3. Indices and cube root

1. Express the following numbers in index form.

(i) Cube of square root of 15

Ans : $(15)^{\frac{3}{2}}$

(ii) Square root of 64

Ans : $(64)^{\frac{1}{2}}$

(iii) Sixth root of 729

Ans : $(729)^{\frac{1}{6}}$

(iv) Eighth root of 256

Ans : $(256)^{\frac{1}{8}}$

(v) Fifth root of 50

Ans : $(50)^{\frac{1}{5}}$

(vi) Ninth root of 81

Ans : $(81)^{\frac{1}{9}}$

2. Write in the form ‘ n^{th} root of a ’ in each of the following numbers.

(i) $(144)^{\frac{1}{12}}$

Ans : 12 th root of 144

(ii) $(5)^{\frac{1}{8}}$

Ans : Eighth root of 5

(iii) $(21)^{\frac{1}{5}}$

Ans : Fifth root of 21

(iv) $(128)^{\frac{1}{7}}$

Ans : Seventh root of 128

(v) $(225)^{\frac{1}{10}}$

Ans : Tenth root of 225

(vi) $(576)^{\frac{1}{3}}$

Ans : Cube root of 576

3. Write the following numbers in the form of rational indices.

(i) Cube root of square of 289

Ans : $\left[(289)^2\right]^{\frac{1}{3}} = (289)^{\frac{2}{3}}$

(ii) Square root of cube of 7

Ans : $\left(7^3\right)^{\frac{1}{2}} = (7)^{\frac{3}{2}}$

(iii) Cube of seventh root of 441

Ans : $\left[(441)^{\frac{1}{7}}\right]^3 = (441)^{\frac{3}{7}}$

(iv) Cube root of square of 196

Ans : $[(196)^2]^{\frac{1}{3}} = (196)^{\frac{2}{3}}$

(v) Ninth root of square of 16

Ans : $[(16)^2]^{\frac{1}{9}} = (16)^{\frac{2}{9}}$

(vi) Cube of fifth root of 400

Ans : $\left[(400)^{\frac{1}{5}}\right]^3 = (400)^{\frac{3}{5}}$

4. Complete the following table :

Sr. No.	Number	Power of the root	Root of the power
(1)	$(24)^{\frac{5}{7}}$		
(2)	$(169)^{\frac{4}{2}}$		
(3)	$(49)^{\frac{3}{10}}$		
(4)	$(800)^{\frac{2}{4}}$		
(5)	$(361)^{\frac{5}{3}}$		
(6)	$(216)^{\frac{3}{3}}$		
(7)	$(55)^{\frac{12}{3}}$		
(8)	$(625)^{\frac{4}{15}}$		

Sr. No.	Number	Power of the root	Root of the power
(1)	$(24)^{\frac{5}{7}}$	Fifth power of seventh root of 24	Seventh root of fifth power of 24
(2)	$(169)^{\frac{4}{2}}$	Fourth power of square root of 169	Square root of fourth power of 169
(3)	$(49)^{\frac{3}{10}}$	Cube of tenth root of 49	Tenth root of cube of 49
(4)	$(800)^{\frac{2}{4}}$	Square of fourth root of 800	Fourth root of square of 800
(5)	$(361)^{\frac{5}{3}}$	Fifth power of cube root of 361	Cube root of fifth power of 361
(6)	$(216)^{\frac{3}{3}}$	Third power of cube root of 216	Cube root of third power of 361
(7)	$(55)^{\frac{12}{3}}$	12^{th} power of cube root of 55	Cube root of 12^{th} power of 55
(8)	$(625)^{\frac{4}{15}}$	Fourth power of 15^{h} root of 625	15^{th} root of fourth power of 625

5. Find the cube roots of the following numbers.

(i) $\sqrt[3]{4096}$

Ans : $\sqrt[3]{4096}$

$$4096 = 2 \times 2048$$

$$= 2 \times 2 \times 1024$$

$$= 2 \times 2 \times 2 \times 512$$

$= 2 \times 2 \times 2 \times 2 \times 256$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 128$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 64$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 32$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 16$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 8$

$= \underline{2 \times 2 \times 2 \times 2} \times \underline{2 \times 2 \times 2 \times 2} \times \underline{2 \times 2 \times 4}$

$= 16 \times 16 \times 16$

$= (16)^3$

$\therefore \sqrt[3]{4096} = 16$

(ii) $\sqrt[3]{2197}$

Ans : $\sqrt[3]{2197}$

$2197 = 13 \times 169$

$= 13 \times 13 \times 13$

$= (13)^3$

$\therefore \sqrt[3]{2197} = 13$

(iii) $\sqrt[3]{85184}$

Ans : $\sqrt[3]{85184}$

$$85184 = 2 \times 42592$$

$$= 2 \times 2 \times 21296$$

$$= 2 \times 2 \times 2 \times 10648$$

$$= 2 \times 2 \times 2 \times 2 \times 5324$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2662$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1331$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 121$$

$$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 11 \times 11 \times 11$$

$$= (4 \times 4 \times 4) \times (11 \times 11 \times 11)$$

$$= (4)^3 \times (11)^3$$

$$= (4 \times 11)^3$$

$$= (44)^3$$

$$\therefore \sqrt[3]{85184} = 44$$

(iv) $\sqrt[3]{3375}$

Ans : $\sqrt[3]{3375}$

$$3375 = 5 \times 675$$

$$= 5 \times 5 \times 135$$

$$= 5 \times 5 \times 5 \times 27$$

$= 5 \times 5 \times 5 \times 3 \times 9$

$= \underline{5 \times 5 \times 5} \times \underline{3 \times 3 \times 3}$

$= (5)^3 \times (3)^3$

$3375 = (5 \times 3)^3$

$= (15)^3$

$\therefore \sqrt[3]{3375} = 15$

(v) $\sqrt[3]{-5832}$

Ans : $\sqrt[3]{-5832}$

First find out the factors of 5832.

$5832 = 2 \times 2916$

$= 2 \times 2 \times 1458$

$= 2 \times 2 \times 2 \times 729$

$= 2 \times 2 \times 2 \times 3 \times 243$

$= 2 \times 2 \times 2 \times 3 \times 3 \times 81$

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 27$

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 9$

$= \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= (2)^3 \times (3)^3 \times (3)^3$

$= (2 \times 3 \times 3)^3$

$= (18)^3$

$-5832 = (-18)^3$

$\therefore \sqrt[3]{5832} = -18$

(vi) $\sqrt[3]{\frac{125}{343}}$

Ans : $\sqrt[3]{\frac{125}{343}}$

$$\frac{125}{343} = \frac{5 \times 5 \times 5}{7 \times 7 \times 7}$$

$$= \frac{(5)^3}{(7)^3}$$

$$= \left(\frac{5}{7}\right)^3$$

$$\therefore \sqrt[3]{\frac{125}{343}} = \frac{5}{7}$$

(vii) $\sqrt[3]{0.006859}$

Ans : $\sqrt[3]{0.006859}$

0.006859

First find out the factors of 6859 .

$6859 = 19 \times 361$

$= 19 \times 19 \times 19$

$6859 = 19^3$

$\therefore 0.006859 = (0.19)^3$

$\therefore \sqrt[3]{0.006859} = 0.19$

(viii) $\sqrt[3]{12167}$

Ans : $\sqrt[3]{12167}$

$12167 = 23 \times 529$

$= 23 \times 23 \times 23$

$= 23^3$

$\therefore \sqrt[3]{12167} = 23$

(ix) $\sqrt[3]{27000}$

Ans : $27000 = 3 \times 9000$

$= 3 \times 3 \times 3000$

$= 3 \times 3 \times 3 \times 1000$

$= \underline{3 \times 3 \times 3} \times \underline{10 \times 10 \times 10}$

$= (3)^3 \times (10)^3$

$= (3 \times 10)^3$

$= (30)^3$

$\therefore \sqrt[3]{27000} = 30$

(x) $\sqrt[3]{-9.261}$

Ans : $\sqrt[3]{-9.261}$

-9.261

First find out the factors of 9261 .

$9261 = 3 \times 3087$

$= 3 \times 3 \times 1029$

$= 3 \times 3 \times 3 \times 343$

$= 3 \times 3 \times 3 \times 7 \times 49$

$= \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

$= (3)^3 \times (7)^3$

$= (3 \times 7)^3$

$= (21)^3$

$9261 = 21 \times 21 \times 21$

$\therefore \; 9.261 = 2.1 \times 2.1 \times 2.1$

$\therefore$ $-9.261 = -2.1 \times -2.1 \times -2.1$

$\therefore \sqrt[3]{-9.261} = -2.1$

6. Find the cube root.

(i) $\sqrt[3]{\frac{54}{250}}$

Ans : $\sqrt[3]{\frac{54}{250}}$

$= \sqrt[3]{\frac{2 \times 27}{2 \times 125}}$

$= \sqrt[3]{\frac{27}{125}}$

$= \frac{\sqrt[3]{27}}{\sqrt[3]{125}}$

$= \frac{\sqrt[3]{(3)^3}}{\sqrt[3]{(5)^3}}$

$= \frac{3}{5}$

$\therefore \sqrt[3]{\frac{54}{250}} = \frac{3}{5}$

(ii) $\sqrt[3]{\frac{256}{108}}$

Ans : $\sqrt[3]{\frac{256}{108}}$

$= \sqrt[3]{\frac{4 \times 64}{4 \times 27}}$

$= \sqrt[3]{\frac{64}{27}}$

$= \sqrt[3]{\frac{(4)^3}{(3)^3}}$

$= \frac{4}{3}$

$\therefore \sqrt[3]{\frac{256}{108}} = \frac{4}{3}$

(iii) $\sqrt[3]{\frac{343}{729}}$

Ans : $\sqrt[3]{\frac{343}{729}}$

$= \sqrt[3]{\frac{7 \times 7 \times 7}{9 \times 9 \times 9}}$

$= \sqrt[3]{\frac{(7)^3}{(9)^3}}$

$= \frac{7}{9}$

$$\therefore \sqrt[3]{\frac{343}{729}} = \frac{7}{9}$$

(iv) $\sqrt[3]{\frac{375}{3000}}$

Ans : $\sqrt[3]{\frac{375}{3000}}$

$$= \sqrt[3]{\frac{3 \times 125}{3 \times 1000}}$$

$$= \sqrt[3]{\frac{125}{1000}}$$

$$= \sqrt[3]{\frac{(5)^3}{(10)^3}}$$

$$= \frac{5}{10}$$

$$\therefore \sqrt[3]{\frac{375}{3000}} = \frac{5}{10}$$

7. If $\sqrt[3]{2744} = 14$ then $\sqrt[3]{2.744}$ ……. ?

Ans : $\sqrt[3]{2.744}$

$$= \sqrt[3]{\frac{2744}{1000}}$$

$$= \sqrt[3]{\frac{14 \times 14 \times 14}{10 \times 10 \times 10}}$$

$$= \sqrt[3]{\frac{14^3}{10^3}}$$

$$= \frac{14}{10}$$

$$= 1.4$$

$$\therefore \sqrt[3]{2.744} = 1.4$$

8. If $\sqrt[3]{3375} = 15$ then $\sqrt[3]{0.003375}$ …….?

Ans : $\sqrt[3]{0.003375}$

$$= \sqrt[3]{\frac{3375}{1000000}}$$

$$= \sqrt[3]{\frac{(15)^3}{(100)^3}}$$

$$= \frac{15}{100}$$

$$= 0.15$$

$$\therefore \sqrt[3]{0.003375} = 0.15$$

9. If $\sqrt[3]{216} = 6$ then $\sqrt[3]{0.000216}$ ……. ?

Ans : $\sqrt[3]{0.000216}$

$$= \sqrt[3]{\frac{216}{1000000}}$$

$$= \sqrt[3]{\frac{(6)^3}{(100)^3}}$$

$$= \frac{6}{100}$$

$$= 0.06$$

$$= \sqrt[3]{0.000216} = 0.06$$

10. Simplify.

(i) $x = (-1000)^{\frac{1}{3}}$ then find the value of x ?

Ans : $x = (-1000)^{\frac{1}{3}}$

$$= [(-1) \times (1000)]^{\frac{1}{3}}$$

$$= (-1)^{\frac{1}{3}} \times (1000)^{\frac{1}{3}}$$

$$= (-1)^{\frac{1}{3}} \times (10 \times 10 \times 10)^{\frac{1}{3}}$$

$$= (-1) \times (10)^{\frac{3}{3}}$$

$= (-1) \times (10)$

$= -10$

(ii) $\frac{\sqrt[3]{1331} + \sqrt[3]{2197}}{\sqrt{36}} = ?$

Ans :

$\frac{\sqrt[3]{1331} + \sqrt[3]{2197}}{\sqrt{36}}$

Here , $1331 = 11 \times 11 \times 11 = (11)^3$

$2197 = 13 \times 13 \times 13 = (13)^3$

$36 = 6 \times 6 = (6)^2$

$\therefore \frac{\sqrt[3]{(11)^3} + \sqrt[3]{(13)^3}}{\sqrt{(6)^2}}$

$= \frac{11 + 13}{6}$

$= \frac{24}{6}$

$= 4$

$\therefore \frac{\sqrt[3]{1331} + \sqrt[3]{2197}}{\sqrt{36}} = 4$

(iii) $\dfrac{[(6)^3 + (7)^3 + (8)^3] - [(3)^3 + (4)^3 + (5)^3]}{\sqrt[3]{8} + \sqrt{49}} = ?$

Ans : $= \dfrac{[216 + 343 + 512] - [27 + 64 + 125]}{2 + 7}$

$= \dfrac{[1071] - [216]}{9}$

$= \dfrac{855}{9}$

$= 95$

$\therefore \dfrac{[(6)^3 + (7)^3 + (8)^3] - [(3)^3 + (4)^3 + (5)^3]}{\sqrt[3]{8} + \sqrt{49}} = 95$

(iv) $\dfrac{(8^2)^{\frac{1}{3}}}{(4^3)^{\frac{1}{2}}} = ?$

Ans : $8^2 = 64$

$\therefore (64)^{\frac{1}{3}} = 4$

$\therefore \left(8^2\right)^{\frac{1}{3}} = 4$ ..... (I)

$4^3 = 64$

$64^{\frac{1}{2}} = 8$

$\therefore \left(4^3\right)^{\frac{1}{2}} = 8$ ..... (II)

$\therefore \dfrac{\left(8^2\right)^{\frac{1}{3}}}{\left(4^3\right)^{\frac{1}{2}}} = \dfrac{4}{8}$ .....[From (I) and (II)]

$= \dfrac{1}{2}$

$\therefore \dfrac{\left(8^2\right)^{\frac{1}{3}}}{\left(4^3\right)^{\frac{1}{2}}} = \dfrac{1}{2}$

11. Find the cube root of – 10648 , fill in the following boxes with appropriate numbers and complete the activity.
To find the cube root of – 10648, let us factorise …… first.

$10648 = \square \times 1331$

$= \square \times 2 \times \square \times \square \times \square \times \square$

$= \square^{3} \times \square^{\square}$

$= \left(\square \times 11\right)^{\square}$

$10648 = \square^{3}$

$\sqrt[3]{10648} = \square$

$\therefore \sqrt[3]{-10648} = \square$

Ans :

First find out the factors of 10648 .

$10648 = \boxed{8} \times 1331$

$= \boxed{2} \times 2 \times \boxed{2} \times \boxed{11} \times \boxed{11} \times \boxed{11}$

$= \boxed{2}^{3} \times \boxed{11}^{\boxed{3}}$

$10648 = (\boxed{2} \times 11)^{\boxed{3}}$

$= \boxed{22}^{3}$

$\sqrt[3]{10648} = \boxed{22}$ $\quad\quad$ $\therefore \sqrt[3]{-10648} = \boxed{-22}$

12. Write the following statement true or false.

1) If a number is written 3 times in a multiplication then the product is called indices of that number.

Ans : False, if a number is written 3 times in a multiplication then the product is called the cube of that number.

2) The cube of a positive number is positive.

Ans : True

3) The cube of a negative number is positive.

Ans : False, the cube of a negative number is negative.

4) m $\frac{a}{b}$ means 'bth power of a^{th} root of m'

Ans : False , m $\frac{a}{b}$, means 'a^{th} power of bth root of m'

5) A positive number has only one square root.

Ans : False, A positive number has two square roots.

6) zero, is the square root of one number, itself.

Ans : True

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