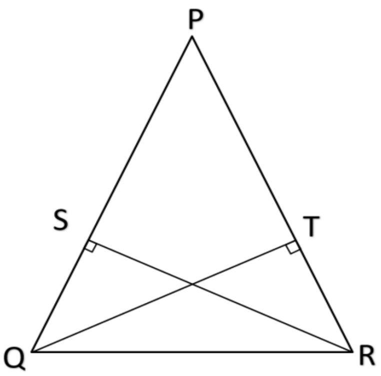
$\angle BAC = \angle DCA$ …… (interior alternate angles made by the line p and line q)

seg AC $\cong$ seg CA …… (common side)

$\angle BAC = \angle DAC$ …….. (interior alternate angles made by the line l and line m)

$\therefore \Delta ABC \cong \Delta CDA$ ……. (A – S – A test)

20. In the given figure, seg PQ $\cong$ seg PR and $\angle QSR = \angle RTQ = 90^0$ then show seg PS $\cong$ seg PT.



Solution : In ΔQTP and ΔRSP,

seg PQ $\cong$ seg PR …… (given)

$\angle QTP = \angle RSP$ …… (each 90^0)

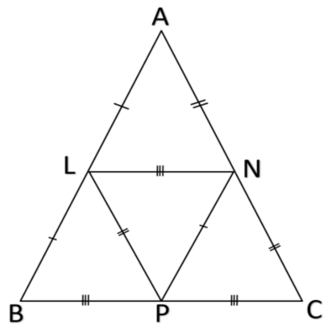
$\angle TPQ = \angle SPR$ ……. (common angle)

$\therefore \Delta QTP \cong \Delta RSP$ …… (S – A – A test)

$\therefore$ seg PT $\cong$ seg PS ……. (corresponding sides of congruent triangles)

$\therefore$ seg PS $\cong$ seg PT

21. In the adjacent figure, write the names of each pair are congruent by S – S – S test.



Solution : (i) In ΔALN and ΔLBP,

seg AL ≅ seg LB , seg LN ≅ seg BP

and seg AN ≅ seg LP.

∴ ΔALN ≅ ΔLBP ……. (S – S – S test)

(ii) In ΔALN and ΔNPC ,

seg AL ≅ seg NP , seg AN ≅ seg NC

and seg LN ≅ seg PC

∴ ΔALN ≅ ΔNPC ……. (S – S – S test)

(iii) In ΔALN and ΔLPN,

seg AL ≅ seg PN, seg AN ≅ seg LP

and seg LN ≅ seg LN

∴ ΔALN ≅ ΔLPN ……. (S – S – S test)

From (i) , (ii) , (iii) , ΔALN , ΔLBP , ΔNPC and ΔLPN are congruent to each other. So 6 pairs of congruence's are formed such as :

(i) ΔALN ≅ ΔLBP	**(ii) ΔALN ≅ ΔNPC**
(iii) ΔALN ≅ ΔLPN	**(iv) ΔLBP ≅ ΔNPC**
(v) ΔLBP ≅ ΔLPN	**(vi) ΔNPC ≅ ΔLPN**

22. One to one correspondence ABC ↔ XYZ of vertices of the two triangles are congruent. Write the all adjacent congruent parts.

Ans : ΔABC and ΔXYZ are congruent in the correspondence ABC ↔ XYZ, we get three pairs of adjacent congruent sides and three pairs of adjacent angle :

(i) Pairs of adjacent congruent sides :

Side AB ≅ Side XY

Side BC ≅ Side YZ

Side AC ≅ Side XZ

(ii) Pairs of adjacent congruent angles :

∠A ≅ ∠X

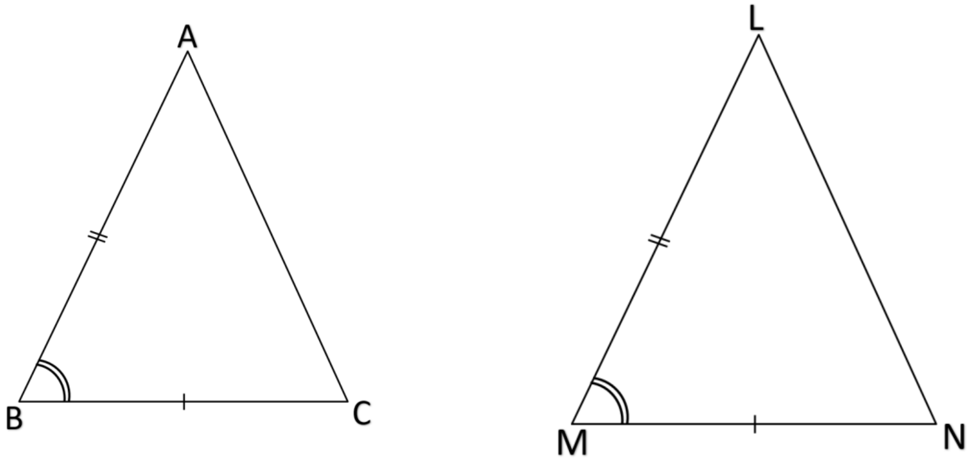
∠B ≅ ∠Y

∠C ≅ ∠Z

23. Explain S – A – S test with diagram and one to one correspondence between the vertices.

Ans : S – A – S test : If two sides and the included angle of a triangle are congruent with two corresponding sides and the included angle of the other triangle then the triangles are congruent with each other.

Explanation :



In ΔABC and ΔLMN ,

In the correspondence ABC ↔ LMN ,

seg BC ≅ seg MN i. e. l(BC) = l(MN)

∠B ≅ ∠M i. e. m∠B = m∠M

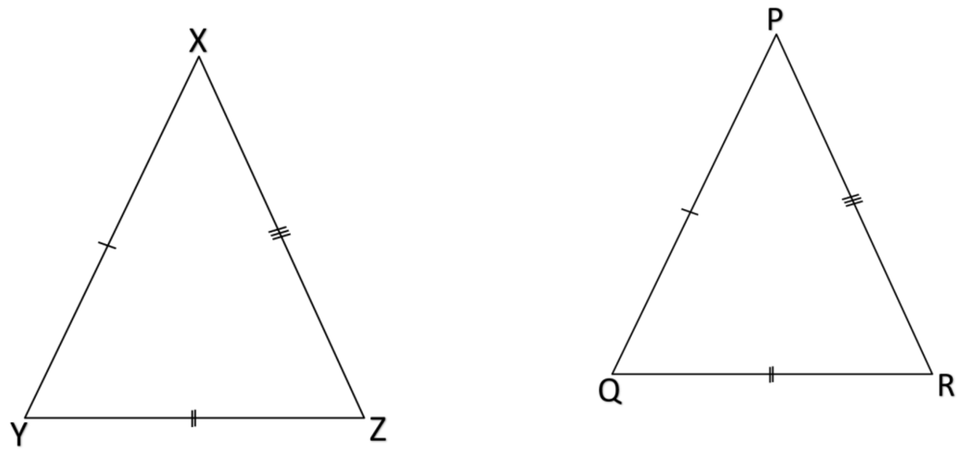
seg AB ≅ seg LM i. e. l(AB) = l(LM)

∴ ΔABC ≅ ΔLMN …….. (S – A – S test)

24. Explain S – S – S test with diagram and one to one correspondence between the vertices.

Ans : S – S – S test : If three sides of a triangle are congruent with three corresponding sides of the other triangle , then the two triangles are congruent.

Explanation :



In ΔXYZ and ΔPQR ,

In the correspondence XYZ ↔ PQR ,

seg XY ≅ seg PQ i . e. l (XY) = l(PQ)

seg YZ ≅ seg QR i . e. l (YZ) = l(QR)

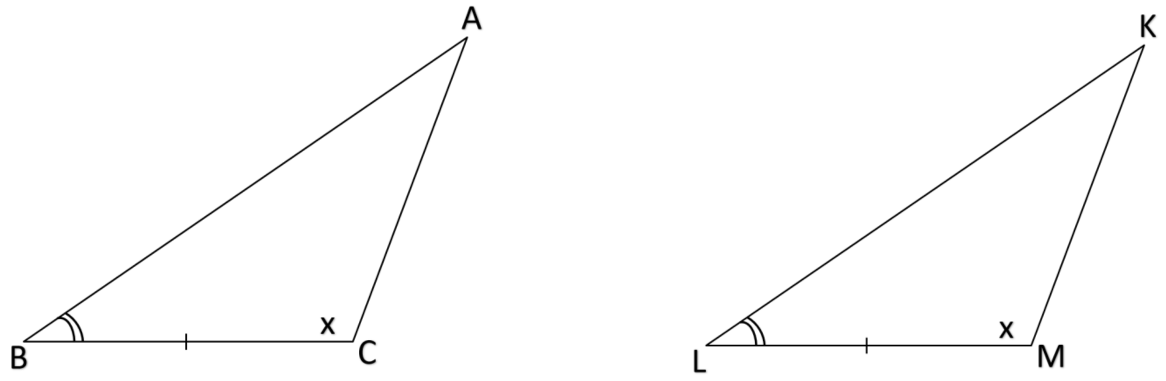
seg XZ ≅ seg PR i . e. l (XZ) = l(PR)

∴ ΔXYZ ≅ ΔPQR ……(S – S – S test)

25. Explain A – S – A test with diagram and one to one correspondence of vertices.

Ans : A – S – A test : If two angles of a triangle and a side included by them are congruent with two corresponding angles and the side included by them of the other triangle, then the triangles are congruent with each other.

Explanation :



In ΔABC and ΔKLM ,

In the correspondence ABC ↔ KLM

$\angle B \cong \angle L$ i. e. $m\angle B = m\angle L$

seg BC ≅ seg LM i . e. $l(BC) = l(LM)$

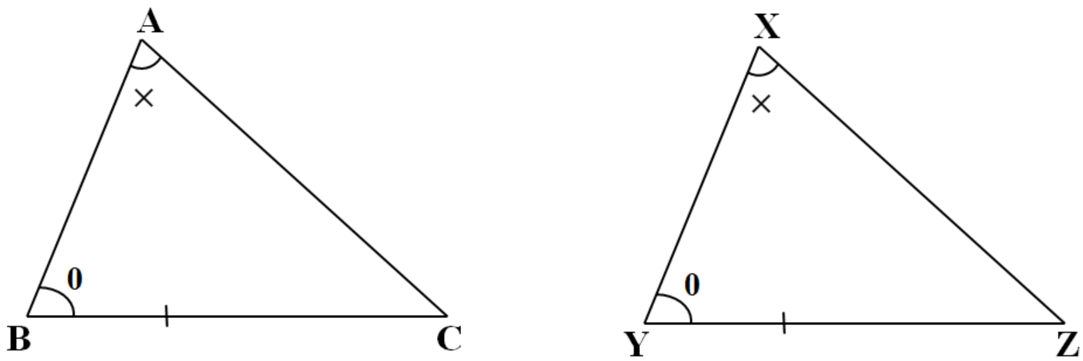
$\angle C \cong \angle M$ i. e. $m\angle C = m\angle M$

∴ ΔABC ≅ ΔKLM ……. (A – S – A test)

26. Explain A – A – S test with diagram and one to one correspondence of vertices.

Ans : A – A – S test : If two angles of a triangle and a side not included by them are congruent with corresponding angles and a corresponding side not included by them of the other triangle then the triangles are congruent with each other.

Explanation :



In ΔABC and ΔXYZ,

In the correspondence ABC ↔ XYZ

∠A ≅ ∠X i. e. $m\angle A = m\angle X$

∠B ≅ ∠Y i. e. $m\angle B = m\angle Y$

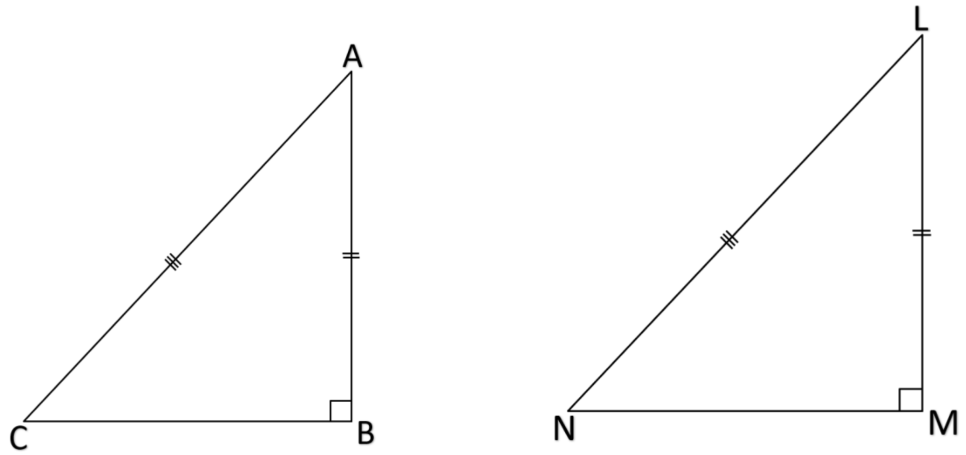
seg BC ≅ seg YZ i . e. l (BC) = l(YZ)

∴ ΔABC ≅ ΔXYZ ……. (A – A – S test)

27. Explain hypotenuse - side test with diagram.

Ans : Hypotenuse - side test : If the hypotenuse and a side of a right angled triangle are congruent with the hypotenuse and the corresponding side of the other right angled triangle, then the two triangles are congruent with each other.

Explanation :



In right angled ΔABC and ΔLMN.

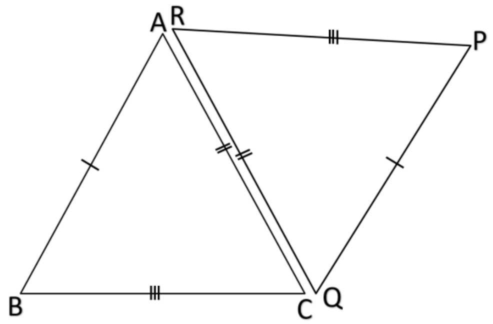
seg AB ≅ seg LM i . e. l (AB) = l(LM)

hypt AC ≅ hypt LN i . e. l (AC) = l(LN)

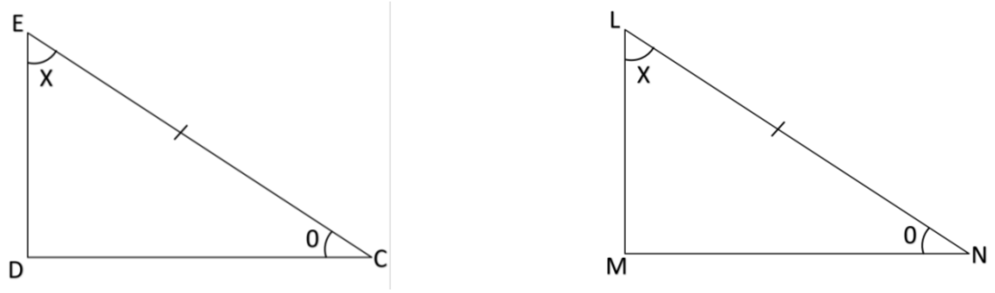
∴ ΔABC ≅ ΔLMN ……. (Hypotenuse - side test)

28. In the given figures parts of triangles bearing identical marks are congruent. State the test and one to one correspondence of vertices by which the triangles in each pair are congruent.

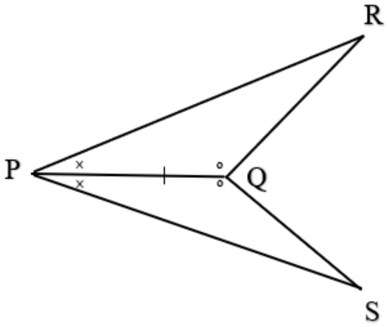
(i)



(ii)



(iii)



Ans : (i) The given triangles are congruent by S – S – S test in the correspondence ABC ↔ PQR.

(ii) The given triangles are congruent by A – S – A test in the correspondence LNM ↔ ECD.

(iii) The given triangles are congruent by A – S – A test in the correspondence PQR ↔ PQS.

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